EVOLUTIONARY TECHNIQUES AND ITS POSSIBILITY TO IDENTIFY CATASTROPHIC EVENTS

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Abstract. This paper introduces an overview of possible use of evolutionary techniques on catastrophic events detection. Catastrophic events here means Thom's catastrophes that are part of chaotic dynamics and can be used to model bifurcations, that appears in the nonlinear behavior of various dynamical systems. Participation summarize yet obtained results as well as demonstration of another possible EAs use to detect Thom's catastrophes.

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INTRODUCTION

Our world, mostly consisting of a nonlinear systems, is full of our i.e. human technology, that is less or more reliable. Technological systems are mostly, as their natural counterparts, nonlinear and complex and very often show chaotic as well as catastrophic behavior. The term "catastrophe" here means catastrophe according to Thom's catastrophe theory [1]-[3], that describe sudden changes in the system behavior under slightly changing (usually) external conditions. This changes, depending on one or more parameters, can be modeled like the special *N* dimensional surfaces in the so-called parameter space, see Figure 2 - 3.

As an example of such systems (and catastrophic events), can be mentioned systems like electrical networks (blackout, …), economical systems (black Friday, NY stock market 1929, …), weather systems (Lorenz model of weather born via series of bifurcations modeled by Thom's catastrophes, [1]), civil construction falling (bridge collapse, etc), complex systems (self-criticality and spontaneous system "reconstruction" leading to the better energetically stability) and more.

Different mathematical models, of which one possible is just mentioned Thom's catastrophe theory, model such events.

Our aim in this paper is to show, that it is possible to use evolutionary algorithms (EAs) to identify such events on mathematical models of such systems and/or it is possible to use EAs to design technological systems in such a way that possibility to reach regimes exhibiting sudden changes in their behavior (i.e. catastrophe events) is minimized.

CHAOS, CATASTROPHES, AND SUDDEN CHANGES

When hearing the word "chaos", people who are not specialists in this field of science will normally imagine a process, which is of purely random nature and lacks any internal rules. Few people realize that "being chaotic" means strictly obeying precise rules where there is often no room for randomness. As indicated in the historical outline, chaos is a discipline which obtained its name only in the 20th century but whose roots date back to the 18th and 19th centuries, associated with the finding that even simple problems generate very complex and unpredictable behavior. For historical reasons, Hamiltonian systems were the first systems to be studied, represented then by celestial mechanics problems. Many rules, which are valid for a wide class of Hamiltonian systems generating chaotic behavior, were discovered, and were even found to apply to some dissipative chaotic systems as well.

Chaos and Bifurcation

Chaos can be easily visualized via so called bifurcation diagrams that show system dependence on selected control parameters and their setting. As an example can be used Figure 1 with bifurcation

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diagram calculated from logistic equation (1). In this figure is depicted behavior of system with dependence on control parameter A. Figure 1 can be read as this: for parameter value say for example $A = 3.5$ there are possible 4 reachable states, i.e. system is oscillating with period 4, while for $A = 3.7$ there is an infinite number of points which indicate that system behavior – trajectory never repeat. That parts on bifurcation diagrams represent chaotic behavior, while those part with finite number of points (A = 3.5, 3.63, 3.82, …) deterministic one. Moment, when system is changing its behavior (from N periodical to $2\times N$, or to chaotic one) is called bifurcation. Route from deterministic behavior to chaos via series of bifurcations is one of routes leading to chaos. Increasing of from N periodical to $2\text{x}N$ periodical trajectory is also called period doubling, see [4].

Bifurcations and Catastrophes

Bifurcations just described in the previous section can be modeled by Thom's catastrophe theory. This theory [1]-[3], describe sudden changes in the system behavior under slightly changing (usually) external conditions. These changes, depending on one or more parameters, can be modeled like the special N dimensional surfaces in the so-called parameter space, see Fig. 2 and 3.

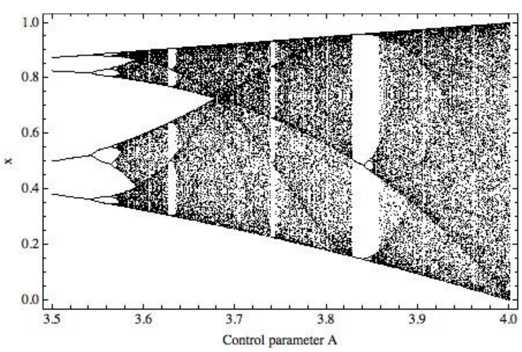

FIGURE 1. Bifurcation diagram of logistic equation.

