

Application of Cooperative Game Theory in Waste Management

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Optimal values of parameters which play the major role in the implementation and financial sustainability of waste-to-energy plants technology can be obtained through the solution of mathematical programs corresponding to the minimization of total costs for the waste treatment. However, decisions based on the idea of total costs minimization can neglect the behavioural aspect of the problems and assume full cooperation between waste producers, which might not occur. The aim of this paper is to present possible approaches to modelling of cooperation between the waste producers in a certain location in the setting with limited or banned landfilling using the apparatus of cooperative game theory. The underlying idea is to justify and formalize the decision-making process leading to a cooperative reduction of the costs for non-recyclable solid waste treatment. In this paper, each application-oriented class of cooperative games is briefly described, and the games modelling interactions between waste producers in exemplary problems with different cooperation restrictions are defined. The Shapley value and its modifications are calculated for these games in order to present possible development of waste producers' costs in a case of cooperation and to demonstrate the distinction between considered classes. The main contribution is an introduction of the newly implemented cooperative game theory approaches enabling analysis and prediction of waste producers' behaviour. These approaches are suitable and prospective for the further research in the field of waste management and can be used in a prediction of the impact of WtE plants building on public finances and assessment of the waste treatment infrastructure sustainability.

1. Introduction

Nowadays, modern society faces numerous problems, and environmental degradation is one of the most discussed among them. Improving life circumstances altogether with rapid technological progress in recent years lead to a noticeable increase in waste production (Hoorweg and Bhada-Tata, 2012). Most of the developed countries have been forced to face this problem by proposing and adopting legislation that embeds an economic system known as the circular economy (Directive (EU) 2018/851). The circular economy aims to create close production system based on the minimization of involvement of new input factors and waste production and maximum reuse of goods and recycling of waste (Morgano et al., 2018). However, the problem with the sustainability of the circular economy concept occurs, since not all waste can be recycled. Due to this fact, building waste-to-energy (WtE) plants can be considered as a possible solution to this issue. For example, integration of WtE plants into an existing central district heating system has been proved to be a financially sustainable application of non-recyclable waste energy recovery technology (Janošťák et al., 2019).

The energy recovery approach to solid waste treatment can be rather efficient but complex. The complexity of its implementation is justified by the necessity of consideration of numerous factors impacting the financial sustainability of WtE plants. Main variables involved in the process of optimal decision-making related to WtE plants financial sustainability are locations of waste treatment facilities, their capacities and gate fees (Šomplák et al., 2013). In Kúdela et al. (2019) a multi-stage multi-period stochastic mixed-integer programming model is defined to minimize overall costs for modernization of waste processing infrastructure in order to achieve goals established by the government directive for solid waste treatment. Optimization approaches considered in the two previously cited works provide the possibility to determine the attractiveness of potential sites for the construction of new waste treatment facilities and present important contribution towards efficient waste management. However, obtained results are based on the assumption of full cooperation between waste producers which may not occur. Due to this fact, the behaviour of waste producers presented in the area has to be modelled in order to adjust and correct information obtained through total costs optimization and to define a more realistic outcome. None of the above-mentioned models states what impact such methods of solid waste treatment will have on waste producers' budgets and how costs for waste treatment will be distributed in a case of cooperation. However, there exists a branch of applied mathematics that deals with exactly this kind of issues.

Game theory, by its definition, focuses on mathematical models of complex interactions among rational participants (players) of the formalized conflict (game). Game theory has become an essential framework in the past years due to its ability to describe natural and logical development and anticipate possible outcomes of conflicts, in which decision-makers with different goals are involved and can affect each other (Myerson, 1991). Conflicts of interest in the problems of waste management are widespread. For example, waste treatment facilities pursue maximization of their income by setting optimal gate fees, which will attract waste producers with more distant locations. In some cases, establishing transportation subsidies altogether with slight gate fee increase can present an undeniable competitive advantage. In the problem considered in the article, each waste producer pursues minimization of costs related to waste treatment and a conflict between them is inevitable since capacities of the WtE plants are limited, and every waste producer prefers a more economical variant of waste treatment. Cooperative branch of game theory has proven itself as a powerful tool for establishing efficient cooperation strategies, even though it can meet certain obstacles such as time complexity, especially in large-scale games. It can be considered as a suitable approach for modelling waste producers' conflict since there are no legal barriers, that can prevent waste producers from at least partial cooperation.

