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Hyperelastic Material Characterization: How the Change in Mooney-Rivlin Parameter Values Effect the Model Curve

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Abstract. Mooney-Rivlin is the most frequently used model from all models used for mechanical characterization of the hyperelestic materials. Simplicity, applicability in a large range of strains are the key reasons for regular use of this model. However, depending on the number of parameters, the Mooney model can take several forms. While, nine parameter being the highest order noticed, two parameter model is the most commonly found form in the current research domain.

Since two parameter model used repetitively, we investigated the effect of incremental change in two material constant values one at a time, on model curve. As Drucker stability criterion is governing the extreme values of material parameters, changes in the model curves are discussed related to it.

Resultant effects on stress-strain curves due to change in parameter values were examined and physical effect on the characterization is interpreted accordingly.

Introduction

Hyperelastic materials are mechanically characterized using one or the other material model selected from forty odd models available to date. As stated by Jadhav et al. [1] the selection depends on, application, related variables, and amount of available data.

Though there are many models, out of all these models, Mooney-Rivlin model is a frequently used in hyperelastic material research [2-5]. This is partially due to the simplicity and its applicability in a large range of strains [6].

When it comes to the discussion of this model, depending on the number of parameters, the model may take different forms and shapes. This is because, as observed by Kumar et al. [7], the general shape of the curve depends on the number of parameters the respective model form has. The Mooney- Rivlin model is a phenomenological model and material constants of the model are based on observation through physical response [8].

The stability of a particular model is important to forecast the behaviour of material, and is described by Drucker Stability Criterion [7]. This criterion could define limitations of material parameters. The criteabove rion could be applied to the Mooney model too. The objective of this work is to examine these limitations related to the Mooney two parameter model. Furthermore, results of the work could be used to interpret the relationship of parameters to the material physical properties.

Theory

Hyperelastic materials exhibit large elastic strains under relatively moderate loads. They are considered isotropic, incompressible materials. Stress-strain relationship of these materials shows a non-linear elastic characteristic and is generally independent of strain rate. Considering above mentioned assumptions, through stretch ratios λ_1, λ_2, λ_3, three strain invariants could be defined as

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (1)$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \quad (2)$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 \quad (3)$$

Because of incompressibility, for such materials, the third invariant I_3 is identically leads to unity. Stress-strain relationship of hyperelastic materials is normally found through strain energy density function. The strain energy density function is usually expressed in terms of strain invariants as follows.

$$W = W(\bar{I}_{1,}\bar{I}_{2,}J) = W(\bar{I}_{1,}\bar{I}_2) + U(J) \quad (4)$$

Respective second Piola Kirchhoff stress is given as

$$\bar{S} = \frac{\partial W}{\partial E}(\bar{I}_{1,}\bar{I}_2) \quad (5)$$

Mooney-Rivlin Model

When this is related to Mooney-Rivlin model, the generic form of it is written in terms of strain invariants as follows.

$$W = \sum_{i,j=0}^{N} C_{ij}(\bar{I}_1 - 3)^i(\bar{I}_2 - 3)^j + \sum_{i=1}^{N}\frac{1}{D_i}(J_{ei} - 1)^{2i} \quad (6)$$

Considering incompressibility, this could be further simplified.

$$W = \sum_{i,j=0}^{N} C_{ij}(\bar{I}_1 - 3)^i(\bar{I}_2 - 3)^j \quad (7)$$

Though, theoretically the model could be defined for N number of terms, maximum number of terms found in the research sphere is restricted to nine.

Due to the frequent use of two parameter model, we consider it appropriate to examine the model with two material constants. However, similar approach could be followed in examining the other model forms in further research in this direction.

The two parameter Mooney-Rivlin model could be written as indicated in equation 8.

