Control of Periodically Time-Varying Systems with Delay: An Algebraic Approach vs. Modified Smith Predictors

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Abstract: - The paper deals with comparison of different continuous-time strategies applied to control of single-input single-output (SISO) periodically time-varying systems with delay. The first method is based on the fractional representation in the ring of proper and stable rational functions (R_{PS}), general solutions of Diophantine equations and conditions of divisibility while the other two methods use the modified Smith predictor structures in combination with standard forms for minimum of integral squared time error (ISTE) or design by Coefficient Diagram Method (CDM). The capabilities of all techniques are demonstrated on simulative examples for first and second order periodically time-varying time-delay systems. Moreover, various modifications and improvements, such as control structure with more degrees of freedom or utilization of disturbance controller are also included in the research.

Key-Words: - Time-Varying Systems, Time-Delay, Algebraic Synthesis, Modified Smith Predictor, PI-PD Control, Coefficient Diagram Method

1 Introduction

Plants affected by dead time have attracted attention of control theory researchers for decades. The reason of this interest can be seen in common presence of dead time in real controlled processes and hence in the necessity of quality and easily applicable control algorithms for this type of systems [1], [2], [3], [4]. Unfortunately, time delay always means worse control conditions and, furthermore, the situation is even much more complicated if it is time-varying.

The possible effective and economical solution for systems with relatively small or limited changes of parameters is the application of robust enough fixed-coefficient controllers. The worthwhile closed-loop configuration for compensation of time delay has been well known as Smith predictor since 1959. Recently, many new modifications of Smith predictor with improved properties have been introduced [5], [6], [7]. Another way how to overcome dead time lies in combination of its approximation and following utilization of an algebraic control design method. The advantageous solution represents fractional approach developed in [8], [9] and applied for robust control of time-delay systems e.g. in [10].

The main aim of this paper is to provide thorough analysis and comparison of methods which are applicable to control of SISO periodically time-varying systems with delay. The outputs given by continuous-time controller designed in R_{PS} under assumption of one degree of freedom (1DOF) and two degrees of freedom (2DOF) control loop configuration [10], [11], [12] are compared with those obtained with the use of the modified Smith predictor designed by [5] and also the modified PI-PD Smith predictor with or without presence of a disturbance controller [6]. Some preliminary results on this topic are presented in [13], [14].

The paper is organized as follows. In Section 2, basic description of systems with periodically time-varying parameters is provided. The Section 3 and Section 4 then contain the theoretical backgrounds for an algebraic approach to control design in R_{PS} and modified Smith predictors, respectively. Further, the specific controller calculations, simulative comparisons and analyses for first and second order periodically time-varying systems with delay are presented in an extensive Section 5. And finally, Section 6 offers some conclusion remarks.

2 Description of Systems with Periodically Time-Varying Parameters

Number of real natural and industrial processes has a time-varying behaviour. For the purpose of this paper, the controlled objects are modelled as SISO linear continuous-time dynamical systems with harmonically time-varying parameters and delay. The plants are described by differential equation:

$$\begin{aligned} & a_n(t)y^{(n)}(t)+a_{n-1}(t)y^{(n-1)}(t)+\cdots \\ & \cdots+a_1(t)y'(t)+a_0(t)y(t)= \\ & =b_m(t)u^{(m)}(t-\Theta(t))+b_{m-1}(t)u^{(m-1)}(t-\Theta(t))+\cdots \\ & \cdots+b_1(t)u'(t-\Theta(t))+b_0(t)u(t-\Theta(t)) \end{aligned} \tag{1}$$

with initial conditions:

$$\begin{aligned} & y(0)=y'(0)=\cdots=y^{(n-1)}=0 \\ & u(0)=u'(0)=\cdots=u^{(m-1)}=0 \end{aligned} \tag{2}$$

and $m<n$. The coefficients in (1) are periodically t-variant according to:

$$\begin{aligned} & b_m(t)=\beta_m+\lambda_{bm}\sin\left(\omega_{bm}t\right) \\ & a_n(t)=\alpha_n+\lambda_{an}\sin\left(\omega_{an}t\right) \\ & \Theta(t)=\theta+\lambda_\Theta\sin\left(\omega_\Theta t\right) \end{aligned} \tag{3}$$

where β_m,α_n,θ are real constants; $\lambda_{bm},\lambda_{an},\lambda_\Theta$ amplitudes and $\omega_{bm},\omega_{an},\omega_\Theta$ angular velocities. Obviously, the selection $\lambda_{bm}=\lambda_{an}=\lambda_\Theta=0$ or $\omega_{bm}=\omega_{an}=\omega_\Theta=0$ would represent time-invariant system.

However, a convenient non-standard hybrid "transfer functions", whose coefficients depend both on complex variable *s* and on time *t*, can be used for description of such systems instead of differential equations. The notation is than simplified to:

$$\begin{aligned} & G(s,t)=\frac{b_m(t)s^m+b_{m-1}(t)s^{m-1}+\cdots+b_1(t)+b_0(t)}{a_n(t)s^n+a_{n-1}(t)s^{n-1}+\ldots+a_1(t)++a_0(t)}e^{-\Theta(t)s} \\ & m<n \end{aligned} \tag{4}$$

again with parameters (3).

Just for example, the pair of controlled systems investigated in this paper has the differential equations and the corresponding hybrid "transfer functions" as follows:

$$\begin{aligned} & y'(t)+\left[0.2+0.02\sin\left(2t\right)\right]y(t)= \\ & =\left[1+0.1\sin t\right]u(t-\left[5+1.5\sin(3t)\right]); \\ & G(s,t)=\frac{1+0.1\sin t}{s+\left[0.2+0.02\sin\left(2t\right)\right]}e^{-(5+1.5\sin(3t))s} \end{aligned} \tag{5}$$

$$\begin{aligned} & y''(t)+\left[3+0.3\sin\left(5t\right)\right]y'(t)+\left[2+0.2\sin\left(6t\right)\right]y(t)= \\ & =\left[2+0.2\sin(3t)\right]u(t-\left[1+0.5\sin(4t)\right]); \\ & G(s,t)=\frac{2+0.2\sin(3t)}{s^2+\left[3+0.3\sin\left(5t\right)\right]s+\left[2+0.2\sin\left(6t\right)\right]}e^{-(1+0.5\sin(4t))s} \end{aligned} \tag{6}$$

3 An Algebraic Approach to Control Design in R_{PS}

The key idea of the proposed control design method consists in determining the stabilizing controller in accordance with demanded properties of the control loop for a fixed nominal system and subsequently in the application of obtained regulator to a perturbed time-variant plant. The asymptotic tracking of the reference value, disturbance rejection or disturbance attenuation belong among the most commonly required features. The applied fractional approach developed in [8] and [9] and discussed in [11], [12], [15] enables relatively deep insight into control tuning and a more elegant expression of all suitable controllers. This synthesis supposes the description of linear systems in R_{PS} as a ratio of two rational fractions which is bounded with conventional transfer function by transformation:

$$\begin{aligned} & G(s)=\frac{b(s)}{a(s)}=\frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}}=\frac{B(s)}{A(s)} \\ & n=\max\left\{\deg(a),\deg(b)\right\},\ m>0 \end{aligned} \tag{7}$$

The scalar positive parameter $m>0$ which enters into the synthesis process can be later conveniently used as a "tuning knob" influencing final control behavior.

