

Techniques for Predictor Design in Multivariable Predictive Control

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Abstract: - Model Based Predictive Control (MBPC) or only Predictive Control is one of the control methods which have developed considerably over a few past years. It is mostly based on discrete models of controlled systems. Model of a controlled system is used for computation of predictions of the systems output on the basis of past inputs, outputs and states and designed sequence of future control increments. This paper is focused in comparison of various approaches to computation of multi-step-ahead predictions using a multivariable input – output model. Computational aspects of derivation of predictions can be limiting especially in adaptive predictive control. Many processes are affected by external disturbances that can be measured. Inclusion of measurable disturbances into prediction equations for different approaches was also elaborated.

Key-Words: - predictive control, multivariable systems, multi-step-ahead prediction, Diophantine equations, CARIMA model

1 Introduction

Model Based Predictive Control (MBPC) or only Predictive Control [1], [2], [3] is one of the control methods which have developed considerably over a few past years. Predictive control is essentially based on discrete or sampled models of processes. Computation of appropriate control algorithms is then realized especially in the discrete domain.

Theoretical research in the area of predictive control has a great impact on the industrial world and there are many applications of predictive control in industry. Its development has been significantly influenced by industrial practice. At present, predictive control with a number of real industrial applications belongs among the most often implemented modern industrial process control approaches. First predictive control algorithms were implemented in industry as an effective tool for control of multivariable industrial processes with constraints more than twenty five years ago. The use of predictive control was limited on control of namely rather slow processes due to the amount of computation required. At present, with the computing power available today, this is not an essential problem. A fairly actual applications of predictive control are presented in [4], [5], [6], [7]. An extensive surveys of industrial applications of predictive control are presented in [8], [9], [10], [11].

The term Model Predictive Control designates a class of control methods which have common particular attributes [12], [13]:

- Mathematical model of a systems control is used for prediction of future control of a systems output.
- The input reference trajectory in the future is known.
- A computation of the future control sequence includes minimization of an appropriate objective function (usually quadratic one) with the future trajectories of control increments and control errors.
- Only the first element of the control sequence is applied and the whole procedure of the objective function minimization is repeated in the next sampling period.

The principle of MBPC is shown in Fig. 1, where $u(t)$ is the manipulated variable, $y(t)$ is the process output and $w(t)$ is the reference signal, N_1 , N_2 and N_u are called minimum, maximum and control horizon. This principle is possible to define as follows:

1. The process model is used to predict the future outputs $\hat{y}(t)$ over some horizon N . The predictions are calculated based on information up to time k and on the future control actions that are to be determined.

2. The future control trajectory is calculated as a solution of an optimisation problem consisting of an objective function and constraints. The cost function comprises future output predictions, future reference trajectory, and future control actions.
3. Although the whole future control trajectory was calculated in the previous step, only first element $u(k)$ is actually applied to the process. At the next sampling time the procedure is repeated. This is known as the Receding Horizon concept.

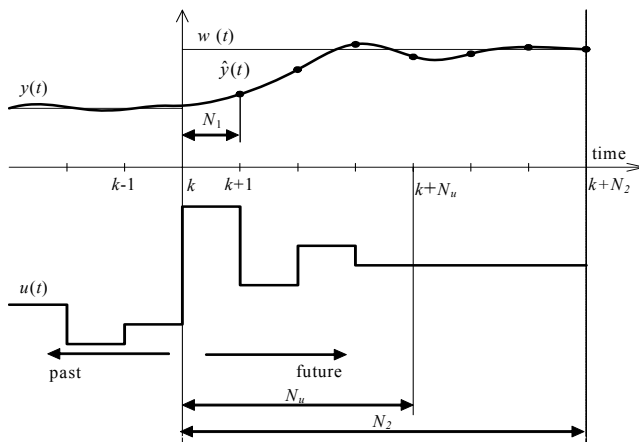


Fig. 1. Principle of MBPC

The computation of a control law of MBPC is mostly based on minimization of the following criterion

$$J(k) = \sum_{j=N_1}^N e(k+j)^2 + \lambda \sum_{j=1}^{N_u} \Delta u(k+j)^2 \quad (1)$$

where $e(k+j)$ is a vector of predicted control errors, $\Delta u(k+j)$ is a vector of future increments of manipulated variables, N is length of the prediction horizon, N_u is length of the control horizon and λ is a weighting factor of control increments.

A predictor in a vector form is given by

$$\hat{y} = G\Delta u + y_0 \quad (2)$$

where \hat{y} is a vector of system predictions along the horizon of the length N . The first element in the equation (2) represents the forced response of the system. Δu is a vector of control increments and G is a matrix of the dynamics which contains values of the step sequence. y_0 is the free response vector.

The first task is computation of predictions for an arbitrary prediction horizon. Dynamics of most of processes require horizons of length where it is not possible to compute predictions in a simple straightforward way. For a particular model, it is

necessary to compute prediction equations. The most often used models in applications and academic papers are state – space and input output models. For state-space models computation of predictions is rather obvious. For input – output models there are several approaches how to compute prediction equations. All the approaches result to the same prediction equations. But computational demands for particular methods are different. Of course, the main computational problem in predictive control is solving the optimization problem. But in adaptive control [14],[15] when it is necessary to compute prediction equations in each sampling period the computational time consumption can be important. It can also be important while using the prediction for other purposes than for the predictive control. Some of the methods also are not algorithmically understandable and clear.

One of the main advantages of predictive control is its simple applicability for control of multi – input multi – output (MIMO) systems. It is one of the most effective approaches to control of multivariable systems since multivariable systems can be handled in a straightforward manner. The aim of the paper is then to compare various approaches to computation of multi – step ahead predictions using a multivariable input – output model.

Many processes are affected by external disturbances caused by the variation of variables that can be measured. This situation is typical in processes whose outputs are affected by variations of the load regime. Known disturbances can be taken explicitly into account in predictive control. Another aim of the paper is then to deal with incorporation of measurable disturbances to prediction equations using various methods of computation of prediction equations.

2 Model of the System

Let us consider a two input – two output system. The two – input/two – output (TITO) processes are the most often encountered multivariable processes in practice and many processes with inputs/outputs beyond two can be treated as several TITO subsystems [16].

A general transfer matrix of a two-input–two-output system with significant cross-coupling between the control loops is expressed as:

$$G(z) = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \quad (3)$$

$$Y(z) = G(z)U(z) \quad (4)$$

where $U(z)$ and $Y(z)$ are vectors of the manipulated variables and the controlled variables, respectively.

$$U(z) = [u_1(z), u_2(z)]^T \quad Y(z) = [y_1(z), y_2(z)]^T \quad (5)$$

It may be assumed that the transfer matrix can be transcribed to the following form of the matrix fraction:

$$G(z) = A^{-1}(z^{-1})B(z^{-1}) = B_1(z^{-1})A_1^{-1}(z^{-1}) \quad (6)$$

where the polynomial matrices $A \in R_{22}[z^{-1}]$, $B \in R_{22}[z^{-1}]$ are the left coprime factorizations of matrix $G(z)$ and the matrices $A_1 \in R_{22}[z^{-1}]$, $B_1 \in R_{22}[z^{-1}]$ are the right coprime factorizations of $G(z)$. The model can be also written in the form

$$A(z^{-1})Y(z) = B(z^{-1})U(z) \quad (7)$$

As an example a model with polynomials of second order was chosen. This model proved to be effective for control of several TITO laboratory processes [17], where controllers based on a model with polynomials of the first order failed. The model has sixteen parameters. The matrices A and B are defined as follows

$$A(z^{-1}) = \begin{bmatrix} 1 + a_1z^{-1} + a_2z^{-2} & a_3z^{-1} + a_4z^{-2} \\ a_5z^{-1} + a_6z^{-2} & 1 + a_7z^{-1} + a_8z^{-2} \end{bmatrix} \quad (8)$$

$$B(z^{-1}) = \begin{bmatrix} b_1z^{-1} + b_2z^{-2} & b_3z^{-1} + b_4z^{-2} \\ b_5z^{-1} + b_6z^{-2} & b_7z^{-1} + b_8z^{-2} \end{bmatrix} \quad (9)$$

The most popular MPC method is Generalized Predictive Control (GPC) which was introduced in [18], [19]. A widely used model in GPC is the CARIMA model which we can obtain from the nominal model (7) by adding a disturbance model

$$A(z^{-1})y(k) = B(z^{-1})u(k) + \frac{C(z^{-1})}{\Delta} e_s(k) \quad (10)$$

where $e_s(k)$ is a non-measurable random disturbance that is assumed to have zero mean value and constant covariance and the operator delta is an integrator. The matrix $C(z^{-1})$ will be further considered as 2x2 identity matrix. Application of this model enables to achieve integral action.

