On the Infinite-Dimensional Model Multiple-Parameter Estimation using Feedback-Relay Identification Test

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Abstract. The objective of this contribution is twofold. First, it demonstrates a case study on applying the standard single-run relay-feedback parameter identification test to a representative of infinite-dimensional systems. Namely, a delayed mathematical model of a circuit heating laboratory appliance process is used. Second, an initial estimation of the model parameters is done via the parameter identification of another – simpler – model. The transition between these two models adopts the idea of dominant spectrum assignment that is solved by using the well-established Levenberg-Marquardt algorithm. Finally, the remaining model parameters are estimated by solving another nonlinear optimization problem in the frequency domains. As transfer function denominator parameters are set independently to the numerator ones, the proposed technique significantly reduces the number of additional relay experiments. Numerical results indicate that the method needs improvements regarding time-response as well as frequency-response accuracy.

Keywords: Relay-based identification, Infinite-dimensional model, Initial estimation, Pole assignment, Levenberg-Marquardt algorithm.

1 Introduction

One of the very popular frameworks for process model parameters identification is based on the closed-loop experiment with a relay placed instead of the controller [1]. It was implemented in practice, especially in (but not limited to) the process and chemical industry [2]. This original test used the on/off (or ideal) relay with two-level symmetrical output without hysteresis. As the experiment output, one (ultimate) point of the process model frequency response is estimated via the so-called describing function (DF), which gives rise to the determination of only two unknown parameters. Hence, dozens of enhanced techniques and methods have been developed [3] since the groundbreaking work. For instance, time-domain [4] and frequency-domain [5] data evaluation have been investigated, various relay types have been applied [6, 7], multiple tests have enabled to provide more information about the process dynamics [8], or even these extended data have been extracted from the single test [9].

Infinite-dimensional models (IDMs) are characterized by the infinite number of impulse-response modes, or equivalently, by the infinite number of state-matrix eigenvalues that correspond to the solution of the characteristic function. Let us call these eigenvalues poles for simplicity. IDMs can inherently describe dynamics of complex high-order processes, even if the model dynamics is of a low order (measured by order of derivatives). Time-delay models (TDMs) constitute typical representatives of the IDMs family [10]. They comprise delays in dynamics (i.e., the so-called internal delay) besides the habitual delay in the input-output (IO) relation. Unfortunately, according to the authors' best knowledge, there have been published only a limited number of results on relay-based identification of TDMs. A simple four-parameter model (SFPM) was identified using the standard relay and a priori knowledge of the IO delay [11]. The Fourier transform and a relay with saturation [7] were applied in [12] to identify parameters of a five-parameter TDM representing a simplified model of a circuit heating laboratory process. However, a more precise heating process model (HPM) based on the use of heat and mass balance equations included nine parameters [13] (in a selected single-input single-output relation). The method of moments applied to the series of the SFPM and a high-order finite-dimensional non-periodic subsystem can be found in [14].

Although the SFPM is assumed to be a "universal" model, it cannot estimate a complex system dynamics. However, one can take it as the first-attempt approach to modeling processes with unknown dynamics. Its advantage is all of its four parameters can be detected under a single relay experiment. Two parameters can be obtained from the ultimate cycle data and the corresponding DF, while two remaining ones can be read from the shape of the limit cycles and its numerical integration (or from the process step response). Contrariwise, the HPM or another multi-parameter TDM requires either a multiple relay test (which is time-consumptive) or to solve a computationally demanding nonlinear optimization problem that usually arises from an advanced single-relay test.

This contribution is primarily focused on applying the standard relay test to the nine-parameter HPM under a single run. This goal can be achieved via a priori performance of identifying the SFPM, the dynamics of which is given by its poles. These poles are then attempted to be identical to the poles of the HPM, which yields the initial guess of the characteristic function parameters. To solve this partial task, the Levenberg-Marquardt algorithm (LMA) is used. By adopting the IO delay value estimation technique and the computation of the steady state from the relay test, there are only two remaining HPM parameters that do not depend on model poles. Hence, these remaining parameters are then estimated via the standard test using the knowledge of the DF. This last task can be formulated using a relatively simple nonlinear problem in the frequency domain.

The rest of the paper is organized as follows. Section 2 introduces the problem in question in more detail and provides the reader with the HPM and the standard relay identification test via the DF. In Section 3, the SFPM and its basic spectral properties are given first. Then, the LMA – as the crucial numerical tool – is described. The proposed method for the identification of the HPM using the SFPM and the DF is eventually summarized, and its algorithm is provided. A numerical example that performs all the steps of the proposed method is presented in Section 4. A comparison of the actual and estimated HPM is given in the time domain and the frequency domain. Finally, section 5 concludes this study.