A general feedback system is shown in Fig. 1. It should be emphasized that all functions and signals depicted in this figure are considered to belong to R_{PS}.

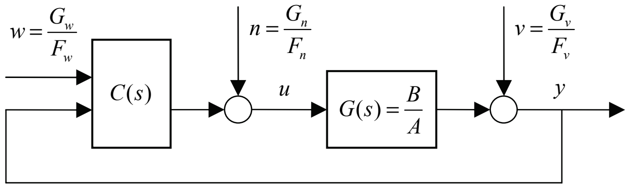

Fig. 1: General Feedback Control System

The circuit can have separated feedback $C_b(s)=\frac{Q(s)}{P(s)}$ and feedforward $C_f(s)=\frac{R(s)}{P(s)}$ part. In such case one speaks about control systems with two degrees of freedom (2DOF, FBFW) and, assuming zero disturbances $(n = v = 0)$, control signal u is generated according to the law:

$$u = \begin{pmatrix} C_f & C_b \end{pmatrix} \begin{pmatrix} w \\ -y \end{pmatrix} = C_f w - C_b y \tag{8}$$

If $C(s) = C_b(s) = C_f(s) = \dfrac{Q(s)}{P(s)}$ then Fig. 1 represents conventional control system with one degree of freedom (1DOF, FB). This loop works with tracking error e in compliance with:

$$u = C_b(w - y) = C_b e \tag{9}$$

Signals w, n, v represent reference value, load disturbance in the input and disturbance in the output of the control plant, respectively. Usually, w and n are considered as step signals and disturbance v is modelled to have a harmonic shape. Hence, the denominators of these signals in R_{PS} are:

$$F_w = F_n = \frac{s}{s+m}; \quad F_v = \frac{s^2 + \omega^2}{(s+m)^2} \tag{10}$$

where ω is angular velocity and $m > 0$.

The first and definitely the most important requirement is to ensure the stability of control loop from Fig. 1. Stabilizing controllers are given by ratio:

$$\frac{Q}{P} = \frac{Q_0 - AT}{P_0 + BT} \tag{11}$$

where T is free in R_{PS}, $P_0 + BT \neq 0$ and P_0, Q_0 is a particular solution of Diophantine equation:

$$AP + BQ = 1 \tag{12}$$

The formula (11) says that there exists either infinite amount of stabilizing controllers or none and it is called (Bongiorno-)Youla-Kučera parameterization of controllers.

Another important property is the convergence of tracking error e to zero. Working on an assumption that no disturbances affect the control system in Fig. 1 $(n = v = 0)$ it follows for circuits given by (9) and (8), respectively:

$$e = \frac{AP}{AP + BQ} \frac{G_w}{F_w} \tag{13}$$

$$e = \left(1 - \frac{BR}{AP + BQ}\right) \frac{G_w}{F_w} \tag{14}$$

Algebraic analysis of (13), (14) and substitution of (12) to (13), (14) results in fact that for zero tracking error:

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} \left[s \cdot e(s) \right] = 0 \tag{15}$$

the expression F_w must disappear from denominators of (13), (14). Therefore F_w must divide product AP for structure 1DOF (9), possibly F_w must divide $(1 - BR)$ for structure 2DOF (8). The second condition implies another Diophantine equation:

$$F_w Z + BR = 1 \tag{16}$$

Utilizing this technique, the controller can be designed also for rejection of disturbances n and v. The situation during synthesis is similar, only slightly more complicated [12].

An illustration of the practical controller computation is shown in the Section 5.

4 Modified Smith Predictors

The Smith predictor is a popular, well-known and relatively effective structure for dead time compensation. The main advantage of the Smith predictor is that the time delay term is eliminated from the characteristic equation of the closed-loop system. Typically a PI or PID controller is used. However, the Smith predictor fails for unstable or integrating processes under presence of disturbance and it is very sensitive to modelling errors. Some of classical Smith predictor drawbacks have been reduced by its modifications [5], [6], [7], [16], which have brought the improvement setpoint and disturbance responses for many situations. For the purpose of this paper, two modifications have been selected to be compared with the algebraic control design. This section provides only the very basic theoretical background with references. The examples of controller calculations are given in the following Section 5.

4.1 Modified Smith Predictor Design by CDM

The controller design using the Coefficient Diagram Method (CDM) was proposed in [5]. This method uses the improved Smith predictor structure with the trio of controllers according to Fig. 2. Furthermore, w, n, y denote reference signal, disturbance in the input of the controlled plant, and output signal, respectively.

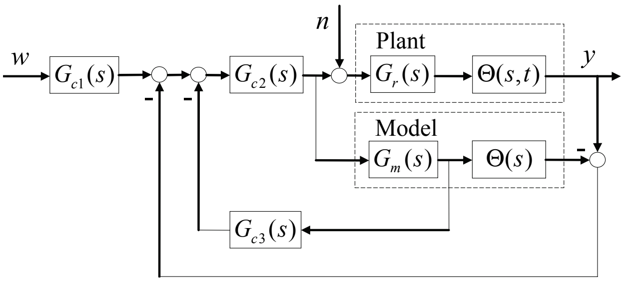

Fig. 2: The Modified Smith Predictor Structure (CDM)

The CDM design is based on the four studies:

- Coefficient diagram: It is a semilogarithmic diagram which allows investigating the stability and response of systems in a single graph. The vertical axis logarithmically shows the coefficients of characteristic polynomial, stability indices, stability limits and equivalent time constant while the horizontal axis represents the order values corresponding to each of coefficients.
- Modification of Kessler standard form: The form developed by Kessler in 1960 has decreased the oscillations and overshoots compared to the original Graham's ITAE form. In this approach, a new form called "Standard Manabe Form" is used. This design should result in quite stable and robust responses with small settling time.
- Lipatov stability analysis: The effect of coefficient variations can not be seen clearly for higher order systems. The conditions for stability or instability of such systems, based on Lipatov's work, are included in CDM design technique.
- Obtaining characteristic polynomial: A method similar to pole placement is applied. However, the main difference is the Manabe form.