Difference equations of the incremental form without the unknown term are as follows

$$y_1(k+1) = (1-a_1)y_1(k) + (a_1-a_2)y_1(k-1) + a_2y_1(k-2) - a_3y_2(k) + (a_3-a_4)y_2(k-1) + a_4y_2(k-2) + b_1\Delta u_1(k) + b_2\Delta u_1(k-1) + b_3\Delta u_2(k) + b_4\Delta u_2(k-1)$$

$$y_2(k+1) = (1-a_7)y_2(k) + (a_7-a_8)y_2(k-1) + a_8y_2(k-2) - a_5y_1(k) + (a_5-a_6)y_1(k-1) + a_6y_1(k-2) + b_5\Delta u_1(k) + b_6\Delta u_1(k-1) + b_7\Delta u_2(k) + b_8\Delta u_2(k-1) \quad (11)$$

3 Computation of Predictions

In academic literature there are described several methods for computation of prediction equations for models based on transfer function. This paper will be focused in most often used approaches: methods based on Diophantine equations [1], methods based on matrix operations [12] and straightforward computation on the basis of the CARIMA model [20]. Particular methods will be described in the following subsections.

3.1 Method Based on Matrix Operations

A general difference equation for one – step ahead prediction can be written as follows

$$y(k+1) = A_1y(k) + \dots + A_{n_a}y(k-n_a) + B_1\Delta u(k) + B_2\Delta u(k-1) + \dots + B_{n_b}\Delta u(k-n_b+1) \quad (12)$$

where n_a is the order of the matrix A and n_b is the order of the matrix B .

Now we can formulate equation for j -step ahead prediction.

$$\hat{y}(k+j) = H^{[j]} \Delta u + P^{[j]} \Delta u + Q^{[j]} y \quad (13)$$

It consists of three terms. As for the notation, the arrow pointing right is used for strictly future – not including current value and arrow pointing left is used for past including current value. The particular terms are then past output values, past control increments and future control increments. The matrices H , P and Q are matrices of coefficients. Initialization of the matrices H , P and Q for i equal to one is as follows

$$H^{[1]} = [B_1, 0, 0, 0, \dots] \quad P^{[1]} = [B_2, B_3, \dots, B_{n_b}] \quad Q^{[1]} = [A_1, A_2, \dots, A_{n_a+1}] \quad (14)$$

Further a recursive substitution will be used to find prediction (15)

$$\hat{y}(k+j+1) = H^{[j]} \Delta u + P^{[j]} \Delta u + Q^{[j]} y \quad (15)$$

Following expressions which can be substituted into prediction equation (15) can be derived

$$Q^{[j]} y = [Q^{[j]}, 0] \begin{bmatrix} y(k+1) \\ y \\ \dots \\ y \\ \dots \\ y \end{bmatrix} = Q_1^{[j]} y(k+1) + [Q_2^{[j]}, \dots, Q_{n_a}^{[j]}, 0] y \quad (16)$$

$$\begin{aligned} \mathbf{P}^{[j]} \Delta \mathbf{u} &= \begin{bmatrix} \mathbf{P}^{[j]}, 0 \\ \Delta \mathbf{u} \\ \leftarrow k-1 \end{bmatrix} = \\ &= \mathbf{P}_1^{[j]} \Delta \mathbf{u}(k) + \left[\mathbf{P}_2^{[j]}, \dots, \mathbf{P}_{n_s-1}^{[j]}, 0 \right] \Delta \mathbf{u} \end{aligned} \quad (17)$$

Prediction equation (15) then takes this form

$$\begin{aligned} \hat{\mathbf{y}}(k+j+1) &= \left[\mathbf{P}_1^{[j]}, \mathbf{H}^{[j]} \right] \Delta \mathbf{u} + \left[\mathbf{P}_2^{[j]}, \dots, \mathbf{P}_{n_s-1}^{[j]}, 0 \right] \Delta \mathbf{u} + \\ &+ \left[\mathbf{Q}_2^{[j]}, \dots, \mathbf{Q}_{n_s}^{[j]}, 0 \right] \mathbf{y} + \mathbf{Q}_1^{[j]} \mathbf{y}(k+1) \end{aligned} \quad (18)$$

After substitution of one step ahead prediction we obtain

$$\begin{aligned} \hat{\mathbf{y}}(k+j+1) &= \left[\mathbf{P}_1^{[j]}, \mathbf{H}^{[j]} \right] \Delta \mathbf{u} + \left[\mathbf{P}_2^{[j]}, \dots, \mathbf{P}_{n_s-1}^{[j]}, 0 \right] \Delta \mathbf{u} + \\ &+ \left[\mathbf{Q}_2^{[j]}, \dots, \mathbf{Q}_{n_s}^{[j]}, 0 \right] \mathbf{y} + \mathbf{Q}_1^{[j]} \left[\mathbf{H}^{[1]} \Delta \mathbf{u} + \mathbf{P}^{[1]} \Delta \mathbf{u} + \mathbf{Q}^{[1]} \mathbf{y} \right] \end{aligned} \quad (19)$$

Common terms can be grouped together

$$\begin{aligned} \hat{\mathbf{y}}(k+j+1) &= \underbrace{\left[\mathbf{P}_1^{[j]} + \mathbf{Q}_1^{[j]} \mathbf{H}^{[1]}, \mathbf{H}^{[j]} \right]}_{\mathbf{H}^{[j+1]}} \Delta \mathbf{u} + \\ &+ \underbrace{\left\{ \left[\mathbf{P}_2^{[j]}, \dots, \mathbf{P}_{n_s-1}^{[j]}, 0 \right] + \mathbf{Q}_1^{[j]} \mathbf{P}^{[1]} \right\}}_{\mathbf{P}^{[j+1]}} \Delta \mathbf{u} + \\ &+ \underbrace{\left\{ \left[\mathbf{Q}_2^{[j]}, \dots, \mathbf{Q}_{n_s}^{[j]}, 0 \right] + \mathbf{Q}_1^{[j]} \mathbf{Q}^{[1]} \right\}}_{\mathbf{Q}^{[j+1]}} \mathbf{y} \end{aligned} \quad (20)$$

The recursion is then performed according to the following expressions

$$\begin{aligned} \mathbf{H}^{[j+1]} &= \left[\mathbf{P}_1^{[j]} + \mathbf{Q}_1^{[j]} \mathbf{H}^{[1]}, \mathbf{H}^{[j]} \right] \\ \mathbf{P}^{[j+1]} &= \left[\mathbf{P}_2^{[j]}, \dots, \mathbf{P}_{n_s-1}^{[j]}, 0 \right] + \mathbf{Q}_1^{[j]} \mathbf{P}^{[1]} \\ \mathbf{Q}^{[j+1]} &= \left[\mathbf{Q}_2^{[j]}, \dots, \mathbf{Q}_{n_s}^{[j]}, 0 \right] + \mathbf{Q}_1^{[j]} \mathbf{Q}^{[1]} \end{aligned} \quad (21)$$

Initialization of the matrices \mathbf{H} , \mathbf{P} , \mathbf{Q} for our TITO example is then following

$$\mathbf{H} = \begin{bmatrix} b_1 & b_3 \\ b_5 & b_7 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} b_2 & b_4 \\ b_6 & b_8 \end{bmatrix} \quad (22)$$

$$\mathbf{Q} = \begin{bmatrix} 1-a_1 & -a_3 & a_1-a_2 & a_3-a_4 & a_2 & a_4 \\ -a_5 & 1-a_7 & a_5-a_6 & a_7-a_8 & a_6 & a_8 \end{bmatrix}$$

The recursion then proceeds according to the expressions (21).