The majority of works devoted to conflicts related to environmental studies are based on non-cooperative games and the solution concept of the Nash Equilibrium. In Gao et al. (2017), these concepts are applied to a design of the shale gas supply chain minimizing greenhouse gas emissions. This equilibrium concept has also been used for developing a strategy for phosphogypsum pollution reduction (Xu et al., 2019) and comparison of the efficiency of green loans and subsidies for clean production innovations (Li et al., 2018). Cooperative game theory has not been applied so widely compared to the non-cooperative branch. In Lejano et al. (2018), it has been used to study international negotiations about carbon emission reduction. In Liang et al. (2019), cooperative games have been defined for modelling of the renegotiation of environmental Public-Private Partnership (funding model for public infrastructure projects). Both mentioned works in a cooperative framework can be classified as canonical coalitional games, according to the classification proposed by Saad et al. (2009). The research gap in the application of the two other application-oriented classes of cooperative games, which are coalition formation games and coalitional graph games, can be identified.

The canonical coalitional games approach to waste producers' conflict has been presented firstly in Osička (2016). Except for this work, literature about the application of cooperative game theory for the considered problem is scarce. The main contribution and novelty of the paper is the implementation of all three above-mentioned application-oriented classes of cooperative games in the exemplary waste management problems and their comparison. The characteristic functions values and suitable modifications of the well-known Shapley value will be calculated in order to present the impact of settings with distinct cooperation restrictions on waste producers' costs and importance of communication in waste management problems. These results justify the application of newly implemented coalition formation games and coalitional graph games in waste producers' conflict and prove that they extend the scope of possible game theory application in waste management. Altogether all three considered classes enable future comprehensive analysis of the interaction between waste producers on real data. Also, they provide the ability to predict the possible impact of waste energy recovery possibilities on their budgets and, in perspective, on the financial sustainability of the waste treatment facilities.

2. Methods

In general, a cooperative or coalitional game is uniquely defined by pair (N, v) , where N is a set of players, and v is a coalition value function that assigns each coalition (binding agreement of players to act as a single entity)

$S \subseteq N$ it is worth in the game. In this section, the application-oriented classification of coalitional games will be briefly described, and the formal definition of the characteristic function describing waste producers' conflict will be presented. Definitions of theoretical concepts used in the section can be found in Myerson (1991).

2.1 Coalitional games classes

According to Saad et al. (2009), there are three distinct application-oriented classes of cooperative games: canonical coalitional games, coalitional graph games and coalition formation games. The canonical coalitional game has to be in the characteristic form, superadditive (subadditive for the cost game) and its main objectives are to study the possibility of forming the grand coalition $v(N)$ and fair and stable allocations of the value produced by a grand coalition between players. Coalition formation games focus on a study of the negotiation process leading to the formation of certain coalition structures (partitions of N) and on the description of the resulted structure properties. Coalition formation games can be divided into two basic types: static and dynamic. In this article, only the former type will be considered. Static games approach dwells in studying of the game (N, v, \mathcal{B}) , where \mathcal{B} is a coalition structure predefined by some external factor. In coalitional graph games, communication possibilities between players are presented by a graph associated with the cooperative game (Myerson, 1977). Due to this fact, these games are also called communication games. They represent another possible point of view on the individual expectation of players from the cooperation depending not only on his role in collaboration but also on his role in the communication process between the players.