The reader interested in CDM can find the all necessary background and details of the procedure in [5], [17], [18] and related literature.

4.2 Modified PI-PD Smith Predictor for Control of Processes with Long Dead Time

The modification of the classical Smith predictor presented in [6] comes from the structure with three controllers shown in Fig. 3, where G_{c1} is a PI controller, G_{c2} is a PD (or only P where it is appropriate) controller and G_{c3} is the disturbance controller introduced in [16]. Again, w, n, y represent reference signal, disturbance in the input of the controlled plant, and output signal, respectively.

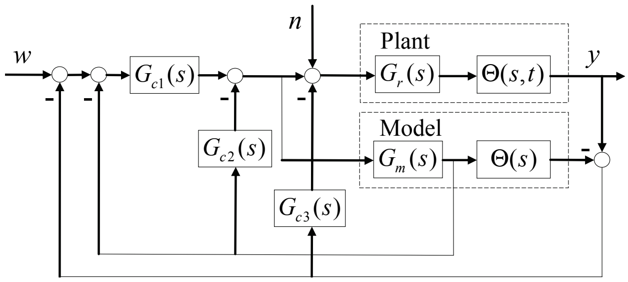

Fig. 3: The Modified Smith Predictor Structure (PI-PD)

The new structure, which replaces the conventional controller by a PI-PD structure, should outperform a PID controller in some common SISO systems. Generally, the synthesis is based on usage of standard forms for obtaining the optimal closed-loop transfer function parameters in the meaning of integral squared time error (ISTE) criterion, i.e. a simple algebraic approach to control system design is applied.

5 Design of Controllers and Simulative Comparison

5.1 Example 1 – First Order Time-Varying Plant with Delay

A controlled plant is given by:

$$y'(t) + \left[0.2 + 0.02\sin\left(2t\right)\right]y(t) = \\ = \left[1 + 0.1\sin t\right]u(t - \left[5 + 1.5\sin(3t)\right]); \qquad (17)$$

$$G(s,t) = \frac{1 + 0.1\sin t}{s + \left[0.2 + 0.02\sin\left(2t\right)\right]} e^{-(5+1.5\sin(3t))s}$$

and its nominal mathematical model, for the control design purpose, is assumed as:

$$G(s) = \frac{1}{s + 0.2} e^{-5s} = \frac{5}{5s + 1} e^{-5s} \qquad (18)$$

The controllers have been tuned successively for all three compared methods (R_{PS}, CDM and PI-PD) in order to obtain visually acceptable results without or with only small overshot and short settling time. For the sake of better comparability, responses with the same time of reaching the reference value were chosen (about 14.6s in this case).

5.1.1 Control Design in R_{PS}

Regarding to an algebraic approach to control design in R_{PS}, the time-delay term from the nominal system (18) has to be approximated, for example using the first order Padé approximation:

$$G(s)=\frac{1}{s+0.2}e^{-5s}\approx\frac{-2.5s+1}{(s+0.2)(2.5s+1)}= =\frac{-s+0.4}{s^2+0.6s+0.08}=\frac{b_1s+b_0}{s^2+a_1s+a_0} \quad (19)$$

After transposition of all transfer functions in R_{PS} the basic "stabilizing" Diophantine equation (12) can be written in the form:

$$\frac{s^2+a_1s+a_0}{(s+m)^2}\frac{p_1s+p_0}{s+m}+\frac{b_1s+b_0}{(s+m)^2}\frac{q_1s+q_0}{s+m}=1 \quad (20)$$

Its particular solution is given by:

$$\begin{aligned} p_1 &= 1 \\ p_0 &= \frac{3m^2b_0b_1-a_0b_0b_1-3mb_0^2+a_1b_0^2-b_1^2m^3}{a_1b_0b_1-b_0^2-a_0b_1^2} \\ q_1 &= \frac{3m-a_1-p_0}{b_1} \\ q_0 &= \frac{m^3-a_0p_0}{b_0} \end{aligned} \quad (21)$$

Consequently, all stabilizing controllers can be obtained with the assistance of Youla-Kučera parameterization (11):

$$\begin{aligned} P &= P_0+BT=\frac{s+p_0}{s+m}+\frac{b_1s+b_0}{(s+m)^2}T \\ Q &= Q_0-AT=\frac{q_1s+q_0}{s+m}-\frac{s^2+a_1s+a_0}{(s+m)^2}T \end{aligned} \quad (22)$$

where T is an arbitrary member of R_{PS}. Supposing the step changes in reference signal (and thus $F_w=\frac{s}{s+m}$), it is now necessary to choose such controller from the set (22) in order to F_w divides AP. Hence, it has to be found $T=t_0$ so that term s can be separated from the numerator of P. After simple adjustment it follows that complying t_0 is the one and only, i.e. $t_0=-\frac{p_0m}{b_0}$. By its substitution into (22) the numerator and denominator of the controller, which will not only stabilize the controlled plant in closed-loop system but it will also guarantee the asymptotic tracking of the reference signal, are obtained:

$$\begin{aligned} P &= \frac{s^2+s\left(p_0+m-p_0m\frac{b_1}{b_0}\right)}{(s+m)^2}=\frac{s^2+\tilde{p}_1s}{(s+m)^2} \\ Q &= \frac{s^2\left(q_1+\frac{p_0m}{b_0}\right)+s\left(q_0+q_1m+a_1\frac{p_0m}{b_0}\right)+a_0\frac{p_0m}{b_0}+q_0m}{(s+m)^2}= \\ &= \frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{(s+m)^2} \end{aligned} \quad (23)$$

As can be seen, the final feedback controller of PID type is described by transfer function:

$$\frac{Q}{P}=\frac{\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{s^2+\tilde{p}_1s} \quad (24)$$

with parameters:

$$\begin{aligned} \tilde{p}_1 &= p_0+m-p_0m\frac{b_1}{b_0} \\ \tilde{q}_2 &= q_1+\frac{p_0m}{b_0} \\ \tilde{q}_1 &= q_0+q_1m+a_1\frac{p_0m}{b_0} \\ \tilde{q}_0 &= q_0m+a_0\frac{p_0m}{b_0} \end{aligned} \quad (25)$$

It is obvious that all controller coefficients are generally nonlinear functions of parameter $m>0$.