3.2 Method Based on Diophantine Equations

It is possible to compute j-step ahead prediction from the model (10)

$$\hat{\mathbf{y}}(k+j) = \frac{\mathbf{B}}{\mathbf{A}} \mathbf{u}(k+j) + \frac{\mathbf{C}}{\Delta \mathbf{A}} \mathbf{e}_s(k+j) \quad (23)$$

From the last term of this expression can be separated terms with positive powers of z where \mathbf{E} is a polynomial matrix of the order j minus one and \mathbf{F}

is a polynomial matrix of the same order as the polynomial matrix \mathbf{A} .

$$\frac{\mathbf{C}(z^{-1})}{\Delta \mathbf{A}(z^{-1})} = \mathbf{E}_j(z^{-1}) + z^{-j} \frac{\mathbf{F}_j(z^{-1})}{\Delta \mathbf{A}(z^{-1})} \quad (24)$$

After substitution to equation (23) we can obtain the predictor in the form

$$\hat{\mathbf{y}}(k+j) = \frac{\mathbf{B}}{\mathbf{A}} \mathbf{u}(k+j) + \mathbf{E}_j \mathbf{e}_s(k+j) + \frac{\mathbf{F}_j}{\Delta \mathbf{A}} \mathbf{e}_s(k) \quad (25)$$

From the original equation (23) we can compute the disturbance and substitute to equation (25)

$$\begin{aligned} \hat{\mathbf{y}}(k+j) &= \frac{\mathbf{B}}{\mathbf{C}} \left[\frac{\mathbf{C}}{\Delta \mathbf{A}} - z^{-j} \frac{\mathbf{F}_j}{\Delta \mathbf{A}} \right] \Delta \mathbf{u}(k+j) + \frac{\mathbf{F}_j}{\mathbf{C}} \mathbf{y}(k) + \\ &+ \mathbf{E}_j \mathbf{e}_s(k+j) \end{aligned} \quad (26)$$

After substitution we obtain

$$\hat{\mathbf{y}}(k+j) = \frac{\mathbf{B}\mathbf{E}_j}{\mathbf{C}} \Delta \mathbf{u}(k+j) + \frac{\mathbf{F}_j}{\mathbf{C}} \mathbf{y}(k) + \mathbf{E}_j \mathbf{e}_s(k+j) \quad (27)$$

Now let us make two simplifications: a white noise case will be considered and future noise values will be further omitted.

$$\hat{\mathbf{y}}(k+j) = \mathbf{B}\mathbf{E}_j \Delta \mathbf{u}(k+j) + \mathbf{F}_j \mathbf{y}(k) \quad (28)$$

The matrix \mathbf{G} is defined as follows

$$\hat{\mathbf{y}}(k+j) = \mathbf{G}_j \Delta \mathbf{u}(k+j) + \mathbf{F}_j \mathbf{y}(k) \quad \mathbf{G}_j = \mathbf{B}\mathbf{E}_j \quad (29)$$

For the design of the j – step ahead predictor the following Diophantine equation is solved

$$\mathbf{I} = \mathbf{E}_j \Delta \mathbf{A} + z^{-j} \mathbf{F}_j \quad (30)$$

Further it is necessary to solve a recursion of the Diophantine equation (30). Particular matrices in the Diophantine equation can be expanded as follows

$$\begin{aligned} \tilde{\mathbf{A}}(z^{-1}) &= \Delta \mathbf{A}(z^{-1}) = \mathbf{I}_{mxm} + \mathbf{A}_1 z^{-1} + \mathbf{A}_2 z^{-2} + \dots + \\ &+ \mathbf{A}_{n_s} z^{-n_s} + \mathbf{A}_{n_s+1} z^{-(n_s+1)} \end{aligned} \quad (31)$$

$$\mathbf{E}_j(z^{-1}) = \mathbf{E}_{j,0} + \mathbf{E}_{j,1} z^{-1} + \mathbf{E}_{j,2} z^{-2} + \dots + \mathbf{E}_{j,j-1} z^{j-1} \quad (32)$$

$$\mathbf{F}_j(z^{-1}) = \mathbf{F}_{j,0} + \mathbf{F}_{j,1} z^{-1} + \mathbf{F}_{j,2} z^{-2} + \dots + \mathbf{F}_{j,j-1} z^{j-1} \quad (33)$$

Let us consider the Diophantine equation corresponding to prediction $\hat{\mathbf{y}}(k+j+1)$

$$\mathbf{I} = \mathbf{E}_{j+1}(z^{-1}) \tilde{\mathbf{A}}(z^{-1}) + z^{-(j+1)} \mathbf{F}_{j+1}(z^{-1}) \quad (34)$$

It is possible to subtract the Diophantine equation (30) from the Diophantine equation (34) and obtain the following expression

$$0 = (\mathbf{E}_{j+1}(z^{-1}) - \mathbf{E}_j(z^{-1}))\tilde{\mathbf{A}}(z^{-1}) + z^{-j}(\mathbf{F}_{j+1}(z^{-1}) - \mathbf{F}_j(z^{-1})) \quad (35)$$

Now it is possible to define the following term

$$\mathbf{E}_{j+1}(z^{-1}) - \mathbf{E}_j(z^{-1}) = \tilde{\mathbf{R}}(z^{-1}) + R_j z^{-1} \quad (36)$$

After substitution

$$0 = \tilde{\mathbf{R}}(z^{-1})\tilde{\mathbf{A}}(z^{-1}) + z^{-j}(\mathbf{R}_j\tilde{\mathbf{A}}(z^{-1}) + \mathbf{F}_{j+1}(z^{-1}) - \mathbf{F}_j(z^{-1})) \quad (37)$$

it is obvious that $\tilde{\mathbf{R}}(z^{-1}) = 0$ in order to obtain the zero matrix on the left side of the equation (37). The matrix \mathbf{E} can be then computed recursively according to the following expression

$$\mathbf{E}_{j+1}(z^{-1}) = \mathbf{E}_j(z^{-1}) + R_j z^{-j} \quad (38)$$

Following expressions can be obtained from the equation (37)

$$R_j = F_{j,0} \\ F_{j+1,i} = F_{j,i+1} - R_j \tilde{A}_{i+1} \quad \text{for } i = 0 \dots \delta(\mathbf{F}_{j+1}) \quad (39)$$

Initial conditions for the recursion are as follows

$$\mathbf{E}_1 = \mathbf{I} \\ \mathbf{F}_1 = z(\mathbf{I} - \tilde{\mathbf{A}}) \quad (40)$$

By making the polynomial matrix

$$\mathbf{E}_j(z^{-1})\mathbf{B}(z^{-1}) = \mathbf{G}_j(z^{-1}) + z^{-j}\mathbf{G}_{jp}(z^{-1}) \quad (41)$$

the prediction equation can be written as

$$\hat{\mathbf{y}}(k+j) = \mathbf{G}_j(z^{-1})\Delta\mathbf{u}(k+j) + \mathbf{G}_{jp}(z^{-1})\Delta\mathbf{u}(k-1) + \mathbf{F}_j(z^{-1})\mathbf{y}(k) \quad (42)$$

The last two terms of the equation (42) depend on past values of the process output and input and represent the free response of the process. The first term depends on future values of control increments and represents the forced response. Equation (42) can be rewritten as

$$\hat{\mathbf{y}}(k+j) = \mathbf{G}_j\Delta\mathbf{u}(k+j) + \mathbf{y}_{0,j} \quad (43)$$

where

$$\mathbf{y}_{0,j} = \mathbf{G}_{jp}(z^{-1})\Delta\mathbf{u}(k-1) + \mathbf{F}_j(z^{-1})\mathbf{y}(k) \quad (44)$$

The predictions can be expressed in condensed form as (2).