2.2 Formal definition

According to Osička (2016), the general conflict of waste processors can be described as a TU-game in the characteristic function with $v(S)$ defined as follows. Let $M = \{1, \dots, m\}$ be a set of WtE plants; w_1^c, \dots, w_m^c denote their capacities and c_1^g, \dots, c_m^g denote their gate fees. The set of producers is $N = \{1, \dots, n\}$. Their waste productions are w_1^p, \dots, w_n^p . Transportation costs are given by the matrix $[c_{i,j}^t]$, where $c_{i,j}^t$ represents the cost of transportation from the producer $i \in N$ to the plant $j \in M$. The amount of waste sent by the producer $i \in N$ to the WtE plant $j \in M$ in tonnes is denoted by $x_{i,j}$. The characteristic function $v(S), S \subseteq N$ is defined by Eq(1)–Eq(8).

$$v(S) = \arg \min_{x_{i,j}: i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} (c_{i,j}^t + c_j^g) x_{i,j} \quad (1)$$

$$s. t. \sum_{i \in S} x_{i,j} \leq w_j^c - \sum_{i \in N \setminus S} x'_{i,j}, \forall j \in M, \quad (2)$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \forall i \in S, \quad (3)$$

$$x_{i,j} \geq 0, \forall i \in S, \forall j \in M, \quad (4)$$

$$\{x'_{i,j}: i \in N \setminus S, j \in M\} = \arg \min_{x_{i,j}: i \in N \setminus S, j \in M} \sum_{j \in M} \sum_{i \in N \setminus S} (c_{i,j}^t + c_j^g) x_{i,j} \quad (5)$$

$$s. t. \sum_{i \in N \setminus S} x_{i,j} \leq w_j^c, \forall j \in M, \quad (6)$$

$$\sum_{j \in M} x_{i,j} = w_i^p, \forall i \in N \setminus S, \quad (7)$$

$$x_{i,j} \geq 0, \forall i \in N \setminus S, \forall j \in M. \quad (8)$$

Waste treatment costs are presented as a sum of transportation costs and gate fees multiplied by an amount of treated waste. The main assumption of the whole model is that all produced waste can be treated by WtE plants (their capacities are sufficient). This value function corresponds to the minimum of the sum of total costs of participants of coalitions, that have made their decision right after the coalition of all outsiders has minimized its total costs. It presents worst-case scenario costs estimation in a setting with banned landfill and reflects general principles of the cooperation between decision-makers in waste producers' conflicts. Due to this fact, the above-defined characteristic function will present the foundation for application of all considered game classes.

3. Exemplary case study

Three exemplary problems are presented. A setting of each problem corresponds to a particular coalitional game class described in Section 2. Each problem has the same data for the sake of better presentation of cooperation restriction impact on a game outcome. Resulting costs are compared by means of the Shapley value (fair allocation rule) computed for each problem, and their difference is briefly discussed. Definitions of the Shapley value for each game class can be found in Myerson (1991). The data was invented for the purpose of the discussion that follows and does not reflect any real region.

3.1 A problem without cooperation restriction

The first exemplary problem is represented in Figure 1, where used notation fully corresponds to the previously given description. Canonical coalitional games approach is suitable in this case since no cooperation restrictions are assumed. From the practical point of view, such setting can be explained in the following way. In the case of full cooperation, waste producer 2 would willingly choose the more expensive services of the WtE plant 1 in order to reduce total costs by leaving free capacity of WtE plant 2 to waste producers 1 and 3. Increased expenses of waste producer 2 will be then compensated by waste producers 1 and 3 from the money they spared, because even with such compensation their costs will be less, than in a case with absence of cooperation. For the set $N = \{1,2,3\}$ of waste producers and characteristic function v presented in subsection 2.2, the canonical coalitional game of waste producers is defined as a pair (N, v) .

