are arranged to monic forms $g(s)$ (with unit coefficients by the highest power of s) such that

$$g_j = h_j / h_n \quad j = 0, 1, \ldots, n \tag{48}$$

where $n = \deg h$.

The second polynomial m ensuring properness of the controller is chosen as

$$m(s) = 1 \tag{49}$$

for both UFOTDS and ITDS with the TN expansion,

$$m(s) = s + \frac{2}{\tau_d} \tag{50}$$

for both UFOTDS and ITDS with the Padé approximation,

$$m(s) = s + \frac{1}{\tau_2} \tag{51}$$

for the USOTDS with the TN expansion,

$$m(s) = \left(s + \frac{1}{\tau_2}\right)\left(s + \frac{2}{\tau_d}\right) \tag{52}$$

for the USOTDS with the Padé approximation,

$$m(s) = s + \frac{1}{\tau} \tag{53}$$

for both SFOPITDS and UFOPITDS with the TN expansion, and,

$$m(s) = \left(s + \frac{2}{\tau_d}\right)\left(s + \frac{1}{\tau}\right) \tag{54}$$

for both UFOPITDS and SFOPITDS with the Padé approximation.

The above forms of m lead to the polynomial d with coefficients containing only the selectable parameter φ with all other coefficients depending on parameters of polynomials b and a. Consequently, a location of the closed loop poles can be affected by the selectable parameter φ.

The transfer functions of controllers with degrees of polynomials in their numerators and denominators given by (35) are

$$Q(s) = \frac{q_1}{p_0}, \quad R(s) = \frac{r_1 s + r_0}{p_0 s} \tag{55}$$

for both UFOTDS and ITDS with the TN expansion,

$$Q(s) = \frac{q_2 s + q_1}{s + p_0}, \quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)} \tag{56}$$

for both UFOTDS and ITDS with the Padé approximation, and, for the USOTDS, SFOPITDS and UFOPITDS with the TN expansion. Further,

$$Q(s) = \frac{q_3 s^2 + q_2 s + q_1}{s^2 + p_1 s + p_0}$$
$$R(s) = \frac{r_3 s^3 + r_2 s^2 + r_1 s + r_0}{s(s^2 + p_1 s + p_0)} \tag{57}$$

for the USOTDS, SFOPITDS and UFOPITDS with the Padé approximation.

In all cases, the parameters q in numerators of controllers are computed from parameters t according to (37).

For clarity, derived formulas for computation of parameters p_0 and t the controller derived for all considered cases together with conditions of the controllers' stability are introduced in the form of tables.

Table 1. Controller parameters for UFOTDS

TN expansion
$p_0 = \frac{\tau \tau_d (g_1 + \tau_d g_0) + \tau}{\tau - \tau_d}$ $t_0 = \frac{\tau}{K} g_0, \quad t_1 = \frac{1}{K}\frac{\tau}{\tau_d}(p_0 - 1)$
$p_0 > 0$ for $\tau_d < \tau$
Padé approximation
$p_0 = \frac{\tau\left[2 g_2 + \tau_d (g_1 + \frac{\tau_d}{2} g_0)\right] + 2}{2\tau - \tau_d}, \quad t_0 = \frac{\tau}{K} g_0$ $t_1 = \frac{1}{K}\left[p_0 + \tau(g_1 + \tau_d g_0)\right],$ $t_2 = \frac{1}{K}\left[\tau(p_0 - g_2) - 1\right]$
$p_0 > 0$ for $\tau_d < 2\tau$

Table 2. Controller parameters for ITDS

TN expansion
$p_0 = 1 + \tau_d (g_1 + K \tau_d)$ $t_0 = r_0 = \frac{1}{K} g_0, \quad t_1 = \frac{1}{K}(g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d
Padé approximation

$p_0 = g_2 + \frac{\tau_d}{4}(2g_1 + \tau_d g_0)$, $t_0 = r_0 = \frac{1}{K} g_0$ $t_1 = \frac{1}{K}(g_1 + \tau_d g_0)$, $t_2 = \frac{\tau_d}{4K}(2g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d