In our TITO example, the matrix $\tilde{\mathbf{A}}$ has the following form

$$\tilde{\mathbf{A}}(z^{-1}) = \mathbf{A}\mathbf{A}(z^{-1}) = \begin{bmatrix} 1 + (a_1 - 1)z^{-1} + (a_2 - a_1)z^{-2} - a_2z^{-3} & a_3z^{-1} + (a_4 - a_3)z^{-2} - a_4z^{-3} \\ a_5z^{-1} + (a_6 - a_5)z^{-2} - a_6z^{-3} & 1 + (a_7 - 1)z^{-1} + (a_8 - a_7)z^{-2} - a_8z^{-3} \end{bmatrix} \quad (45)$$

Initial conditions of the recursion are

$$\mathbf{E}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (46)$$

$$\mathbf{F}_1 = z(\mathbf{I} - \tilde{\mathbf{A}}) = [\mathbf{f}_0 \quad \mathbf{f}_1 \quad \mathbf{f}_2] \quad (47)$$

$$\mathbf{f}_0 = \begin{bmatrix} 1 - a_1 & -a_3 \\ -a_5 & 1 - a_7 \end{bmatrix} \quad (48)$$

$$\mathbf{f}_1 = \begin{bmatrix} a_1 - a_2 & a_3 - a_4 \\ a_5 - a_6 & a_7 - a_8 \end{bmatrix} \quad (49)$$

$$\mathbf{f}_2 = \begin{bmatrix} a_2 & a_4 \\ a_6 & a_8 \end{bmatrix} \quad (50)$$

Initialization of the matrix of the free response and the matrix of the dynamics are following

$$\mathbf{x} = [\mathbf{f}_0 \quad \mathbf{f}_1 \quad \mathbf{f}_2 \quad \mathbf{B}_2] \quad (51)$$

$$\mathbf{G} = \mathbf{B}_1 \quad (52)$$

where

$$\mathbf{B}_1 = \begin{bmatrix} b_1 & b_3 \\ b_5 & b_7 \end{bmatrix} \quad (53)$$

$$\mathbf{B}_2 = \begin{bmatrix} b_2 & b_4 \\ b_6 & b_8 \end{bmatrix} \quad (54)$$

The recursion then proceeds according to previously introduced steps.

$$\mathbf{R} = \mathbf{f}_0 \quad (55)$$

$$\mathbf{f}_0 = \mathbf{f}_1 - \mathbf{R}(a_1 - 1) \quad (56)$$

$$\mathbf{f}_1 = \mathbf{f}_2 - \mathbf{R}(a_2 - a_1) \quad (57)$$

$$\mathbf{f}_2 = \mathbf{R}a_2 \quad (58)$$

$$\mathbf{E} = [\mathbf{E} \quad \mathbf{R}] \quad (59)$$

Extension of the matrices of the dynamics and the free response is as follows:

$$\mathbf{G} = \begin{bmatrix} \mathbf{G} \\ \mathbf{B}_1\mathbf{E}(i+1) + \mathbf{B}_2\mathbf{E}(i) \end{bmatrix} \quad (60)$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x} \\ \mathbf{f}_0 \quad \mathbf{f}_1 \quad \mathbf{f}_2 \quad \mathbf{B}_2\mathbf{E}(i+1) \end{bmatrix} \quad (61)$$

3.3 Method Based on Direct Computation from CARIMA Model

This method is based on an analytical derivation of certain predictions and subsequent recursive derivation of later predictions. The number of predictions which are necessary to compute directly depends on the order of the system. The a priori analytical computation, which is required, enables to reduce number of matrix operations which are necessary to perform during the matrix methods.

The differential equations (11) can be rewritten into the matrix form (62)

$$y(k+1) = A_1 y(k) + A_2 y(k-1) + A_3 y(k-2) + B_1 \Delta u(k) + B_2 \Delta u(k-1) \quad (62)$$

where

$$A_1 = \begin{bmatrix} 1-a_1 & -a_3 \\ -a_5 & 1-a_7 \end{bmatrix} \quad (63)$$

$$A_2 = \begin{bmatrix} a_1-a_2 & a_3-a_4 \\ a_5-a_6 & a_7-a_8 \end{bmatrix} \quad (64)$$

$$A_3 = \begin{bmatrix} a_2 & a_4 \\ a_6 & a_8 \end{bmatrix} \quad (65)$$

$$B_1 = \begin{bmatrix} b_1 & b_3 \\ b_5 & b_7 \end{bmatrix} \quad (66)$$

$$B_2 = \begin{bmatrix} b_2 & b_4 \\ b_6 & b_8 \end{bmatrix} \quad (67)$$

It was necessary to directly compute three step ahead predictions in a straightforward way by establishing of previous predictions to later predictions. The model order defines that computation of one step ahead prediction is based on three past values of the system output.

$$\begin{aligned} \hat{y}(k+1) &= A_1 y(k) + A_2 y(k-1) + A_3 y(k-2) + B_1 \Delta u(k) + B_2 \Delta u(k-1) \\ \hat{y}(k+2) &= A_1 y(k+1) + A_2 y(k) + A_3 y(k-1) + B_1 \Delta u(k+1) + B_2 \Delta u(k) \\ \hat{y}(k+3) &= A_1 y(k+2) + A_2 y(k+1) + A_3 y(k) + B_1 \Delta u(k+2) + B_2 \Delta u(k+1) \end{aligned} \quad (68)$$

It is possible to divide computation of the predictions to recursion of the free response and recursion of the matrix of the dynamics. The free response vector can be expressed as:

$$y_0 = \begin{bmatrix} p(1,1) & p(1,2) & p(1,3) & p(1,4) & p(1,5) & p(1,6) & p(1,7) & p(1,8) \\ p(2,1) & p(2,2) & p(2,3) & p(2,4) & p(2,5) & p(2,6) & p(2,7) & p(2,8) \\ p(3,1) & p(3,2) & p(3,3) & p(3,4) & p(3,5) & p(3,6) & p(3,7) & p(3,8) \\ p(4,1) & p(4,2) & p(4,3) & p(4,4) & p(4,5) & p(4,6) & p(4,7) & p(4,8) \\ p(5,1) & p(5,2) & p(5,3) & p(5,4) & p(5,5) & p(5,6) & p(5,7) & p(5,8) \\ p(6,1) & p(6,2) & p(6,3) & p(6,4) & p(6,5) & p(6,6) & p(6,7) & p(6,8) \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_1(k-1) \\ y_2(k-1) \\ y_1(k-2) \\ y_2(k-2) \\ \Delta u_1(k-1) \\ \Delta u_2(k-1) \end{bmatrix} = \begin{bmatrix} P(1,1) & P(1,2) & P(1,3) & P(1,4) \\ P(2,1) & P(2,2) & P(2,3) & P(2,4) \\ P(3,1) & P(3,2) & P(3,3) & P(3,4) \end{bmatrix} \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \end{bmatrix} = P \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \end{bmatrix} \quad (69)$$