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) \quad (8)$$

Through this equation, the stress-strain relationships for three deformation modes could be obtained. Hence, uniaxial deformation can be written as in equation 9.

$$\sigma = 2(C_{10} + \lambda^{-1} * C_{01})(\lambda - \lambda^2) \quad (9)$$

Biaxial and Pure shear relationships are given by equations 10 and 11 respectively.

$$\sigma = 2(C_{10} + \lambda^2 * C_{01})(\lambda - \lambda^{-5}) \quad (10)$$

$$\sigma = 2(C_{10} + C_{01})(\lambda - \lambda^{-3}) \quad (11)$$

The Drucker stability criterion can be stated as follows.

$$\frac{\partial \sigma_{ij}}{\partial \varepsilon_{ij}} \geq 0 \text{ and } \partial\sigma_{ij}\partial\varepsilon_{ij} \geq 0 \quad (12)$$

Referring Mooney-Rivlin two parameter model, relationship between material constants could be stated as

$$C_{10} + C_{01} \geq 0 \text{ and } C_{01} \geq 0 \tag{13}$$

Furthermore, C_{10} and C_{01} elastic coefficients can be given as

$$G = 2(C_{10} + C_{01}) \tag{14}$$

Where, G is small-strain shear modulus or modules of rigidity.

$$G = \rho RT / M_c \tag{15}$$

Furthermore, shear modules could be linked with crosslinking density through equation 15, given here. In the equation, R is the universal gas constant, ρ is the density of material and T is the temperature, M_C corresponding chain molecular wt.

Materials and Methods

Initial material constant comparisons were done using readily available data. However, set of experiments are planned in order to get our own sets of data which could be subsequently used for similar comparisons. For the time being, as it is often used for hyperelastic material characterization, we also used Treloar data for comparison. Therefore, material used for Treloar tests are valid here as well. According to sources [9], unfilled natural rubber (8% Sulphur) had been originally used for these tests.

Results and Discussion

Data fitting of two Treloar data sets, uniaxial and biaxial combined in to Mooney-Rivlin two parameter model, gives two values 0.1788 and 0.0037 for C10 and C01 respectively. The objective of this work was to oscillate the parameters intentionally around these figures and observe the resultant effect on model curves. Changes to the parameters were done in both plus and minus directions. One parameter was fixed at correct value given above while changes were done to the other. Selected combinations of material constants are given in tabulated form below.

Table 1. Constant value combinations with fixed C10

	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**	**Trial 6**
C10	0.1788	0.1788	0.1788	0.1788	0.1788	0.1788
C01	-0.003	-0.002	-0.001	0.001	0.002	0.003

Table 2. Constant value combinations with fixed C01

	Trial 7	**Trial 8**	**Trial 9**	**Trial 10**	**Trial 11**	**Trial 12**
C10	-0.3	-0.2	-0.1	0.1	0.2	0.3
C01	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037

Resultant graphs for this analysis are given in figure 1 to 6. From them, figures 1 to 3 depict the effect due to the variation of material constant C01 with C10 kept constant, while 4-6 give the results of variation of C10 with C01 constant.

From the first set of graphs (Fig. 1-3), at first glance one can see that, all uniaxial curves are as not responding to the change in parameter values. Furthermore, all curves seems somewhat deviating from the actual data set with the progression of strains. Considering this result, it is possible to mention that small changes to the parameter C01 in both plus and minus directions as not considerably affecting the overall results. However, when consider biaxial curves, picture is somewhat different. In this case, as it shows, minus values for the material constant C01 are certainly creating an unstable set of curves. Though curves are unstable at the negative side, with constant reaching towards plus direction, the curve seems moving to the stability. When material constant is increased, at certain point, the graph matches actual data values. This is the point where the value of C01 equate the actual value obtained through data fitting. Further increase of material constant make the curve moving