Table 3. Controller parameters for USOTDS

TN expansion
$p_0 = \frac{\tau_1(g_2 + \tau_d g_1 + \tau_d^2 g_0) + 1}{\tau_1 - \tau_d}$ $t_0 = \frac{\tau_1}{K} g_0$, $t_1 = \frac{1}{K}[p_0 + \tau_1 g_1 + \tau_1(\tau_2 + \tau_d) g_0]$ $t_2 = \frac{1}{K}\frac{\tau_1\tau_2}{\tau_d}\left[p_0 - g_2 - \frac{1}{\tau_1}\right]$
$p_0 > 0$ for $\tau_d < \tau_1$
Padé approximation
$p_0 = \frac{2g_3 + \tau_1\left[2g_2 + \tau_d\left(g_1 + \frac{\tau_d}{2} g_0\right)\right] + \frac{2}{\tau_1}}{2\tau_1 - \tau_d}$ $p_1 = g_3 + \frac{1}{\tau_1}$ $t_0 = \frac{\tau_1}{K} g_0$, $t_1 = \frac{1}{K}\left[p_0 + \tau_1\left(g_1 + (\tau_2 + \tau_d) g_0\right)\right]$ $t_2 = \frac{1}{K}\left[\left(\frac{4\tau_1\tau_2}{\tau_d} + \tau_1 - \tau_2\right)p_0 - \right.$ $\left. -\left(\frac{4\tau_2}{\tau_d} + 1\right)\left(g_3 + \tau_1 g_2 + \frac{1}{\tau_1}\right) - \tau_1\tau_2 g_1\right]$ $t_3 = \frac{\tau_2}{K}\left[\tau_1(p_0 - g_2) - g_3 - \frac{1}{\tau_1}\right]$
$p_0 > 0$ for $\tau_d < 2\tau_1$

Table 4. Controller parameters for SFOPITDS

TN expansion
$p_0 = g_2 + \tau_d(g_1 + \tau_d g_0)$, $t_0 = \frac{1}{K} g_0$ $t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$, $t_2 = \frac{\tau}{K}(g_1 + \tau_d g_0)$
$p_0 > 0$ for all τ_d
Padé approximation
$p_0 = g_2 + \frac{\tau_d}{4}(2g_1 + \tau_d g_0)$, $p_1 = g_3$
$t_0 = \frac{1}{K} g_0$, $t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{4K}[(2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau\tau_d g_0]$ $t_3 = \frac{\tau\tau_d}{4K}(2g_1 + \tau_d g_0)$
$p_1 > 0$ for all τ_d, $p_0 > 0$ for all τ_d

Table 5. Controller parameters for UFOPITDS

TN expansion
$p_0 = \frac{(\tau + \tau_d)[g_2 + \tau_d(g_1 + \tau_d g_0)] + 2\tau}{\tau - \tau_d}$
$t_0 = \frac{1}{K} g_0$, $t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{K}\frac{2\tau g_2 + \tau(\tau + \tau_d)(g_1 + \tau_d g_0) + 2}{\tau - \tau_d}$
$p_0 > 0$ for $\tau_d < \tau$
Padé approximation
$p_0 = \frac{4g_3 + (2\tau + \tau_d)\left(g_2 + \frac{\tau_d}{2} g_1 + \frac{\tau_d^2}{4} g_0\right) + \frac{4}{\tau}}{2\tau - \tau_d}$ $p_1 = g_3 + \frac{2}{\tau}$ $t_0 = \frac{1}{K} g_0$, $t_1 = \frac{1}{K}[g_1 + (\tau + \tau_d) g_0]$ $t_2 = \frac{1}{K}\left[\left(\frac{4\tau}{\tau_d} - 1\right)p_0 - \frac{8}{\tau_d} g_3 - \left(\frac{4\tau}{\tau_d} + 1\right) g_2 - \right.$ $\left. -\tau g_1 - \frac{8}{\tau\tau_d}\right]$ $t_3 = \frac{1}{K}\left[\tau(p_0 - g_2) - 2g_3 - \frac{2}{\tau}\right]$
$p_1 > 0$ for all τ_d, $p_0 > 0$ for $\tau_d < 2\tau$

4 Simulation Results

All simulations were performed by MATLAB-Simulink tools. In all cases, the unit step reference w was introduced at the time t = 0 and the step disturbances v_1 and v_2 were subsequently injected after settling of the control responses.

4.1 UFOTDS

The parameters in the transfer function (1) has been chosen as $K = 1$ and $\tau = 4$.

The responses in Fig.2 document applicability of the TNE for the UFOTDS with a small value of τ_d. Further, the responses illustrate necessity of a higher value of φ to achieving of an aperiodic character of responses. Smaller values of φ lead to their oscillatory character. An effect of the parameter β_1 can be seen in Fig.3. Its increasing value speeds the control but causes expressive overhoots.