All the elements $P(i,j) \ i=1 \dots 3, j=1 \dots 4$ have to be directly computed to initialize the recursion. The next row of the matrix P is repeatedly computed on the basis of the three previous predictions until the prediction horizon is achieved. As an illustrative example it is given the computation of the next element of the first column:

$$P(4,1) = \begin{bmatrix} p(7,1) & p(7,2) \\ p(8,1) & p(8,2) \end{bmatrix} = A_1 P(3,1) + A_2 P(2,1) + A_3 P(1,1) \quad (70)$$

The recursion of the matrix G is similar. The next element of the first column is repeatedly computed and the remaining columns are shifted. This procedure is performed repeatedly until the prediction horizon is achieved. If the control horizon is lower than the prediction horizon a number of columns in the matrix is reduced. The technique is apparent from the equations (71) and (72).

$$G \Delta u = \begin{bmatrix} g(1,1) & g(1,2) & 0 & 0 \\ g(2,1) & g(2,2) & 0 & 0 \\ g(3,1) & g(3,2) & g(1,1) & g(1,2) \\ g(4,1) & g(4,2) & g(2,1) & g(2,2) \\ g(5,1) & g(5,2) & g(3,1) & g(3,2) \\ g(6,1) & g(6,2) & g(4,1) & g(4,2) \end{bmatrix} \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \\ \Delta u_1(k+1) \\ \Delta u_2(k+1) \end{bmatrix} = \begin{bmatrix} G(1,1) & 0 \\ G(2,1) & G(1,1) \\ G(3,1) & G(2,1) \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \end{bmatrix} \quad (71)$$

$$\begin{aligned} G(4,1) &= \begin{bmatrix} g(7,1) & g(7,2) \\ g(8,1) & g(8,2) \end{bmatrix} = A_1 G(3,1) + A_2 G(2,1) + A_3 G(1,1) = \\ &= \begin{bmatrix} 1-a_1 & -a_3 \\ -a_5 & 1-a_7 \end{bmatrix} \begin{bmatrix} g(5,1) & g(5,2) \\ g(6,1) & g(6,2) \end{bmatrix} + \\ &+ \begin{bmatrix} a_1-a_2 & a_3-a_4 \\ a_5-a_6 & a_7-a_8 \end{bmatrix} \begin{bmatrix} g(3,1) & g(3,2) \\ g(4,1) & g(4,2) \end{bmatrix} + \\ &+ \begin{bmatrix} 1-a_1 & -a_3 \\ -a_5 & 1-a_7 \end{bmatrix} \begin{bmatrix} g(1,1) & g(1,2) \\ g(2,1) & g(2,2) \end{bmatrix} \end{aligned} \quad (72)$$

4 Simulation Results

Predictions of the following systems behaviour is given here as an example.

$$A(z^{-1}) = \begin{bmatrix} 1 - 0.5827z^{-1} + 0.1745z^{-2} & -0.0220z^{-1} + 0.1797z^{-2} \\ 0.0167z^{-1} - 0.0886z^{-2} & 1 - 0.4564z^{-1} - 0.0830z^{-2} \end{bmatrix} \tag{73}$$

$$B(z^{-1}) = \begin{bmatrix} -0.0035z^{-1} + 0.0955z^{-2} & 0.1484z^{-1} + 0.2197z^{-2} \\ 0.2783z^{-1} + 0.3107z^{-2} & -0.0371z^{-1} - 0.3489z^{-2} \end{bmatrix} \tag{74}$$

Fig. 1 shows the plant's step response

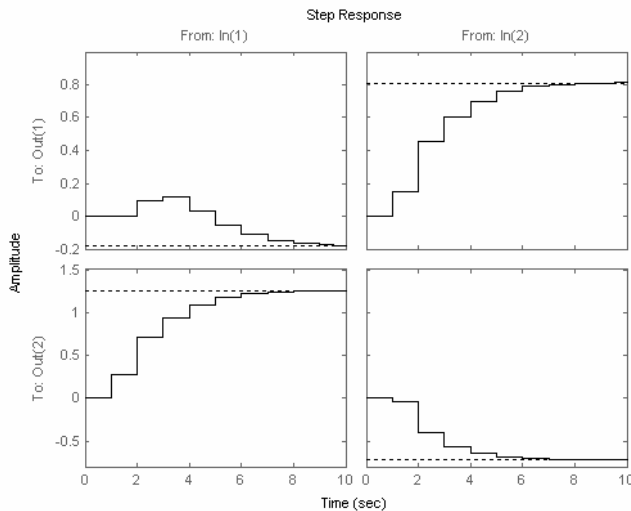


Fig. 1 Step response of the plant

As the input of the system it was chosen a random signal with zero mean value. Values of the signal were generated a priori. Predictions are computed so that such a number of outputs is predicted which corresponds to the value of the prediction horizon. Only the first predicted value is taken and the procedure is repeated.

Results obtained for particular methods were compared each other. In all cases were obtained identical courses of systems output predictions. It means that each different method makes the same final prediction equations. The results were also compared to results of simulation, which means simple input – output response. Courses of the outputs were also identical and correctness of all methods was proved. In the following figures are results of simulation and prediction (only one figure for prediction is presented because as it was mentioned the results were identical). Both prediction and control horizons were set to 10.

All three methods resulted to the same prediction equations. But computational demands for particular methods are different. Computational time demands on CPU of personal computer for particular methods were evaluated. Algorithms for recursive

computation of matrices of coefficients, where particular methods differ, were considered. The sequences of commands were tested using Matlab. During these experiments only the simulation programs for particular methods run on the computer. Measurement for each method was performed thirty times and the results were averaged. Individual time periods of computation for particular methods did not differ significantly so that the results can be considered as authentic. Results of the time measurements are in the following table

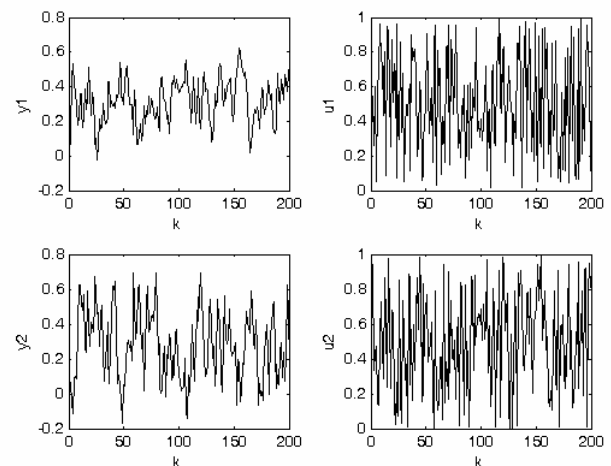


Fig. 2 Prediction

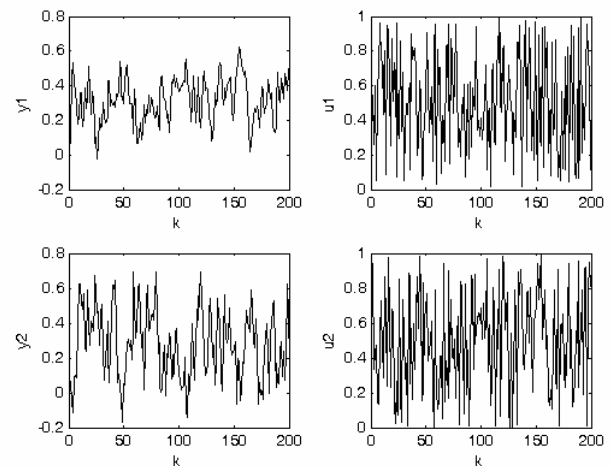


Fig. 3 Simulation

Table. 1 Time periods of computation

Method based on matrix operations	Method based on Diophantine equations	Direct computation from CARIMA model
0,0109 s	0,0154 s	0,0007 s