If 2DOF control structure described by (8) is considered, it is necessary to solve one more Diophantine equation (16), here in the concrete form:

$$\frac{s}{s+m}\frac{z_1s+z_0}{s+m}+\frac{b_1s+b_0}{(s+m)^2}r_0=1 \quad (26)$$

with the useful term of particular solution $r_0=\frac{m^2}{b_0}$ and with general solution $R=r_0+\frac{s}{s+m}\tilde{T}$, where $\tilde{T}$

is again free in R_{PS} (e.g. $\tilde{T}=0$). It leads to the feedforward part:

$$\frac{R}{P}=\frac{r_0}{\frac{s^2+\tilde{p}_1 s}{(s+m)^2}}=\frac{r_0 s^2+2r_0 ms+r_0 m^2}{s^2+\tilde{p}_1 s}=\frac{\tilde{r}_2 s^2+\tilde{r}_1 s+\tilde{r}_0}{s^2+\tilde{p}_1 s} \quad (27)$$

Finally, the calculation of 1DOF regulator using the tuning parameter $m=0.301$ for the system (19) gives the transfer function:

$$\frac{Q}{P}=\frac{0.2537s^2+0.1525s+0.0205}{s^2+0.8577s} \quad (28)$$

and for the case of 2DOF structure of control system and $m=0.335$ it follows:

$$\begin{aligned}\frac{Q}{P}&=\frac{0.38s^2+0.2307s+0.0315}{s^2+1.12s}\\ \frac{R}{P}&=\frac{0.2806s^2+0.188s+0.0315}{s^2+1.12s}\end{aligned} \quad (29)$$

5.1.2 Modified Smith Predictor Design by CDM

In CDM as the second method the settling time was preset to $t_s=8.07s$ and disturbance rejection structure was selected. Obviously, the transfer function of controlled system model (without TD) is assumed in the form:

$$G_m(s)=\frac{K}{Ts+1}=\frac{5}{5s+1} \quad (30)$$

The trio of resulting controllers is:

$$\begin{aligned}G_{c1}(s)&=1\\ G_{c2}(s)&=\frac{1}{l_1 s}=\frac{1}{5.6156s}\\ G_{c3}(s)&=k_1 s+1=2.6237s+1\end{aligned} \quad (31)$$

The coefficients of these regulators follow from:

$$l_1=\frac{K\tau^2}{2.5T} \quad (32)$$

$$k_1=\tau-\frac{\tau^2}{2.5T} \quad (33)$$

where

$$\tau=t_s/2.1538 \quad (34)$$

5.1.3 Modified PI-PD Smith Predictor

Again, the same controlled system model (without TD) has been supposed, but now in the different mathematical form:

$$G_m(s)=\frac{\beta_0}{s+\alpha_0}=\frac{1}{s+0.2} \quad (35)$$

The transfer functions of the three controllers are:

$$\begin{aligned}G_{c1}(s)&=K_c\left(1+\frac{1}{T_i s}\right)=0.01\left(1+\frac{1}{0.099s}\right)\\ G_{c2}(s)&=K_f=0.2147\\ G_{c3}(s)&=K_o=0\end{aligned} \quad (36)$$

The parameters K_c and T_i have been adjusted by user, while K_f follows from equations:

$$\alpha=\sqrt{\frac{\beta_0 K_c}{T_i}}=0.3178 \quad (37)$$

$$c_1=\alpha T_i=0.03146 \Rightarrow d_1=1.3364 \quad (38)$$

$$K_f=\frac{d_1\alpha-\alpha_0-K_c\beta_0}{\beta_0} \quad (39)$$

The size of d_1 in (38) must be determined on the basis of c_1 according to graph from [6]. For the purpose of this paper, the graphical relation has been approximated by the sixth order polynomial:

$$\begin{aligned}d_1=&-0.0028c_1^6+0.0376c_1^5-0.1766c_1^4+\\ &+0.3076c_1^3+0.0502c_1^2+0.1533c_1+1.3315\end{aligned} \quad (40)$$

Furthermore, the higher value of disturbance controller $G_{c3}(s)=K_o$ generally ensures better disturbance rejection, however there is a trade-off between this rejection and oscillations of the control and output signal. Thus, not only the zero size of K_o as given in (36) has been used for simulation, but also the second option $K_o=0.1$ has been utilized.

5.1.4 Simulative Comparison of the Methods

The controllers designed via all three described methods (R_{PS}, CDM and PI-PD) have been applied during simulative control. Moreover, the control design in R_{PS} has been performed both for 1DOF and 2DOF while the modified PI-PD Smith predictor has been tested using two different values of disturbance controller ($K_o = 0$ and $K_o = 0.1$). First, it has been controlled the nominal mathematical model with fixed parameters (18) and then the time-varying system (17). Thus, it has led to 5 control simulations for nominal and other 5 simulations for perturbed plant. Furthermore, the following common simulation conditions were used: simulation time $T_S = 150s$, reference value 1 with step to 2 in $1/3$ of T_S, load disturbance injected into the plant input $n = -0.1$ in $2/3$ of T_S, and zero disturbance v in the plant output. The results for nominal case are visualized in Fig. 4 and outputs for time-varying system in Fig. 5.

Besides, two integral criteria have been used to evaluate the quality of control. First, it has been an Integral Squared Error (ISE) computed according to:

$$\text{ISE} = \int_0^\infty e^2(t)dt \tag{41}$$

and the second evaluation has been based on Integral Time Squared Error (ITSE) which corresponds to:

$$\text{ITSE} = \int_0^\infty te^2(t)dt \tag{42}$$

The values of ISE and ITSE criteria for fixed nominal and time-varying system are given in Tab. 1 and Tab. 2, respectively.

Table 1: Outcomes of ISE and ITSE calculations for the nominal system (18)

Method	ISE	ITSE
R_{PS} 1DOF	14.93	511.3
R_{PS} 2DOF	14.70	490.4
CDM	16.35	566.1
PI-PD $K_o = 0$	17.58	617.8
PI-PD $K_o = 0.1$	17.26	580.0

Table 2: Outcomes of ISE and ITSE calculations for the time-varying system (17)

Method	ISE	ITSE
R_{PS} 1DOF	14.88	514.2
R_{PS} 2DOF	14.65	492.8
CDM	16.31	570.5
PI-PD $K_o = 0$	17.58	623.4
PI-PD $K_o = 0.1$	17.27	584.9

As can be seen, all methods are able to control given time-varying delay system relatively acceptable. Control design in R_{PS} in combination with 2DOF configuration gives the best responses from the integral criteria point of view, on the top of that without overshot. This technique provides moreover very good rejection of load disturbance. The cost for it is a bit more aggressive control signal. The 1DOF configuration is only a little worse then 2DOF, but it has to cope with the plant using the simpler control system structure. The CDM methodology has taken the third place and it represents a competitive option from the area of modified Smith predictors. The interesting alternative specialized on disturbance rejection is modified PI-PD Smith predictor with "some non-zero" value of disturbance controller (here it has been $K_o = 0.1$). However, as it was already mentioned, the fast disturbance rejection is bought out by greater oscillations in both output and control signal compared to the version with $K_o = 0$ – see especially Fig. 5.