Each point in parameter space represents one of possible system configurations. Those points which are part of so called catastrophic fold (surface, plane, …) are related to system parameter configuration, when system is changing its behavior (moment when bifurcation occur).

When system control parameter is changed then point in the parameter space move and moment when it cross through the catastrophic fold, then behavior of system is changed. This change can mean in reality changes in periodicity as well as switching to chaotic dynamic and/or also more drastic changes in system physical structure and behavior. Such changes then can lead, in reality, to real catastrophes like aircraft crashing, dam failure, collapse of building or power network, etc.

As demonstrated in [1] on Lorenz system (weather model), born of chaos can be understand like a way through the series of bifurcations-Thom's catastrophes. Mutual relation between Thom's catastrophes and bifurcations is thus clear.

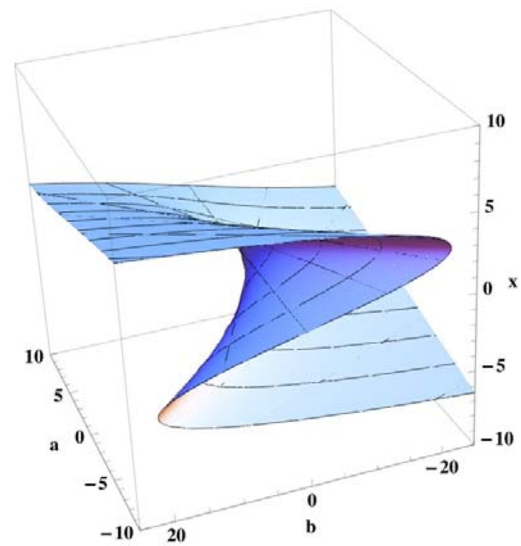

FIGURE 2. Catastrophe Fold.

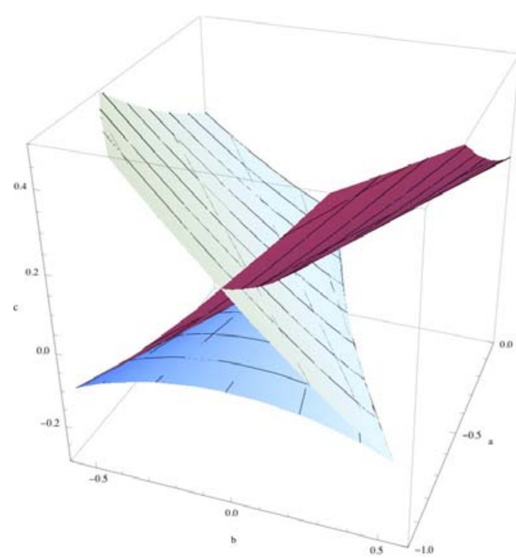

FIGURE 3. Catastrophe Fold.

MOTIVATION

Our aim and motivation is based on previously mentioned facts that between bifurcations and Thom's catastrophes is clear relation and to identify Thom's catastrophes in the dynamical system behavior can be (at least for a certain class of dynamical systems) identified so that bifurcations are located in the "universe" of the possible system behavior. It can also be done by rigorous mathematical analysis, however when the model is too complex or model of system is unknown and it is possible to play with system behavior without danger of technological disaster, then EAs can be used to explore possible system behavior and locate bifurcations i.e. equivalently Thom's catastrophe.

USED ALGORITHMS

For the experiments described here, stochastic optimisation algorithms, such as Differential Evolution (DE) [5], Self Organizing Migrating Algorithm (SOMA) [6] had been used.

Differential Evolution [5] is a population-based optimization method that works on real-number coded individuals. For each individual $\vec{x}_{i,G}$ in the current generation *G*, DE generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a third randomly selected individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as perturbated vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$, otherwise $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$.

Deferential Evolution is robust, fast, and effective with global optimization ability. It does not require that the objective function is differentiable, and it works with noisy, epistatic and time-dependent objective functions.