5 Measurable Disturbances

Many processes are affected by external disturbances caused by the variation of variables that can be measured. Known disturbances can be taken explicitly into account in predictive control. In this case the model must be changed to include the disturbances

$$A(z^{-1})y(k) = B(z^{-1})u(k) + D(z^{-1})v(k) + \frac{C(z^{-1})}{\Delta} e_s(k) \quad (75)$$

where $v(k)$ is a $nx1$ vector of measured disturbances and $D(z^{-1})$ is a polynomial matrix defined as

$$D(z^{-1}) = D_1 z^{-1} + D_2 z^{-2} \quad (76)$$

where

$$D_1 = \begin{bmatrix} d_1 & d_3 \\ d_5 & d_7 \end{bmatrix} \quad (77)$$

$$D_2 = \begin{bmatrix} d_2 & d_4 \\ d_6 & d_8 \end{bmatrix} \quad (78)$$

$$D(z^{-1}) = \begin{bmatrix} d_1 z^{-1} + d_2 z^{-2} & d_3 z^{-1} + d_4 z^{-2} \\ d_5 z^{-1} + d_6 z^{-2} & d_7 z^{-1} + d_8 z^{-2} \end{bmatrix} \quad (79)$$

analogically to equation (41) we can form a polynomial matrix

$$E_j(z^{-1})D(z^{-1}) = L_j(z^{-1}) + z^{-j}L_{jp}(z^{-1}) \quad (80)$$

the prediction equation can now be written as

$$\hat{y}(k+j) = G_j(z^{-1})\Delta u(k+j) + L_j(z^{-1})\Delta v(k+j) + G_{jp}(z^{-1})\Delta u(k-1) + L_{jp}(z^{-1})\Delta v(k-1) + F_j(z^{-1})y(k) \quad (81)$$

The last three terms of the equation (81) depend on past values of the process, output measured disturbances and input variables. They represent the free response of the process considered that the control signals and measured disturbances are constant. The first term depends only on future values of increments of control signal. It is the forced response. The second term depends on the future deterministic disturbances. Analogically as matrix G contains values of the step sequence, the matrix L contains values of the step response of the plant when a unit step is applied to the disturbance signal. Equation (81) can be rewritten as

$$\hat{y}(k+j) = G_j(z^{-1})\Delta u(k+j) + L_j(z^{-1})\Delta v(k+j) + y_{0j} \quad (82)$$

where

$$y_{0j} = G_{jp}(z^{-1})\Delta u(k-1) + L_{jp}(z^{-1})\Delta v(k-1) + F_j(z^{-1})y(k) \quad (83)$$

The predictions can be written in a compact vector form as

$$\hat{y} = G\Delta u + L\Delta v + y_0 \quad (84)$$

In the following sections are described algorithmic modifications of particular methods which have to be implemented in order to incorporate the measurable disturbance into prediction equations.

5.1 Modification of Method Based on Matrix Operations

If we consider the disturbances, the equation (13) takes the form

$$\hat{y}(k+j) = H^{[j]} \underset{\rightarrow k-1}{\Delta u} + P^{[j]} \underset{\leftarrow k-1}{\Delta u} + R^{[j]} \underset{\rightarrow k-1}{\Delta v} + S^{[j]} \underset{\leftarrow k-1}{\Delta v} + Q^{[j]} \underset{\leftarrow k}{y} \quad (85)$$

Where the matrices R and S are matrices of coefficients for future and past disturbances. Initialization of the matrices R and S for j equal to one is as follows

$$R^{[1]} = [D_1, 0, 0, 0, \dots]; \quad S^{[1]} = [D_2, D_3, \dots, D_{n_s}] \quad (86)$$

Following expression can be derived

$$S^{[j]} \underset{\leftarrow k}{\Delta v} = [S^{[j]}, 0] \begin{bmatrix} \underset{\leftarrow k}{\Delta v} \\ \underset{\leftarrow k-1}{\Delta v} \end{bmatrix} = S_1^{[j]} \Delta v(k) + [S_2^{[j]}, \dots, S_{n_s-1}^{[j]}, 0] \underset{\leftarrow k-1}{\Delta v} \quad (87)$$

Prediction equation for $i+1$ step ahead prediction takes the form

$$\hat{y}(k+j+1) = [P_1^{[j]}, H^{[j]}] \underset{\rightarrow k-1}{\Delta u} + [P_2^{[j]}, \dots, P_{n_s-1}^{[j]}, 0] \underset{\leftarrow k-1}{\Delta u} + [S_1^{[j]}, R^{[j]}] \underset{\rightarrow k-1}{\Delta v} + [S_2^{[j]}, \dots, S_{n_s-1}^{[j]}, 0] \underset{\leftarrow k-1}{\Delta v} + [Q_2^{[j]}, \dots, Q_{n_s}^{[j]}, 0] \underset{\leftarrow k}{y} + Q_1^{[j]} y(k+1) \quad (88)$$

after substitution of one step ahead prediction we obtain

$$\begin{aligned} \hat{y}(k+j+1) &= [P_1^{[j]}, H^{[j]}] \underset{\rightarrow k-1}{\Delta u} + [P_2^{[j]}, \dots, P_{n_s-1}^{[j]}, 0] \underset{\leftarrow k-1}{\Delta u} + \\ &+ [S_1^{[j]}, R^{[j]}] \underset{\rightarrow k-1}{\Delta v} + [S_2^{[j]}, \dots, S_{n_s-1}^{[j]}, 0] \underset{\leftarrow k-1}{\Delta v} + [Q_2^{[j]}, \dots, Q_{n_s}^{[j]}, 0] \underset{\leftarrow k}{y} + \\ &+ Q_1^{[j]} \left[H^{[1]} \underset{\rightarrow k-1}{\Delta u} + P^{[1]} \underset{\leftarrow k-1}{\Delta u} + R^{[1]} \underset{\rightarrow k-1}{\Delta v} + S^{[1]} \underset{\leftarrow k-1}{\Delta v} + Q^{[1]} \underset{\leftarrow k}{y} \right] \\ \hat{y}(k+i+1) &= \underbrace{[P_1^{[i]} + Q_1^{[i]} H^{[1]}, H^{[i]}]}_{H^{[i+1]}} \underset{\rightarrow k-1}{\Delta u} + \\ &+ \underbrace{[P_2^{[i]}, \dots, P_{n_s-1}^{[i]}, 0] + Q_1^{[i]} P^{[1]}}_{P^{[i+1]}} \underset{\leftarrow k-1}{\Delta u} + \\ &+ \underbrace{[S_1^{[i]} + Q_1^{[i]} R^{[1]}, R^{[i]}]}_{S^{[i+1]}} \underset{\rightarrow k-1}{\Delta v} + \\ &+ \underbrace{[S_2^{[i]}, \dots, S_{n_s-1}^{[i]}, 0] + Q_1^{[i]} S^{[1]}}_{S^{[i+1]}} \underset{\leftarrow k-1}{\Delta v} + \\ &+ \underbrace{[Q_2^{[i]}, \dots, Q_{n_s}^{[i]}, 0] + Q_1^{[i]} Q^{[1]}}_{Q^{[i+1]}} \underset{\leftarrow k}{y} \end{aligned} \quad (90)$$

The recursion of the matrices R and S is performed according to the following expressions

$$\begin{aligned} R^{[i+1]} &= [S^{[i]} + Q_1^{[i]}R^{[i]}, R^{[i]}] \\ S^{[i+1]} &= [S_2^{[i]}, \dots, S_{n_s-1}^{[i]}, 0] + Q_1^{[i]}S^{[i]} \end{aligned} \quad (91)$$