5.2 Example 2 – Second Order Time-Varying Plant with Delay

In this case a controlled system is given by:

$$\begin{aligned} & y''(t) + \left[3 + 0.3\sin(5t)\right]y'(t) + \left[2 + 0.2\sin(6t)\right]y(t) = \\ & = [2 + 0.2\sin(3t)]u(t - [1 + 0.5\sin(4t)]); \\ & G(s,t) = \frac{2 + 0.2\sin(3t)}{s^2 + \left[3 + 0.3\sin(5t)\right]s + \left[2 + 0.2\sin(6t)\right]} e^{-(1+0.5\sin(4t))s} \end{aligned} \tag{43}$$

and its nominal mathematical model used in control design process is supposed as:

$$\begin{aligned} G(s) &= \frac{2}{s^2 + 3s + 2} e^{-s} = \frac{1}{(s+1)(0.5s+1)} e^{-s} = \\ &= \frac{1}{0.5s^2 + 1.5s + 1} e^{-s} \end{aligned} \tag{44}$$

Similarly to the previous example, the regulators have been tuned successively for all compared techniques. The time of reaching the reference signal was considered to be about 3.9s in all simulations to ensure comparability.

5.2.1 Control Design in R_{PS}

In the first step, the time-delay term has been again approximated using the first order Padé approximation because of suitable form of controlled system transfer function:

$$G(s)=\frac{2}{s^2+3s+2}e^{-s}\approx\frac{2(-0.5s+1)}{(s^2+3s+2)(0.5s+1)}= \\ =\frac{-2s+4}{s^3+5s^2+8s+4}=\frac{b_1s+b_0}{s^3+a_2s^2+a_1s+a_0} \quad (45)$$

The Diophantine equation (12) in R_{PS} now takes the form:

$$\frac{s^3+a_2s^2+a_1s+a_0}{(s+m)^3}\frac{p_2s^2+p_1s+p_0}{(s+m)^2}+ \\ +\frac{b_1s+b_0}{(s+m)^3}\frac{q_2s^2+q_1s+q_0}{(s+m)^2}=1 \quad (46)$$

The whole situation is analogical to the example 1, i.e. the particular solution of the equation (46) has to be computed and consequently, the set of stabilizing controller can be expressed using the Youla-Kučera parameterization:

$$P=P_0+BT=\frac{s^2+p_1s+p_0}{(s+m)^2}+\frac{b_1s+b_0}{(s+m)^3}T \\ Q=Q_0-AT=\frac{q_2s^2+q_1s+q_0}{(s+m)^2}-\frac{s^2+a_1s+a_0}{(s+m)^3}T \quad (47)$$

Assuming the step changes in reference signal ($F_w=\frac{s}{s+m}$) and application of analogical ideas from the previous example lead to exactly the same $T=t_0=-\frac{p_0m}{b_0}$ which ensures the asymptotic tracking. The final third order feedback controller for the tuning parameter $m=1.658$ is given by transfer function:

$$\frac{Q}{P}=\frac{\tilde{q}_3s^3+\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{s^3+\tilde{p}_2s^2+\tilde{p}_1s}= \\ =\frac{1.2832s^3+6.4329s^2+10.3295s+5.1933}{s^3+4.948s^2+11.0609s} \quad (48)$$

For 2DOF control structure the additional Diophantine equation (16):

$$\frac{s}{s+m}\frac{z_2s+z_1s+z_0}{(s+m)^2}+\frac{b_1s+b_0}{(s+m)^3}r_0=1 \quad (49)$$

has to be solved. Thus the tuning parameter $m=1.92$ results in 2DOF controllers:

$$\frac{Q}{P}=\frac{\tilde{q}_3s^3+\tilde{q}_2s^2+\tilde{q}_1s+\tilde{q}_0}{s^3+\tilde{p}_2s^2+\tilde{p}_1s}= \\ =\frac{3.03s^3+15.2509s^2+24.644s+12.5241}{s^3+6.52s^2+20.7559s} \\ \frac{R}{P}=\frac{\tilde{r}_3s^3+\tilde{r}_2s^2+\tilde{r}_1s+\tilde{r}_0}{s^3+\tilde{p}_2s^2+\tilde{p}_1s}= \\ =\frac{1.7695s^3+10.1922s^2+19.5689s+12.5241}{s^3+6.52s^2+20.7559s} \quad (50)$$

5.2.2 Modified Smith Predictor Design by CDM

The settling time for CDM was adjusted to $t_s=2.985s$ and disturbance rejection structure was kept. The model of controlled plant (without TD) has been considered as:

$$G_m(s)=\frac{K}{a_2s^2+a_1s+1}=\frac{1}{0.5s^2+1.5s+1} \quad (51)$$

These assumptions lead to the three controllers:

$$G_{c1}(s)=1 \\ G_{c2}(s)=\frac{1}{l_2s^2+l_1s}=\frac{1}{0.05903s^2+0.2488s} \\ G_{c3}(s)=k_2s^2+k_1s+1=0.3360s^2+1.1371s+1 \quad (52)$$

The controller parameters are:

$$l_2=\frac{K\tau^4}{125a_2} \quad (53)$$

$$l_1=\frac{\frac{K\tau^3}{12.5}-l_2a_1}{a_2} \quad (54)$$

$$k_2 = \frac{\frac{K\tau^2}{2.5} - l_1 a_1 - l_2}{K} \quad (55)$$

$$k_1 = \frac{K\tau - l_1}{K} \quad (56)$$

where τ is given again by (34).