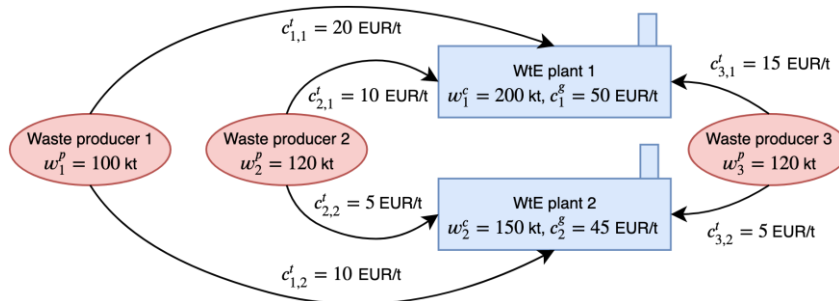


Figure 1: Exemplary problem without cooperation restrictions

3.2 A problem with cooperation restrictions imposing pre-defined coalition structure

The second exemplary problem is represented in Figure 2. The yellow shape presents a natural or legal barrier, which in a certain way, divides waste producers in the considered area.

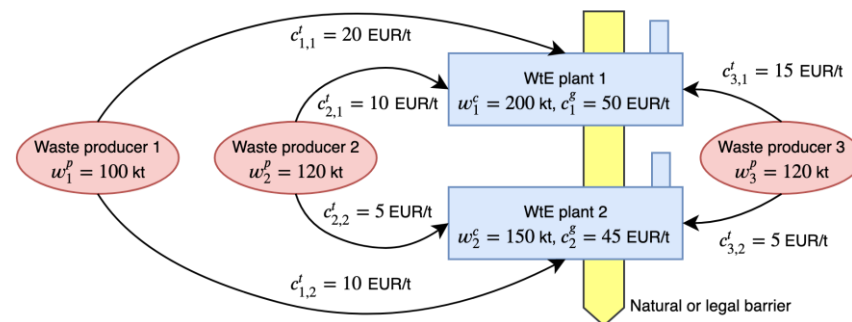


Figure 2: Exemplary problem with cooperation restrictions imposing pre-defined coalition structure

Figure 2 can illustrate a situation, in which exist anti-trust laws, that prohibit cooperation of two waste collection companies. One waste collection company works with waste producers 1 and 2, while another company works with waste producer 3. If cooperation between waste producers requires cooperation between their waste collection companies, existing competition law imposes a cooperation restriction, that can be presented by a coalition structure $\mathcal{B} = \{\{1,2\}, \{3\}\}$. The resulting waste producers' static coalition formation game (N, v, \mathcal{B}) can be viewed as a specific case of canonical coalitional game (N, v) from subsection 3.1 with the same value function and modified solution concepts, which are based on the relative efficiency with respect to \mathcal{B} .

3.3 A problem with cooperation restrictions presented by the communication network

The third exemplary problem is represented by Figure 3, where an underlying communication structure is presented by bidirectional red arrows. In such a setting, waste producers 1 and 3 can communicate only via waste producer 2. Such a setting can describe the following situation. Assume the waste producer 2 owns the only waste collection company in the area. Due to this fact, the waste producers 1 and 3, which do not have their waste collection infrastructure, cannot cooperate without permission of the waste producer 2.

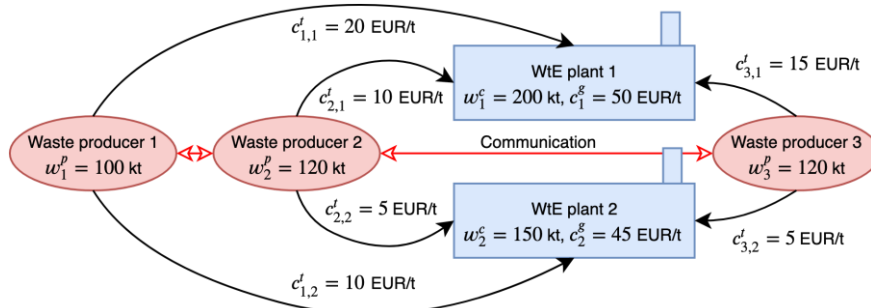


Figure 3: Exemplary problem with cooperation restrictions presented by the communication network