A preference of the PA in comparison with the TN is evident from the controlled output responses in Fig.4 computed under the same conditions. Moreover, the PA enables a use also for higher values of τ_d as shown in Fig.5.

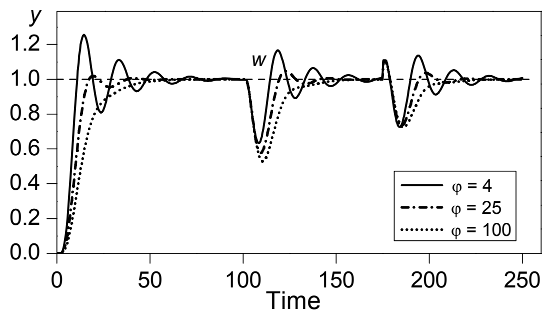

Fig.2. UFOTDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

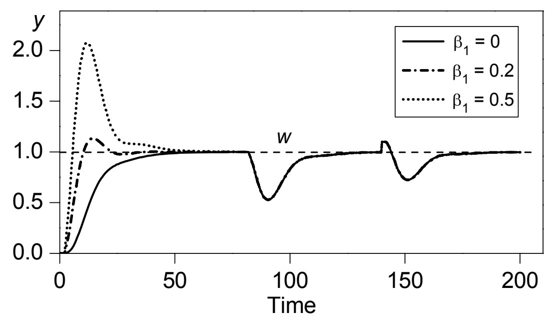

Fig.3. UFOTDS - TNE: Controlled output for various β_1 ($\tau_d = 2$, $\varphi = 100$, $v_1 = -0.2$, $v_2 = 0.1$).

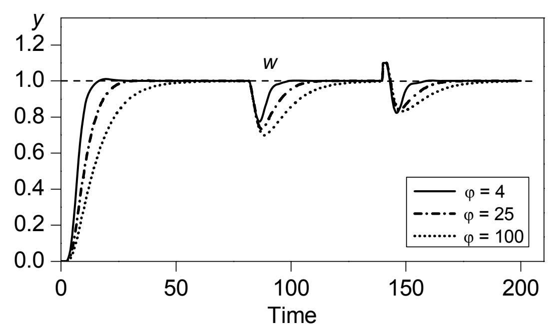

Fig.4. UFOTDS - PA: Controlled output for various φ ($\tau_d = 2$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

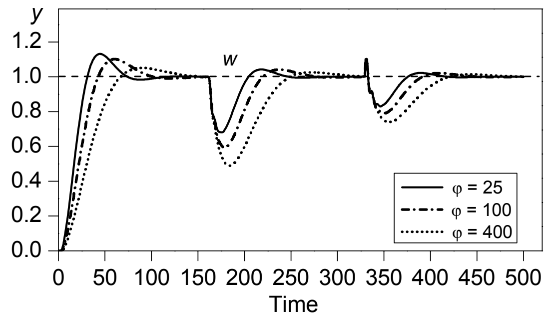

Fig.5. UFOTDS - PA: Controlled output for various φ ($\tau_d = 4$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

4.2 USOTDS

The parameters in the transfer function (2) were chosen as $K = 1$, $\tau_1 = 4$, $\tau_2 = 2$.

Also in this case, an application of the TNE is possible for smaller values of the time delay and for higher values of φ. A higher value of τ_d needs a use of the PA. The simulation results can be seen in Figs.6 and 7.

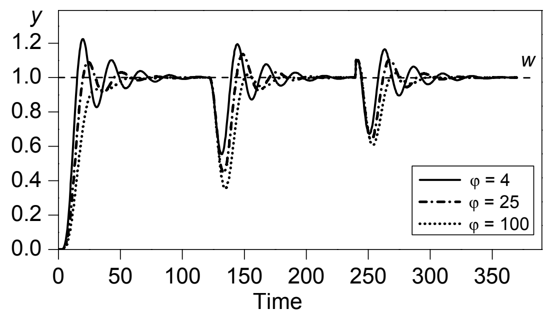

Fig.6. USOTDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_{1,2} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