2 Problem Statement

The framework problem is to apply the standard feedback relay identification test on the nine-parameter HPM under a single run. The estimation of the characteristic quasipolynomial parameters constitutes a particular research question that serves to solve the given problem. The characteristic function solutions are simply the characteristic quasipolynomial zeros, i.e., system poles. This initial guess enables to determine the remaining HPM parameters with a lower effort. Hence, the introduction of HPM and the standard relay [1] test follows.

2.1 Infinite-Dimensional Model of the Thermal Process

The HPM, i.e., a TDM of the laboratory circuit heating process, derived via heat and mass balance laws when considering all the significant process delays, have been published in [13]. A photo of the laboratory appliance is displayed in Fig. 1. The meaning of numbered positions follows.

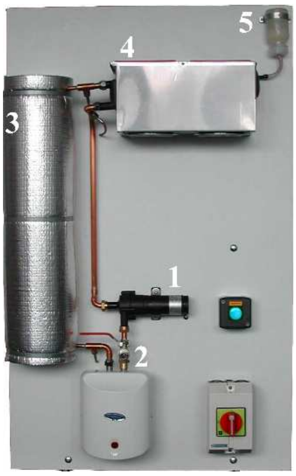

Fig. 1. A photo of the laboratory circuit heating appliance (numbered positions are described in the text body).

The distilled water is driven via a continuously controllable pump {1} through a flow heater {2} to a long insulated coiled pipeline {3}. Then, the heating fluid enters a cooler (air-water plate-and-fin heat exchanger, radiator) {4} with two cooling fans and flows back to the pump. An expansion tank serving for the compensation of the water thermal expansion effect is placed at the top of the appliance {5}.

The manipulated inputs (or measurable disturbances if needed) are the pump input voltage, the heater power, and the input voltage to cooler fans. The outputs are represented by measured outlet temperature from the heater, inlet temperature to the cooler, and the outlet temperature from the cooler.

The eventual model is quite complex, nonlinear, and multivariable. Let us consider the single-input single-output submodel where the heater power is taken as the model input $u(t)$, while the outlet temperature from the heater represents the output $y(t)$. The linearized relation between $u(t)$ and $y(t)$ (in a particular operating point) can be expressed by the delay-differential equation

$$\dddot{y}(t)+a_2\ddot{y}(t)+a_1\dot{y}(t)+a_0 y(t)+a_{0D} y(t-\vartheta)=b_0 u(t-\tau)+b_{0D} u(t-\tau-\tau_0) \quad (1)$$

where $a_0, a_{0D}, a_1, a_2, b_0, b_{0D}$ are real-valued non-delay parameters, ϑ, τ, τ_0 mean the internal (state) delay and delays in the IO relation, respectively, and the dot notation is used for the time derivative. Delay values are naturally non-negative.

Relation (1) is equivalent to the transfer function

$$G_{HPM}(s)=\frac{Y(s)}{U(s)}=\frac{B(s)}{A(s)}=\frac{b_0+b_{0D}\,\mathrm{e}^{-\tau_0 s}}{s^3+a_2 s^2+a_1 s+a_0+a_{0D}\,\mathrm{e}^{-\vartheta s}}\mathrm{e}^{-\tau s} \quad (2)$$

where s is the Laplace transform variable and the denominator quasipolynomial of (2) represents the characteristic quasipolynomial, the roots of which agree with system poles (characteristic values). The number of poles is infinite (i.e., the model is infinite-dimensional) whenever $\vartheta \neq 0$.

A complex two-step parameter identification procedure have been made in [13]. First, non-dynamic parameters have been determined from the measurement of suitable static characteristics. Second, remaining parameters depended on the system dynamics have been identified from step responses. The eventual parameters of (2) have been found as follows.

$$\begin{aligned}&b_0=-2.50\cdot 10^{-7}, b_{0D}=2.27\cdot 10^{-6}, a_0=1.30\cdot 10^{-4}, a_{0D}=-7.22\cdot 10^{-5}, a_1=8.51\cdot 10^{-3},\\&a_2=0.17, \tau=141, \tau_0=1.5, \vartheta=151\end{aligned} \quad (3)$$

Let us take result (3) are the actual (“true”) values for further comparisons.

2.2 Relay-Based Identification Using Describing Function

The relay identification test [1] s the feedback scheme depicted in Fig. 2. If the process is stabilizable by the nonlinear element (relay), the initially excited output reaches sustained oscillations of the constant ultimate amplitude A and period T_u, see Fig. 3 (according to [12]). Process input has the same period yet a different amplitude B. The overall phase lag between $e(t)$ and $y(t)$ is $-\pi$; however, the phase leg between $u(t)$ and $y(t)$ can be within the range $[-\pi, 0)$ (depending on the used relay element).