SOMA [6] is a stochastic optimization algorithm that is modelled on the social behaviour of cooperating individuals. It was chosen because it has been proven that the algorithm has the ability to converge towards the global optimum. SOMA works on a population of candidate solutions in loops called *migration loops*. The population is initialized randomly distributed over the search space at the beginning of the search. In each loop, the population is evaluated and the solution with the highest fitness becomes the leader *L*. Apart from the leader, in one migration loop, all individuals will traverse the input space in the direction of the leader. Mutation, the random perturbation of individuals, is an important operation for evolutionary strategies (ES). It ensures the diversity amongst the individuals and it also provides the means to restore lost information in a population. Mutation is different in SOMA compared with other ES strategies. SOMA uses a parameter called PRT to achieve perturbation. This parameter has the same effect for SOMA as mutation has for GA.

The novelty of this approach is that the PRT Vector is created before an individual starts its journey over the search space. The PRT Vector defines the final movement of an active individual in search space.

The randomly generated binary perturbation vector controls the allowed dimensions for an individual. If an element of the perturbation vector is set to zero, then the individual is not allowed to change its position in the corresponding dimension.

An individual will travel a certain distance (called the path length) towards the leader in *n* steps of defined length. If the path length is chosen to be greater than one, then the individual will overshoot the leader. This path is perturbed randomly.

For an exact description of the algorithms, see [5] for DE and [6] for SOMA

The control parameter settings have been found empirically and are given in Table 1 (SOMA), Table 2 (DE). The main criterion for this setting was to keep the same setting of parameters as much as possible for all simulations and of course the same number of cost function evaluations as well as population size (parameter PopSize for SOMA, NP for DE). Individual length represents number of optimised parameters (parameter *A* in this case…).

COST FUNCTION AND SELECTED CHAOTIC SYSTEM

Main idea of this participation is to identify bifurcation (or equivalently Thom's catastrophes, see [1]) by means of EAs. For this purpose has been selected logistic equation (simplified model of predator-prey system), see (1) and [4].

$$x_{n+1} = Ax_n(1 - x_n) \tag{1}$$

Bifurcation diagram [4] (i.e. system dependence on control parameter) of this system is depicted in Figure 1. Lyapunov exponents of that system, related to parameter *A*, are depicted in Figure 4. Simply: when Lyapunov exponent is negative, system has deterministic behavior, when is positive, system is chaotic. When Lyapunov exponent is = 0 then bifurcation can be observe in the system behavior (compare Figure 1 and 4). Then, to find that moments, we need to locate those parameter *A* for which Lyapunov exponent is = 0. It is enough to use absolute value and then we get Figure 5. Figure 5 represent cost function landscape, where values with 0 are settings of *A* for which bifurcation occur. One can see, that this surface is very erratic – chaotic and thus suitable candidate to find cost values with 0 are heuristics like evolutionary algorithms.

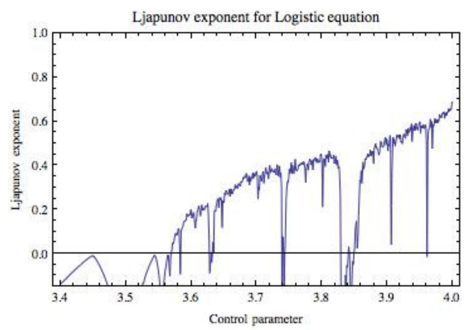

FIGURE 4. Dependence of Lyapunov exponent on parameter *A*, see (2).

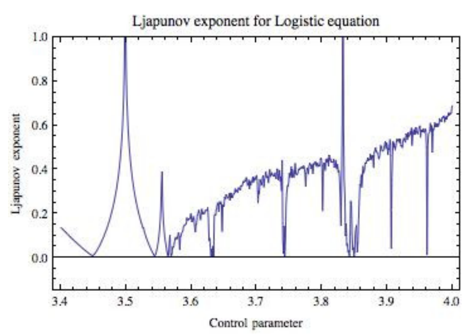

FIGURE 5. An absolute value of dependence of Lyapunov exponent on parameter *A*, see (3).

$$\lambda = \frac{\ln|f'(x_0)| + \ln|f'(x_1)| + \ldots + \ln|f'(x_n)|}{n} \quad (2)$$

$$CV = |(2)| \quad (3)$$

Cost function, used for depicting Figure 5, is given by (3) that is an absolute value of (2), see [4]. Graphically it is depicted in Figure 5. Used EAs were used to search for CV = 0.