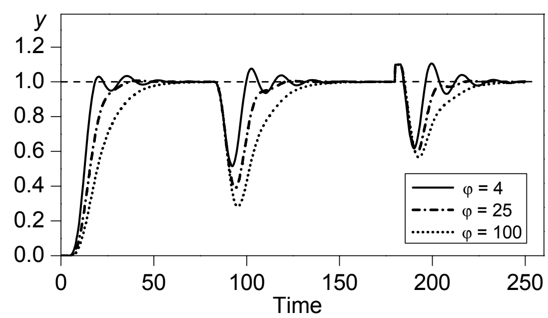

Fig.7. USOTDS - PA: Controlled output for various α ($\tau_d = 3$, $\beta_{1,2,3} = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

The responses in Fig.8 demonstrate their high sensitivity to parameters β. Evidently, on behalf of acceleration of the control, only small values β should be chosen. Their higher values lead to expresive overshoots at the start of the tracking interval.

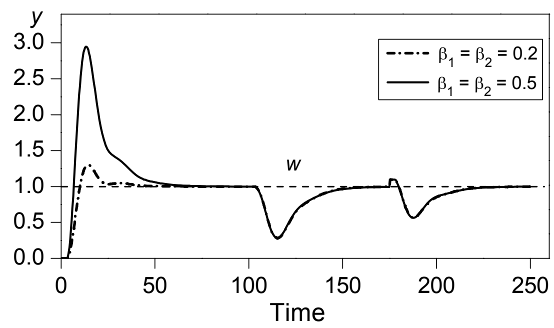

Fig.8. USOTDS - PA: Controlled output for various β_1, β_2 ($\tau_d = 3$, $\varphi = 100$, $\beta_3 = 0$, $v_1 = -0.2$, $v_2 = 0.1$).

4.3 ITDS

In this case, the parameter in (3) has been chosen as $K = 0.2$.

The responses in Fig.9 document applicability of the TNE for the ITDS with smaller values of τ_d. There is not a significant difference in comparison with utilization of the PA as shown in Fig.10. Here, also a selection of the parameter φ is not very important.

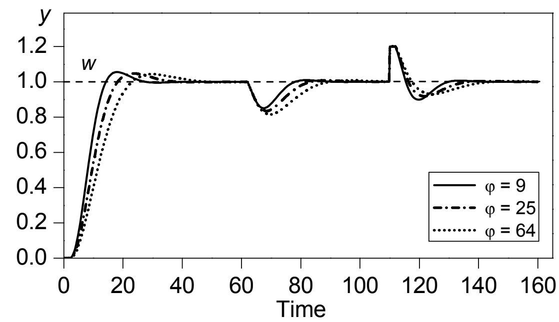

Fig.9. ITDS - TNE: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

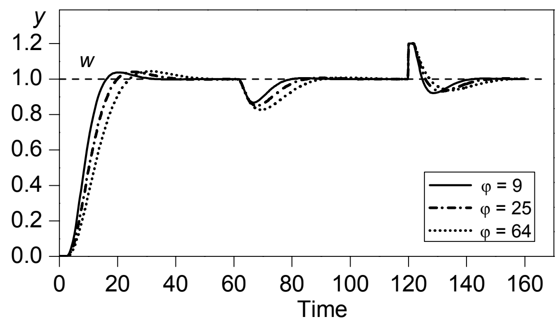

Fig.10. ITDS - PA: Controlled output for various φ ($\tau_d = 2$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

An effect of the parameter β_1 on the controlled output responses can be seen in Fig.11. A reasonable choice of this parameter can accelerate the control responses keeping their apperiodic character.

A difference between both approximations appears for higher values of τ_d as it can be seen in Figs.12, 13 and 14. There, a priority of the PA is evident. It is also clear that a higher value of τ_d requires a use of a higher value of φ.

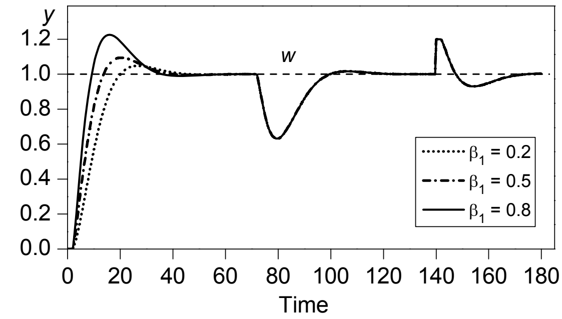

Fig.11. ITDS - TNE: Controlled output for various β_1 ($\tau_d = 5$, $\varphi = 25$, $v_1 = -0.4$, $v_2 = 0.2$).