5.1.3 Modified PI-PD Smith Predictor

A considered form of mathematical model for this instance is:

$$G_m(s) = \frac{\beta_0}{s^2 + \alpha_1 s + \alpha_0} = \frac{2}{s^2 + 3s + 2} \quad (57)$$

Regulators are given as:

$$\begin{aligned} G_{c1}(s) &= K_c\left(1 + \frac{1}{T_i s}\right) = 0.05\left(1 + \frac{1}{0.0609s}\right) \\ G_{c2}(s) &= T_d s + K_f = -0.6251s + 0.3683 \\ G_{c3}(s) &= K_o = 0 \end{aligned} \quad (58)$$

Again, the parameters K_c, T_i have been user-preset and T_d, K_f can be calculated from:

$$\alpha = \sqrt[3]{\frac{\beta_0 K_c}{T_i}} = 1.1798 \quad (59)$$

$$c_1 = \alpha T_i = 0.07185 \Rightarrow \begin{matrix} d_1 = 2.0380 \\ d_2 = 1.4833 \end{matrix} \quad (60)$$

$$T_d = \frac{d_2\alpha - \alpha_1}{\beta_0} \quad (61)$$

$$K_f = \frac{d_1\alpha^2 - \alpha_0 - K_c\beta_0}{\beta_0} \quad (62)$$

The paper [6] offers the diagram depicting the dependence of d_1 and d_2 on c_1. Here, the graphs have been replaced by polynomial relations:

$$\begin{aligned} d_1 &= -0.0049c_1^6 + 0.0693c_1^5 - 0.3613c_1^4 + \\ &+ 0.8238c_1^3 - 0.6236c_1^2 + 0.3832c_1 + 2.0134 \end{aligned} \quad (63)$$

$$\begin{aligned} d_2 &= 0.0008c_1^6 - 0.015c_1^5 + 0.1022c_1^4 - \\ &- 0.3365c_1^3 + 0.531c_1^2 + 0.0396c_1 + 1.4778 \end{aligned} \quad (64)$$

And finally, the other higher value of disturbance controller $G_{c3}(s) = K_o = 0.35$ has been tested as in the previous instance.

5.1.4 Simulative Comparison of the Methods

Again, 5 control simulations for nominal and other 5 simulations for perturbed plant are presented under following simulation conditions: simulation time $T_S = 45s$, reference value 1 with step to 2 in $1/3$ of T_S, load disturbance injected into the plant input $n = -0.5$ in $2/3$ of T_S, and zero disturbance v in the plant output. The Fig. 6 shows the outputs for nominal case while curves for time-varying plant are in Fig. 7. The results of ISE and ITSE criteria are provided in Tab. 3 and Tab. 4.

Table 3: Outcomes of ISE and ITSE calculations for the nominal system (44)

Method	ISE	ITSE
R_{PS} 1DOF	3.777	38.28
R_{PS} 2DOF	3.567	34.57
CDM	4.543	47.24
PI-PD $K_o = 0$	4.898	51.69
PI-PD $K_o = 0.35$	4.815	48.82

Table 4: Outcomes of ISE and ITSE calculations for the time-varying system (43)

Method	ISE	ITSE
R_{PS} 1DOF	3.776	39.51
R_{PS} 2DOF	3.551	35.79
CDM	4.605	48.28
PI-PD $K_o = 0$	4.975	53.63
PI-PD $K_o = 0.35$	4.889	50.84

The successfulness confrontation of individual methods is practically the same as for the first order plant.

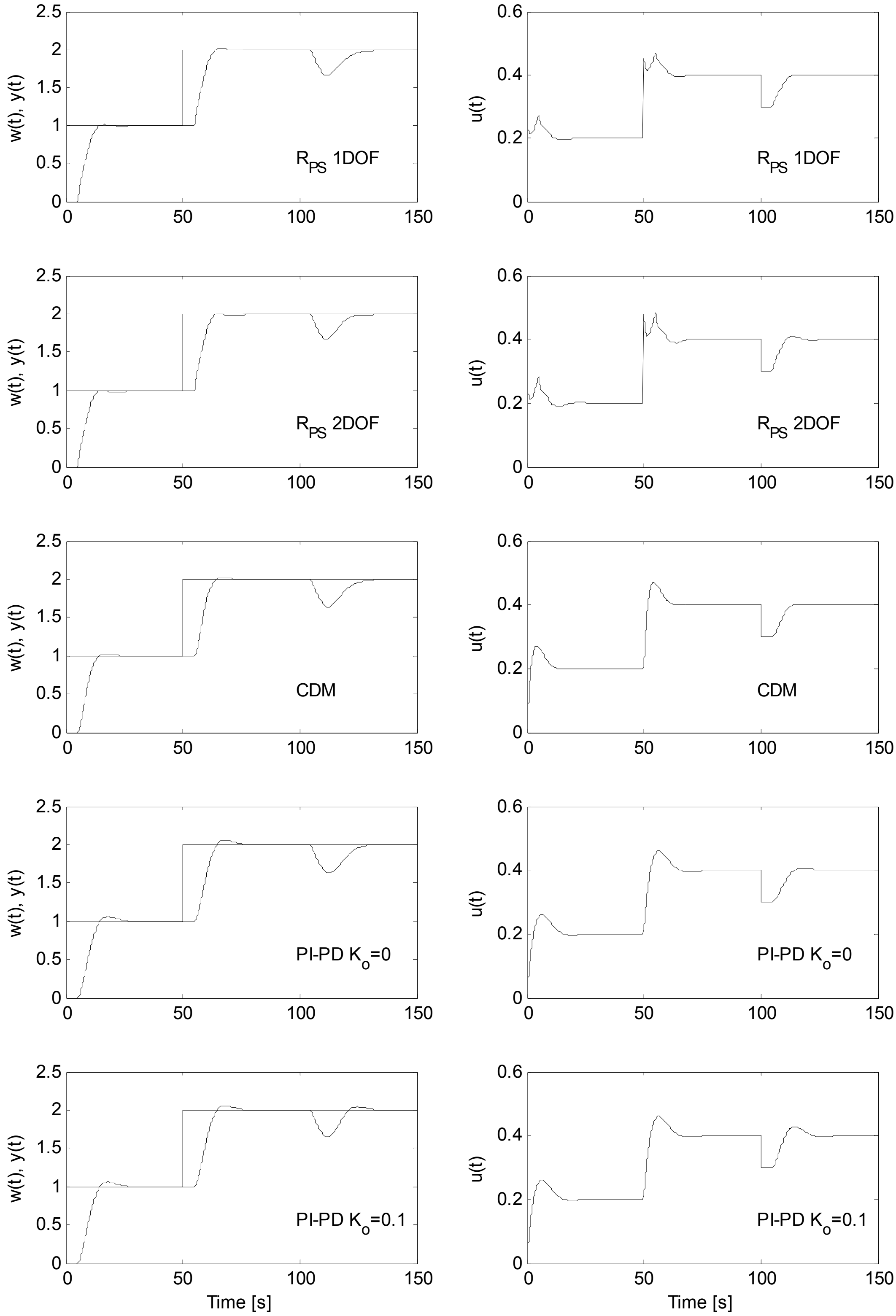

Fig. 4: Output and control signals for the nominal system (18)