EXPERIMENT DESIGN

Experiments has been deign as described in the Table 1 and Table 2. Two basic versions of algorithms SOMA and DE has been used: DERand1Bin and SOMA AllToOne. Each experiment was 50 times repeated and results are reported in the section Results.

All simulations have been done on special grid computer. This grid computer consists of 16 XServers, each 2x2 GHz Intel Xeon, 1 GB RAM, 80 GB HD i.e. 64 CPUs. It has been used for calculations so that each CPU was used like a single processor and thus a rich set of statistically repeated experiments were not a time problem. This does not means that such class of problems can be solved only on special computers. Solved problem reported here can also be done on single PC. Of course in different time scale.

TABLE 1. Soma setting.

PathLength	3
Step	0.11
PRT	0.1
PopSize	100
Migrations	200
MinDiv	-0.1
Individual Length	30

TABLE 2. DE setting

NP	100
F	0.9
CR	0.3
Generations	500

RESULTS

Results obtained in all experiments are reported in Table 3 (SOMA cost function evaluations), Table 4 (DE cost function evaluations) and Table 5 that show minimal, maximal and average cost values given by experimentation. Table 6 shows localized positions of bifurcations.

TABLE 3. SOMA Cost Function Evaluations

Maximum	29504
Average	8950
Minimum	1395

TABLE 4. DE Cost Function Evaluations

Maximum	34582
Average	11843
Minimum	2694

TABLE 5. Cost Function values

EA	Min	Average	Max
DE	5×10^{-15}	1.6×10^{-13}	1.5×10^{-7}
SOMA	4×10^{-14}	2.3×10^{-11}	7×10^{-3}

CONCLUSION

In this participation have been used two different evolutionary algorithms in order to localize possible bifurcations in the chaotic systems. SOMA and DE in basic versions were used on simple system (1), called logistic equation, to localize bifurcations. Simulations were 50x repeated, so in total there has been done 100 simulations.

Based on results reported in the previous section it can be stated, that:

1. **Simulations**. All simulations has been successfully finished by localization of bifurcation event, see Table 6.
2. **Precision**. Located bifurcations were localized with high precision, however not exactly, see Figure 6. Geometrically it can be interpreted so that system is at the edge of catastrophic model (e.g. Figure 2) or almost before crossing the surface (e.g. Figure 3). Our opinion is that such non-precise localization can be useful, when applied on real system and bifurcation can cause real damage, which means that evolutionary search for bifurcations on real system can be stopped very near to bifurcation "position".
3. **Algorithms**. Used evolutionary algorithms were SOMA [5] and DE [6]. It is clear that other EAs can be also used as well as another strategies of DE and SOMA algorithms. This is now in the process.
4. **Chaotic systems**. Beside well known logistic equation can another systems like Ikeda, Burger, Delayed Logistic, etc. be selected. Logistic equation (derived from predator prey system) has been chosen because is well known, and well investigated. Another systems (like those mentioned above) are now in the process of investigation.
5. **Simulation environment and software**. Software used for all calculations, numerical simulations and visualizations was well known *Mathematica* 8. Thanks to fact that *Mathematica* is an integrated environment, time scale for each simulation was in minutes. It is clear that when C or another fast programming languages would be used, then time scale of simulation should be much more shorter and thus real use of this approach on real systems is possible. As reported in the Section 7, all simulations have been done on special grid computer. This grid computer consists of 16 XServers, each 2x2 GHz Intel Xeon, 1 GB RAM, 80 GB HD i.e. 64 CPUs. It has been used for calculations so that each CPU was used like a single processor and thus a rich set of statistically repeated experiments were not a time problem. This does not means that such class of problems can be solved only on special computers. Solved problem reported here can also be done on single PC.
6. **Multiple detection**. In a few cases has been observed that used EAs had in the last population individuals at different position with almost the same cost function that represents multiple solutions, i.e. bifurcations at different positions.
7. **Localized bifurcations**. It is important to note, that only main bifurcations has been localized. Bifurcations at exact positions were undiscovered, which imply space for future research, i.e. better algorithms setting, cost function improvement, etc. Figures 7-9 clearly show, that there is a lot of another bifurcations, that were not localized by used EAs, so the question is what is borderline for EAs to be used on such kind of task.