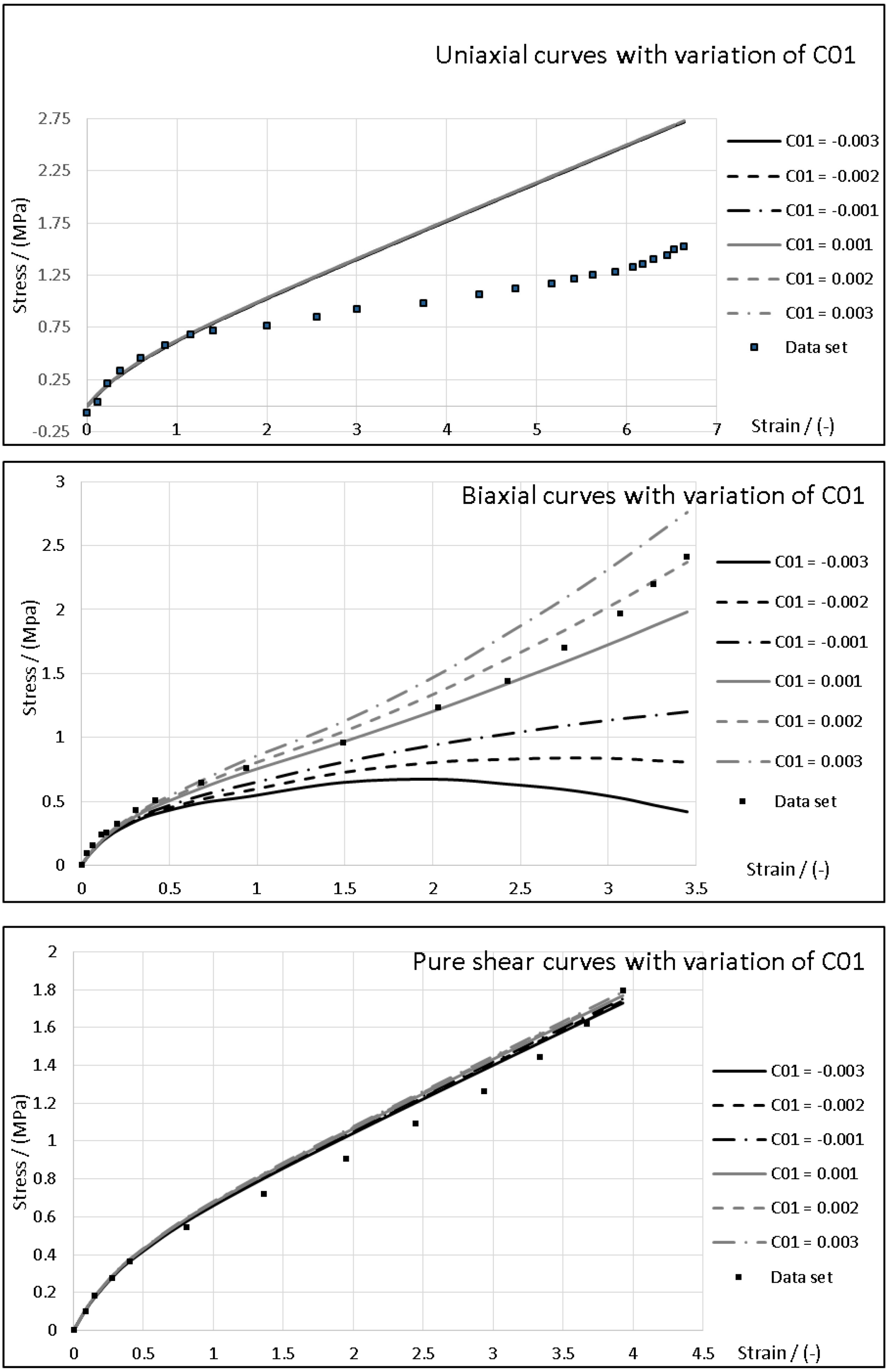

Fig. 1 - Fig. 3: Model curves related of variation in parameter C01

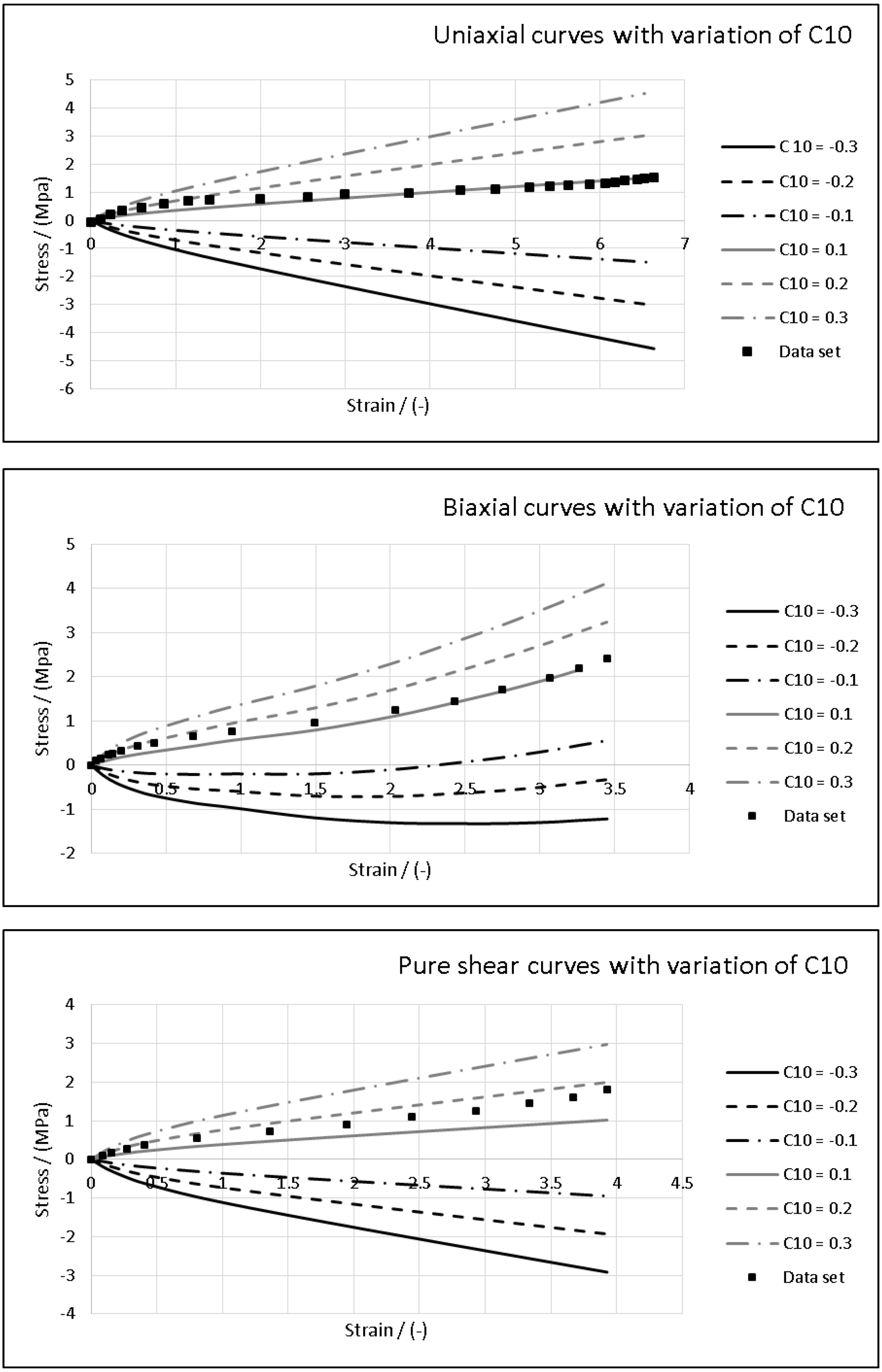

Fig. 4- Fig. 6: Model curves related of variation in parameter C10

away from actual data set at a rapid rate. Finally, in this set of curves, pure shear curves once again remains unmoved by the variation in material constant value.

Drucker criteria is a method used to outline the limitations of parameter values. As material parameters varied, there is a possibility of exceeding these limits and thereby changing the stability of the material. Altogether, if we sum-up the Drucker criteria here, it says that C01 should be positive and at the same time, total of two constants should be above zero. Therefore, first condition seems more dominant in biaxial deformation mode. In this instance, if we are to stick to the second condition of the Drucker criteria, considering the value of C10, second constant C01 should not be less than -0.1788.

The remaining set of graphs which are given in figure 4 to 6 shows characteristic curves for three deformation modes when C10 varied while C01 kept at a constant value. As it can be seen from these graphs, all of them are showing unstable characteristics whenever C10 has negative values. However, biaxial curves in this group seems somewhat improving with larger strains. In general, all three deformation mode curves show similar results and a common picture for the whole range of values of C10.

When we compare deformation mode curves in this category with actual data, uniaxial and biaxial deformations mode curves seems similar and show nearest to the real data when C10 takes a value of 0.1. However, in pure shear mode, the value seems 0.2.

Summary

For the two parameter Mooney-Rivlin model, a comparison was done related to variation in material constants. Firstly, while keeping C10 second material constant was changed. The results of this comparison have shown some mixed curve variations.

Second comparison was done by changing the constant C10 relative to the C01. However, curves for all three modes in this case, give unstable results whenever the constant takes a negative value. Finally, from overall results of comparison, the Drucker criteria could be validated. At the same time, it could be highlighted that there are limitations for the material constant variation. Biaxial mode of deformation seems predominantly affect if violated these limitations.

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