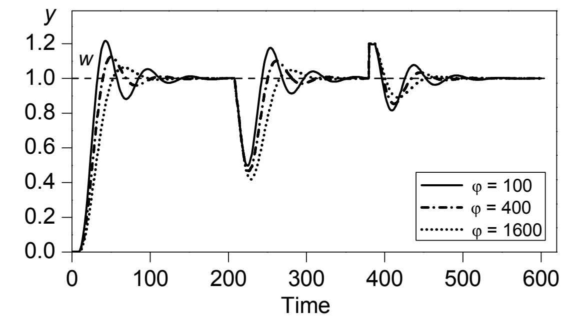

Fig.12. ITDS - TNE: Controlled output for various φ ($\tau_d = 8$, $\beta_1 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

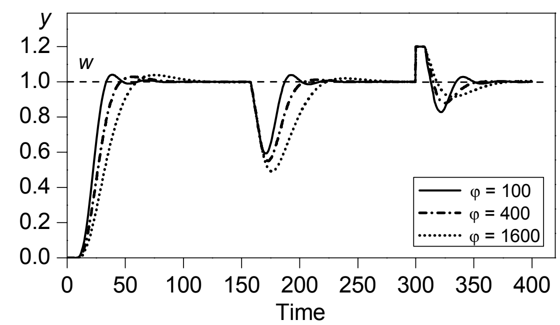

Fig.13. ITDS - PA: Controlled output for various φ ($\tau_d = 8$, $\beta_1 = \beta_2 = 0$, $v_1 = -0.2$, $v_2 = 0.2$).

4.4 SFOPITDS

For this model (and, also for the UFOPITDS), the parameters in (2) have been chosen as $K = 0.2$ and $\tau = 4$. The controlled output responses for various φ are shown in Figs.15 and 16, a comparison between application of the TNE and PA can be seen in Fig.17. The presented results clearly prove a better control quality obtained by the PA. It should be noted that for both SFOPITDS and UFOPITDS zero parameters β were chosen equivalent to the 2DOF control structure. This choice gave best control results.

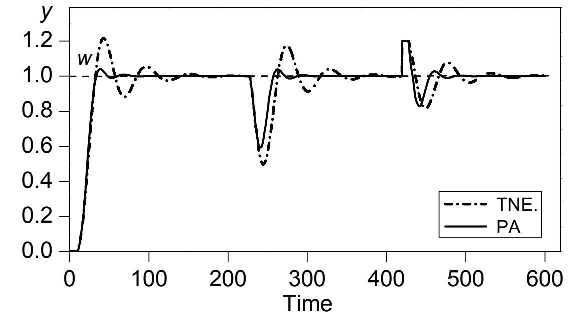

Fig.14. ITDS – Comparison of controlled outputs for TNE and PA ($\tau_d = 8$, $\varphi = 100$, $\beta_1 = \beta_2 = 0$, $v_1 = - 0.2$, $v_2 = 0.2$).

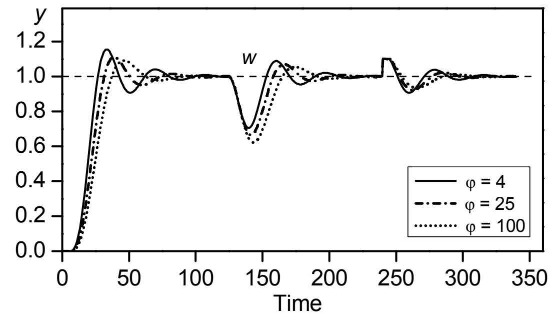

Fig.15. SFOPITDS - TNE: Controlled output for various φ ($\tau_d = 5$, $v_1 = - 0.2$, $v_2 = 0.1$).

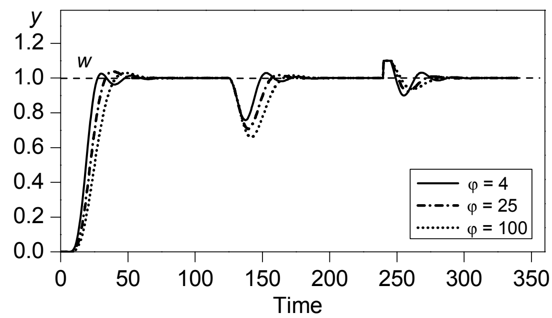

Fig.16. SFOPITDS - PA: Controlled output for various φ ($\tau_d = 5$, $v_1 = - 0.1$, $v_2 = 0.1$).