TABLE 6. Localized bifurcations at positions (aproximately), see Figure 6.

Position	3.543	3.568	3.632	3.742	3.847
SOMA	16	23	38	5	18
DE	12	34	19	8	27

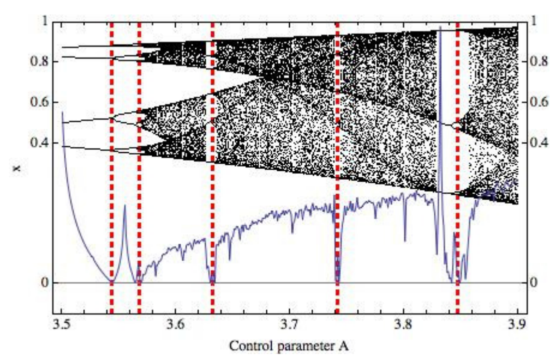

FIGURE 6. Composed picture show identified bifurcations (red dashed lines) that are in exact correlation with Lyapunov exponent, compare Figure 1 and 5 and Table 6.

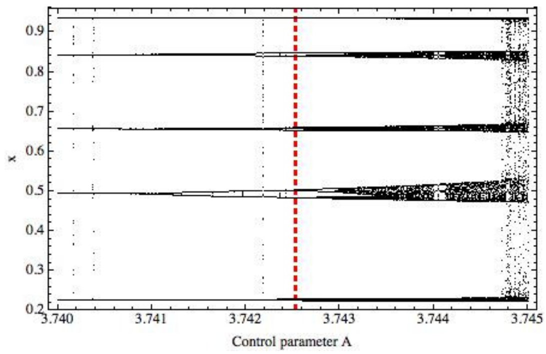

FIGURE 7. Identified and non-identified bifurcations.

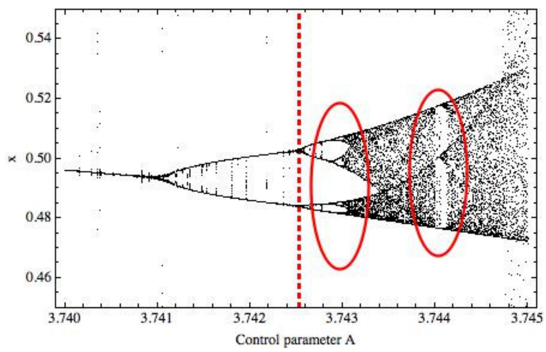

FIGURE 8. Identified and non-identified bifurcations, zoom from Figure 7.

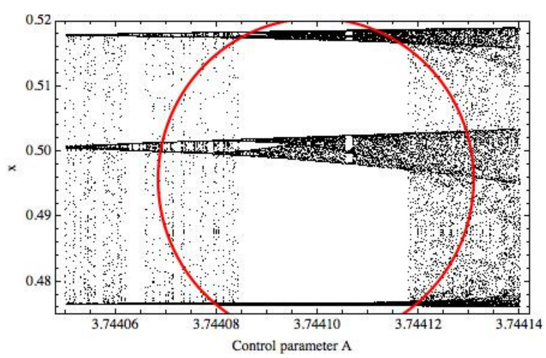

FIGURE 9. Non-identified bifurcations, zoom from the Figure 8.

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REFERENCES

1. R. Gilmore, *Catastrophe Theory for Scientists and Engineers*, John Wiley and Sons, 1993.
2 V. Arnold, *The Theory of Singularities and Its Applications*, Accademia Nazionale Dei Lincei, Pisa, Italy, 1991.
3 T. Poston, I. Stewart, *Catastrophe Theory and its Applications*, Pitman, IEEE Press N.Y., pp. 842-844, 1977.
4 R. Hilborn, *Chaos and Nonlinear Dynamics*, Oxford University Press, 1994.
5 K. Price, R. Storn, *Differential Evolution - A Practical Approach to Global Optimization*, Springer-Verlag, 2005.
6 Zelinka I., SOMA - Self Organizing Migrating Algorithm, *In: Onwubolu , Babu B (eds), New Optimization Techniques in Engineering*, (Springer-Verlag, New York), pp. 167-218, 2004.