Initialization of the matrices R and S for our example is following

$$R = \begin{bmatrix} d_1 & d_3 \\ d_5 & d_7 \end{bmatrix} \quad (92)$$

$$S = \begin{bmatrix} d_2 & d_4 \\ d_6 & d_8 \end{bmatrix} \quad (93)$$

5.2 Modification of Method Based on Diophantine Equations

The matrix of the free response x have to be extended. Initialization of the matrix x is as follows

$$x = [f_0 \quad f_1 \quad f_2 \quad B_2 \quad D_2] \quad (94)$$

Computation of the matrix L is also necessary. The matrix L is initialized as

$$L = D_1 \quad (95)$$

Extension of the matrices L and X is as follows:

$$L = \begin{bmatrix} L \\ D_1 E(i+1) + D_2 E(i) \end{bmatrix} \quad (96)$$

$$x = \begin{bmatrix} x \\ f_0 \quad f_1 \quad f_2 \quad B_2 E(i+1) \quad D_2 E(i+1) \end{bmatrix} \quad (97)$$

5.3 Modification of Method Based on Direct Computation from CARIMA Model

The difference equations (12) after including the disturbance take the form

$$\begin{aligned} y(k+1) &= A_1 y(k) + A_2 y(k-1) + A_3 y(k-2) + \\ &+ B_1 \Delta u(k) + B_2 \Delta u(k-1) + D_1 \Delta v(k) + D_2 \Delta v(k-1) \end{aligned} \quad (98)$$

where

$$D_2 = \begin{bmatrix} d_2 & d_4 \\ d_6 & d_8 \end{bmatrix} \quad (99)$$

$$D_1 = \begin{bmatrix} d_1 & d_3 \\ d_5 & d_7 \end{bmatrix} \quad (100)$$

Three step ahead predictions are computed according to

$$\begin{aligned} \hat{y}(k+1) &= A_1 y(k) + A_2 y(k-1) + A_3 y(k-2) + \\ &+ B_1 \Delta u(k) + B_2 \Delta u(k-1) + D_1 \Delta v(k) + D_2 \Delta v(k-1) \\ \hat{y}(k+2) &= A_1 y(k+1) + A_2 y(k) + A_3 y(k-1) + \\ &+ B_1 \Delta u(k+1) + B_2 \Delta u(k) + D_1 \Delta v(k+1) + D_2 \Delta v(k) \end{aligned} \quad (101)$$

$$\begin{aligned} \hat{y}(k+3) &= A_1 y(k+2) + A_2 y(k+1) + A_3 y(k) + \\ &+ B_1 \Delta u(k+2) + B_2 \Delta u(k+1) + D_1 \Delta v(k+2) + D_2 \Delta v(k+1) \end{aligned}$$

The free response vector is expressed as:

$$y_0 = \begin{bmatrix} p(1,1) & p(1,2) & p(1,3) & p(1,4) & p(1,5) & p(1,6) & p(1,7) & p(1,8) & p(1,9) & p(1,10) \\ p(2,1) & p(2,2) & p(2,3) & p(2,4) & p(2,5) & p(2,6) & p(2,7) & p(2,8) & p(2,9) & p(2,10) \\ p(3,1) & p(3,2) & p(3,3) & p(3,4) & p(3,5) & p(3,6) & p(3,7) & p(3,8) & p(3,9) & p(3,10) \\ p(4,1) & p(4,2) & p(4,3) & p(4,4) & p(4,5) & p(4,6) & p(4,7) & p(4,8) & p(4,9) & p(4,10) \\ p(5,1) & p(5,2) & p(5,3) & p(5,4) & p(5,5) & p(5,6) & p(5,7) & p(5,8) & p(5,9) & p(5,10) \\ p(6,1) & p(6,2) & p(6,3) & p(6,4) & p(6,5) & p(6,6) & p(6,7) & p(6,8) & p(6,9) & p(6,10) \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_1(k-1) \\ y_2(k-1) \\ y_1(k-2) \\ y_2(k-2) \\ \Delta u_1(k-1) \\ \Delta u_2(k-1) \\ \Delta v_1(k-1) \\ \Delta v_2(k-1) \end{bmatrix} = \begin{bmatrix} P(1,1) & P(1,2) & P(1,3) & P(1,4) & P(1,5) \\ P(2,1) & P(2,2) & P(2,3) & P(2,4) & P(2,5) \\ P(3,1) & P(3,2) & P(3,3) & P(3,4) & P(3,5) \end{bmatrix} \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \\ \Delta v(k-1) \end{bmatrix} = P \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \\ \Delta v(k-1) \end{bmatrix} \quad (102)$$

The computation of the matrix L is as follows

$$\begin{aligned} L \Delta v &= \begin{bmatrix} l(1,1) & l(1,2) & 0 & 0 \\ l(2,1) & l(2,2) & 0 & 0 \\ l(3,1) & l(3,2) & l(1,1) & l(1,2) \\ l(4,1) & l(4,2) & l(2,1) & l(2,2) \\ l(5,1) & l(5,2) & l(3,1) & l(3,2) \\ l(6,1) & l(6,2) & l(4,1) & l(4,2) \end{bmatrix} \begin{bmatrix} \Delta v_1(k) \\ \Delta v_2(k) \\ \Delta v_1(k+1) \\ \Delta v_2(k+1) \end{bmatrix} = \\ &= \begin{bmatrix} L(1,1) & 0 \\ L(2,1) & L(1,1) \\ L(3,1) & L(2,1) \end{bmatrix} \begin{bmatrix} \Delta v(k) \\ \Delta v(k+1) \end{bmatrix} \\ L(4,1) &= \begin{bmatrix} l(7,1) & l(7,2) \\ l(8,1) & l(8,2) \end{bmatrix} = A_1 L(3,1) + A_2 L(2,1) + A_3 L(1,1) = \\ &= \begin{bmatrix} 1-a_1 & -a_3 \\ -a_5 & 1-a_7 \end{bmatrix} \begin{bmatrix} l(5,1) & l(5,2) \\ l(6,1) & l(6,2) \end{bmatrix} + \\ &+ \begin{bmatrix} a_1-a_2 & a_3-a_4 \\ a_5-a_6 & a_7-a_8 \end{bmatrix} \begin{bmatrix} l(3,1) & l(3,2) \\ l(4,1) & l(4,2) \end{bmatrix} \\ &+ \begin{bmatrix} 1-a_1 & -a_3 \\ -a_5 & 1-a_7 \end{bmatrix} \begin{bmatrix} l(1,1) & l(1,2) \\ l(2,1) & l(2,2) \end{bmatrix} \end{aligned} \quad (103)$$

5.4 Simulation Example

We will assume a measurable sinusoidal disturbance of known frequency, amplitude and phase. Sinusoids of angular frequencies $\omega_1 = 0,5$ and $\omega_2 = 1$ and amplitudes $A_1 = A_2 = 1$ were applied to the systems outputs according to equation (101). This type of disturbance can occur for example in electrical system where electromagnetic field of AC power lines is superimposed on electromagnetic field of control lines. The matrices of the coefficients D_1 and D_2 were simulated as follows

$$D_1 = \begin{bmatrix} 0,1 & 0,3 \\ 0,2 & 0,4 \end{bmatrix} \quad (105)$$

$$D_2 = \begin{bmatrix} -0,1 & 0,2 \\ 0,3 & 0,4 \end{bmatrix} \quad (106)$$

The objective function (1) was used for computation of control sequence. We considered an unconstrained case even though possibility of incorporation of constraints is very important in predictive control since one of the main advantages of predictive control is its ability to deal effectively with constraints. But the paper is focused on another part of predictive control: computation of predictions. So that the simulated control problem was simplified to be unconstrained. In this case computation of optimal control is a direct problem of linear algebra. Weighting factor λ was set to 0,01.