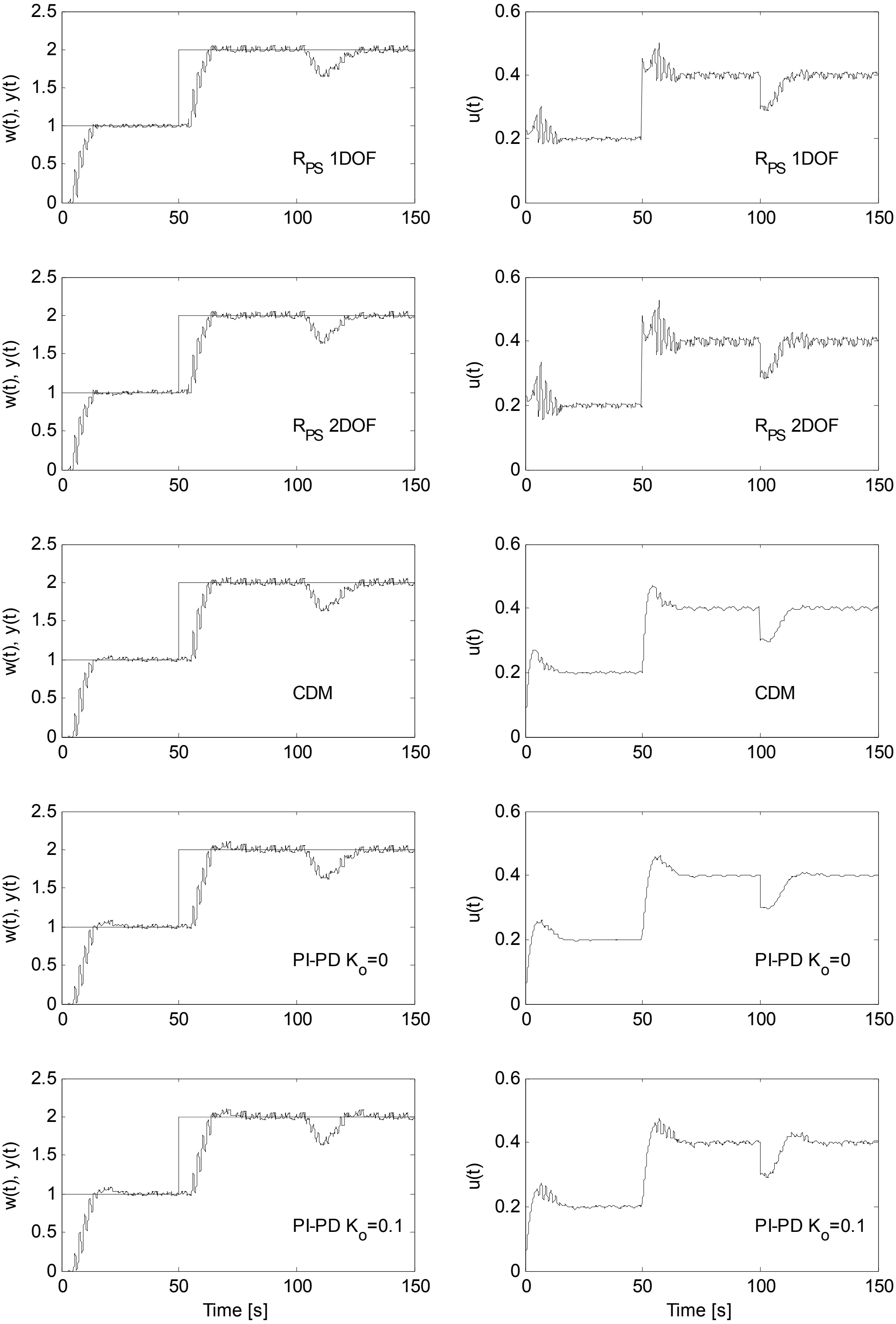

Fig. 5: Output and control signals for the time-varying system (17)