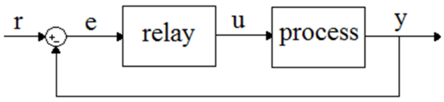

Fig. 2. Relay feedback test scheme.

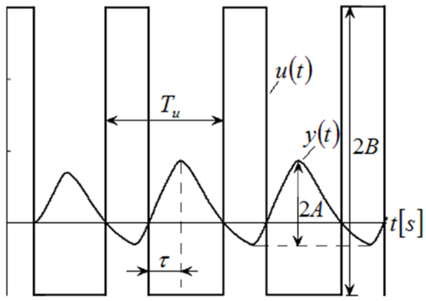

Fig. 3. Sustained oscillations data.

The static characteristics of the asymmetric on/off relay (without hysteresis) can be seen in Fig. 4. The ideal relay satisfies $B = B^{+} = B^{-}$.

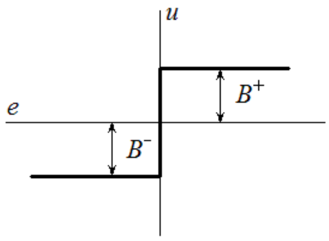

Fig. 4. Asymmetric relay static characteristics [12].

The concept of DF utilizes an approximation of the nonlinear relay behavior, usually via truncation of the Fourier series expansion [15]. The DF of the relay specified by Fig. 4 has the form [16]:

$$k_u = \frac{4B}{\pi A}\sqrt{1-\left(\frac{\delta}{A}\right)^2} \tag{4}$$

where $\delta = 0.5\left|B^+ - B^-\right|$. Hence, for the ideal relay, one has $\delta = 0$. It holds for the sustained oscillations that

$$k_u G(\mathrm{j}\omega_u) = -1 \Leftrightarrow \begin{cases} \left|k_u G(\mathrm{j}\omega_u)\right| = 1 \\ \measuredangle\left(k_u G(\mathrm{j}\omega_u)\right) = -\pi \end{cases} \tag{5}$$

where the ultimate frequency $\omega_u = 2\pi / T_u$, $G(\mathrm{j}\omega)$ stands for the process (model) frequency transfer function, and $\mathrm{j}^2 = -1$, which identifies the critical point -1 of the open-loop Nyquist plot.

Formula (5) can identify only two process-model parameters. However, the IO delay τ can be obtained graphically from Fig. 4. Besides, the model static gain $k = G(0)$ can be computed from

$$k = \frac{\int_t^{t+T_u} y(\xi)\,\mathrm{d}\xi}{\int_t^{t+T_u} u(\xi)\,\mathrm{d}\xi} \tag{6}$$

when using the asymmetric relay (i.e., $\delta \neq 0$), for a sufficiently high t when the oscillations are settled [7, 11]. It is assumed that the measured signal is not affected by dc components, such as static load disturbance. Note that the value of k can also be simply determined from the step response.

To sum up the problem in question, the goal is to use the ideal relay (4) to identify parameters of model (2) via the solution of the condition (5) under a single feedback experiment. Besides, the static gain and the IO delay are estimated using (5) and Fig. 4, respectively. Hence, there remain five parameters to be determined in the model, which gives rise to the main research question.

3 Proposed Technique

3.1 Simple Infinite-Dimensional Model

The leading idea of how to solve the given research question is to identify a simpler TDM (i.e., the SFPM) and then adopt its dynamics (spectrum) to the HPM (2). The SFPM is governed by the transfer function

$$G_{SFPM}(s) = \frac{b_{0,m}}{s + a_{0D,m}\,\mathrm{e}^{-\vartheta_m s}}\,\mathrm{e}^{-\tau_m s} \tag{7}$$

As the static gain of (7) (i.e., $k_m = b_{0,m} / a_{0D,m}$) and τ_m can be obtained as described in subsection 2.2, the remaining two parameters can be determined by the single relay test. When applying (5) on model (7), one gets [11]:

$$\vartheta_m = \frac{\pi - \cos^{-1}\left(k_u k_m \cos\left(\omega_u \tau_m\right)\right)}{\omega_u},\ a_{0D,m} = \frac{\omega_u}{\sin\left(\omega_u \vartheta_m\right) - \tan\left(\omega_u \tau_m\right)\cos\left(\omega_u \vartheta_m\right)} \qquad (8)$$

$$\sin\left(\omega_u \vartheta_m\right) - \tan\left(\omega_u \tau_m\right)\cos\left(\omega_u \vartheta_m\right) \neq 0$$

Remark 1. A disadvantage of the analytic solution (8) is that it inherently supposes that $k_u k_m \cos\left(\omega_u \tau_m\right) \in \left[-1,1\right]$. If this condition does not hold, formulae (8) cannot be used, and a numerical solution of (5) has to be used instead.