This communication structure can be represented as graph $g = (N, A)$, where $N = \{1, 2, 3\}$ is a set of nodes/waste producers and $A = \{\{1, 2\}, \{2, 3\}\}$ is a set of arcs between them obtained from Figure 3. The main objective is to study fair and stable cost allocations of the waste producers coalitional graph game (N, v, g) . This game has to be studied by the mean of graph-restricted game (N, v^g) , associated with the communication game (N, v, g) , in which coalition can obtain its original value only via communication between all of its participants.

3.4 Results and discussion

In Table 1, the values of characteristic functions for the case of the canonical coalitional game and the coalitional graph game. The coalition formation game corresponding to the second exemplary problem is omitted because the characteristic function is the same as in the problem with no cooperation restrictions and does not reflect imposed coalition structure. It will manifest itself later through the Shapley value. The main difference between problems is presented in the column corresponding to coalition $\{1, 3\}$. In coalitional graph game, this coalition cannot obtain its original value from the canonical game and $v^g(\{1, 3\}) = v(\{1\}) + v(\{3\})$.

Table 1: The characteristic functions values in MEUR

Game / Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
(N, v)	7	7.2	7.8	13.75	14.35	14.25	19.75
(N, v^g)	7	7.2	7.8	13.75	14.8	14.25	19.75

The Shapley values for each game are presented in Table 2. In the case of a game with pre-defined coalition structure, the Shapley value is computed separately for the subgames $(\{1, 2\}, v)$ and $(\{3\}, v)$ of (N, v) . In the coalitional graph game, waste producer 2 is in the best position comparing to other cases due to his important role in the communication network. In the end, it can be highlighted, that costs distributed on the basis of the Shapley value are at least the same as they were in a case of absence of cooperation for each player in every problem. This fact indicates the profitability of full cooperation.

Table 2: The Shapley values in MEUR

Game / Waste producer	1	2	3
(N, v)	6.35	6.4	7
(N, v, B)	6.775	6.975	7.8
(N, v^g)	6.425	6.25	7.075

4. Conclusion

In this article, the conflict between waste producers, which are trying to reduce their total costs for non-recyclable waste treatment through cooperation, has been studied by means of cooperative game theory. Three small-scale exemplary problems describing waste producers' conflicts with different cooperation restrictions have

been presented. The differences between problems settings have been illustrated in figures and described from a practical point of view. The games corresponding to each problem were mathematically formalized via means of application-oriented classes of coalitional games. The outcome of each game has been presented and compared by means of the Shapley value. Cooperation in each setting has proven itself as profitable. Results in Table 1 indicate that all considered games have proven themselves as subadditive, i.e. that players are able to reduce their costs through cooperation regardless of cooperation restrictions (or at least to stay at the level of the initial cost). According to Table 1, waste producers can spare 2.25 MEUR through cooperation. Table 2 shows the dependence of waste producers' cost on the producers' underlying communication structure and underlies the importance of game theory approaches, which take into account restricted cooperation in waste management. Future research in this area will be based on usage of the real data for a particular region and devoted to the study of the general profitability of waste producers' cooperation in real conditions. Besides the approaches described in the article, it will be achieved by the application of the concept of the Core and study of dynamic coalition formation games.

Acknowledgements

This work was supported by Grant No. GA 20-00091Y of the Czech Science Foundation. The support is gratefully acknowledged. The authors also acknowledge the financial support provided by the project Sustainable Process Integration Laboratory – SPIL, funded as project No. CZ.02.1.01/0.0/0.0/15_003/0000456, by Czech Republic Operational Programme Research, Development, and Education, Priority 1: Strengthening capacity for quality research.

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