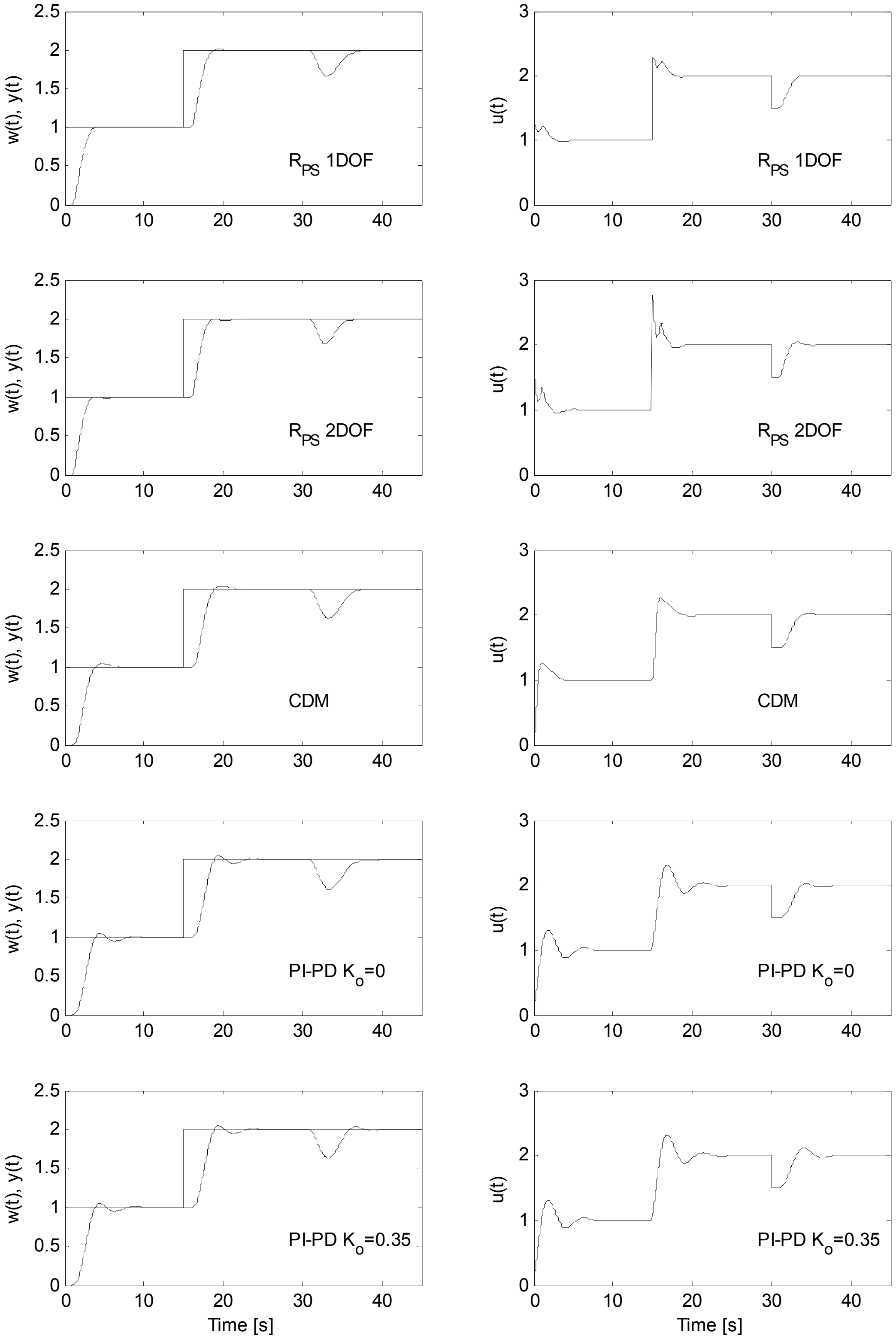

Fig. 6: Output and control signals for the nominal system (44)

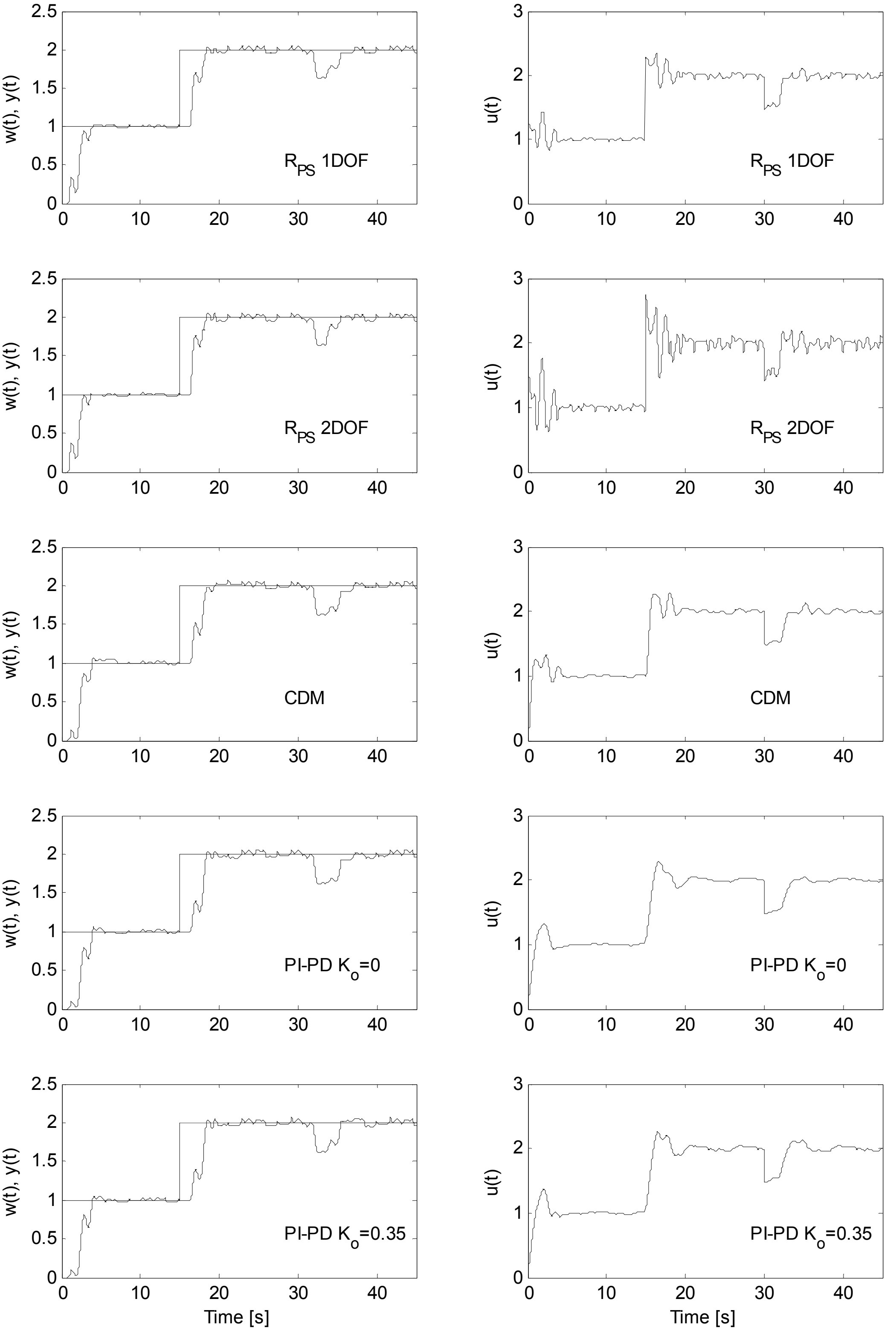

Fig. 7: Output and control signals for the time-varying system (43)

6 Conclusion

The paper has been focused on comparison of three different continuous-time strategies during control of SISO periodically time-varying systems with delay. Application of all analyzed techniques for this category of systems consists in the idea of robustness. The first method is based on the fractional representation in R_{PS}, general solutions of Diophantine equations and conditions of divisibility while the other two methods use the modified Smith predictor structures in combination with standard forms for minimum of ISTE or design by CDM, respectively. The simulation examples from Matlab + Simulink environment for first and second order periodically time-varying systems with delay have shown that all methods are able to control given plants relatively acceptable. The best results were obtained for R_{PS} methodology with 2DOF control loop configuration. Furthermore, disadvantages of both modifications of Smith predictor are more complicated control loop structure and necessity of TD model in the inner loop. Hence, one can claim that the proposed control design in R_{PS} can be considered as an effective method for studied class of systems.

Acknowledgements:
The work was supported by the Ministry of Education, Youth and Sports of the Czech Republic under Research Plan No. MSM 7088352102.

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