In Fig. 4 and Fig. 5 are time responses of control without the disturbance. In Fig. 6. and Fig. 7 are time responses of control with the disturbance when the prediction equations do not include information about the disturbance. In Fig. 8 and Fig. 9 are time responses with the disturbance when the prediction equations take into account the measurable disturbance. It is obvious that the influence of disturbance was rejected.

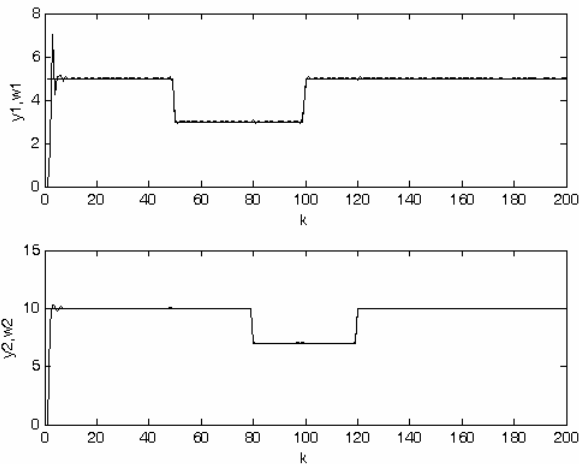


Fig. 4 Control without disturbance

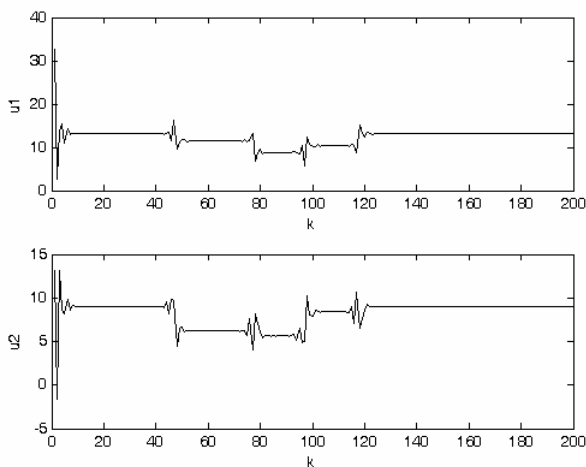


Fig. 5 Control without disturbance – manipulated variables

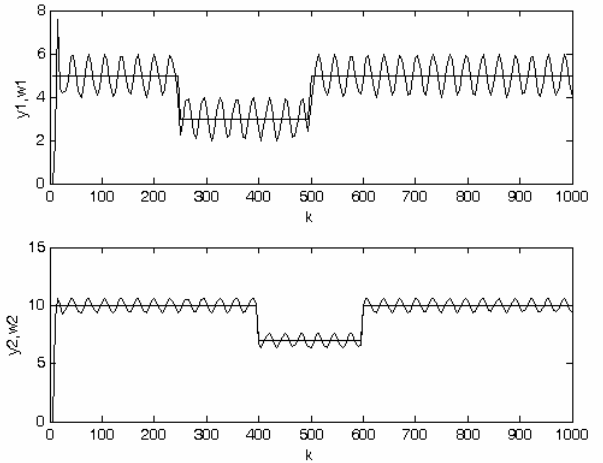


Fig. 6 Control with disturbance without disturbance rejection

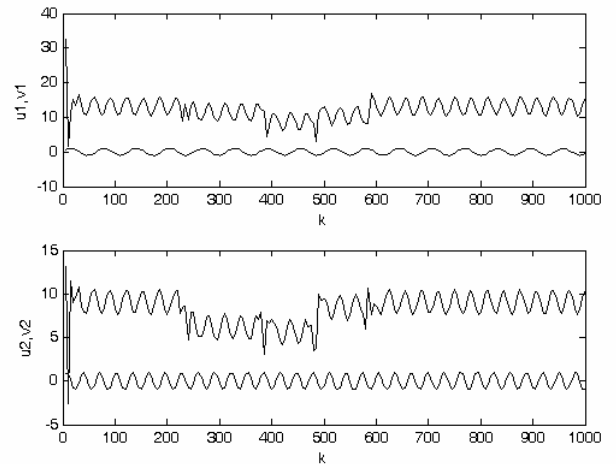


Fig. 7 Control with disturbance without disturbance rejection – manipulated variables and disturbances

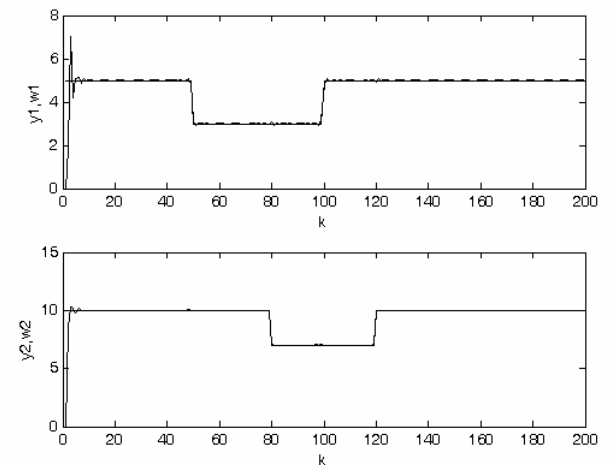


Fig. 8 Control with disturbance with disturbance rejection

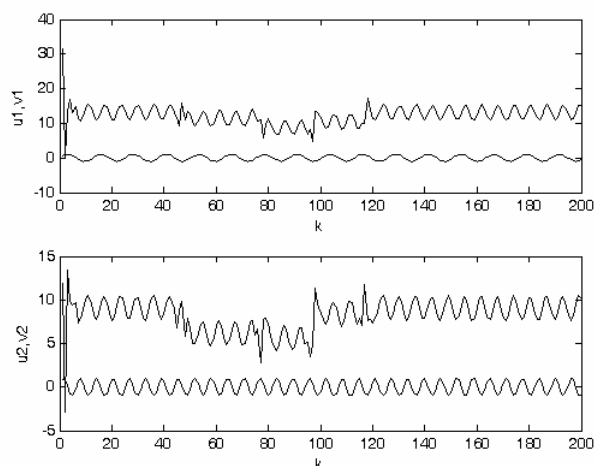


Fig. 9 Control with disturbance with disturbance rejection – manipulated variables and disturbances

6 Conclusion

Three methods of computation of multivariable systems output prediction were algorithmically realized. Correctness of all methods was verified by simulation. Simulation results proved applicability of the methods. The methods were evaluated from the point of view of computational time demands. From this evaluation results that the method based on direct computation of predictions from CARIMA model is significantly faster than the remaining two methods, where comparable results were achieved. If the methods are applied for predictive controllers based on models with fixed parameters then the computation of predictions is necessary to perform only once. In this case saving of computational time does not have significant importance. But in case that the predictive controller is realized as an adaptive predictive controller the computation of predictions is necessary to perform in each sampling period. Saving of computational time then has particular significance. It can also be important while using the prediction for other purposes than for the predictive control.

From the algorithmic point of view the method based on Diophantine equations seems to be less understandable and clear. The method of computation of predictions is not as straightforward as the remaining two methods, where computation is quite clear. A disadvantage of direct computation from CARIMA model is necessity of direct analytical computation of a certain number of predictions.

Computation of prediction equations using all three methods was extended by considering measurable disturbance. The measurable disturbance was included into prediction equations. Simulations proved that the predictive controller is

then able to cope with disturbance. Sinusoidal disturbances of known frequencies were successfully rejected.

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