4.5 UFOPITDS

With regard to a presence of both integrating and unstable parts, the UFOPITDSs belong to hardly controllable systems. However, the control responses in Fig.18 document usability of both TNE and PA for smaller value of τ_d. Higher values of τ_d require a selection of higher values of φ as shown for the PA in Fig.19 However, for higher values of φ, the TNE is unsuitable, as documented in Fig.20.

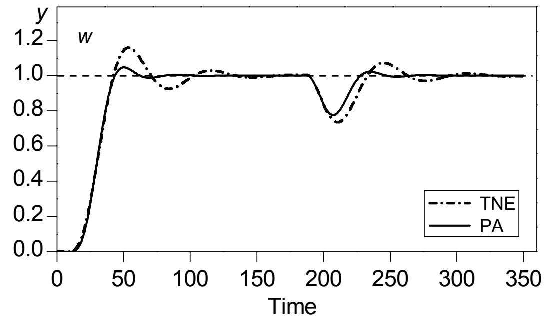

Fig.17. SFOPITDS - Comparison of controlled outputs for TNE and PA ($\tau_d = 8$, $\varphi = 100$, $v_1 = - 0.1$, $v_2 = 0.2$)

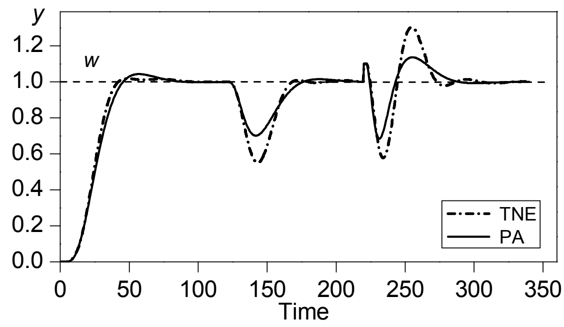

Fig.18. UFOPITDS - Comparison of controlled outputs for TNE and PA ($\tau_d = 2$, $\varphi = 400$, $v_1 = - 0.05$, $v_2 = 0.1$).

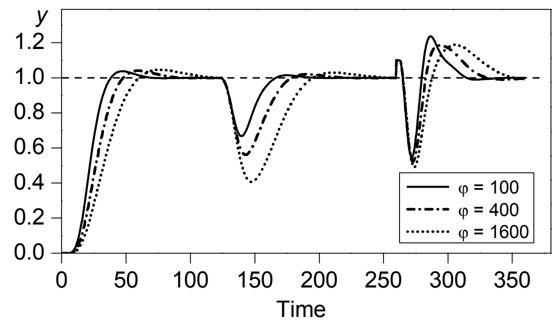

Fig.19. UFOPITDS - PA: Controlled output for various φ ($\tau_d = 3$, $v_1 = - 0.05$, $v_2 = 0.1$).

5 Conclusions

The problem of control design for unstable and integrating time delay systems has been solved and analysed. The proposed method is based in two ways of the time delay approximation. The controller design uses the polynomial synthesis and the controller setting employs the results of the LQ control theory. The presented procedure provides satisfactory control responses in the tracking of a step reference as well as in step disturbances attenuation. The presented results have demonstrate the usability of the method and the control of a good quality also for relatively high ratio of the time delay to the time constant. The procedure makes possible a tuning of the controller parameters by two types of selectable parameters. Using derived formulas, the controller parameters can be automatically computed. From this reason, the method could also be used for an adaptive control.

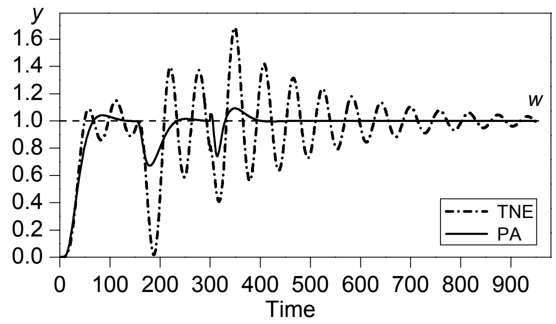

Fig.20. UFOPITDS – Comparison of controlled outputs for TNE and PA (τ_d = 3, φ = 2500, v_1 = - 0.05, v_2 = 0.1).

Acknowledgments

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