The model is asymptotically stable, i.e., its denominator has all its roots in the left-half Gauss plane, if and only if $0 < a_{0D,m}\vartheta_m < \pi / 2$. Moreover, the rightmost pole is real whenever $0 < a_{0D,m}\vartheta_m \leq 1/\mathrm{e}$. Otherwise, there is a complex conjugate pair of poles [8, 11, 17].

3.2 Levenberg-Marquardt Algorithm

The LMA can solve a set of n nonlinear algebraic equations by minimizing the corresponding residual sum of squares [18]. Namely, let the set be

$$\mathbf{f}\left(x_i, \mathbf{p}\right) = \mathbf{0} \qquad (9)$$

where $\mathbf{f} = \left(f_1, f_2, \ldots f_n\right)^T, \mathbf{0} = \left(0, 0, \ldots 0\right)^T$, x_i is a function variable value, and $\mathbf{p} \in \mathbb{R}^m$ stands the parameter set to be found. Superscript T denotes the matrix transpose. Then, the sum

$$\sum_{j=1}^{n}\left(f_j\left(x_i, \mathbf{p}\right)\right)^2 \qquad (10)$$

is minimized via the solution formula

$$\mathbf{p}^{(k+1)} = \mathbf{p}^{(k)} - \left(\left[\mathbf{J}\left(\mathbf{f}\left(x_i, \mathbf{p}^{(k)}\right)\right)\right]^T \mathbf{J}\left(\mathbf{f}\left(x_i, \mathbf{p}^{(k)}\right)\right) + \lambda^{(k)}\mathbf{I}\right)^{-1}\left[\mathbf{J}\left(\mathbf{f}\left(x_i, \mathbf{p}^{(k)}\right)\right)\right]^T \mathbf{f}\left(x_i, \mathbf{p}^{(k)}\right) \qquad (11)$$

where the superscript (k) indicates the iterative step, $\mathbf{J}$ stands for the Jacobian with respect to $\mathbf{p}$, and $\mathbf{I}$ is the identity matrix of size $m \times m$. Factor λ is decreased when (10) decreases, and vice versa.

The algorithm is used twice in this study. First, it serves for the computational setting of denominator parameters of (2) such that dominant poles of (7) coincide with those of (2). Second, once the parameters of the characteristic quasipolynomial are set, numerator parameters of (2) are estimated via the solution of (5). Note that the initial setting of $\mathbf{p}^{(0)}$ is discussed in subsections 4.2 and 4.3.

3.3 Technique Summary

All steps of the proposed technique for single-run parameter identification of the HPM (or, generally, a multiple-parameter TDM) can be summarized by the following algorithm.

Algorithm 1. (Single-run relay identification test for the HPM using dominant pole placement based on the SFPM)

Input: HPM, SFPM with undetermined parameters
Output: Parameters estimation of the HPM.
1: Set $0 < \delta \ll 1$, $B > 0$, and perform the feedback relay experiment as in Fig. 2 until sustained oscillations appear. Read T_u, A.
2: Calculate ω_u, k_u (4), and $k_m = k$ (6). Estimate $\tau_m = \tau$ (Fig. 3).
3: Substitute $b_{0,m} = a_{0D,m}k_m$ in (7) and solve (5) for G_{SFPM} (7) using (8) => $a_{0D,m}, \vartheta_m$.
4: If $a_{0D,m}, \vartheta_m \notin \mathbb{R}$, reset $a_{0D,m}, \vartheta_m$ and go to step 5; else, go to step 7.
5: Set $\lambda^{(0)}$ and the initial denominator parameters of (7).
6: Use the LMA (11) to solve (5) for G_{SFPM} (2) => $a_{0D,m}, \vartheta_m$.
7: Set $\lambda^{(0)}$ and the initial denominator parameters of (2).
8: Use the LMA (11) to set poles of (2) identical to those of (7) => $a_2, a_1, a_0, a_{0D}, \vartheta$.
9: Substitute $b_0 = (a_0 + a_{0D})k - b_{0D}$ in (2).
10: Set $\lambda^{(0)}$ and the initial numerator parameters b_{0D}, τ_0 of (2).
11: Use the LMA (11) to solve (5) for G_{HPM} (2) => b_0, b_{0D}, τ_0.

A numerical example demonstrating all the algorithm steps follows.

4 Numerical Results

Consider model (2)-(3) as the exact heat-process model for this example purposes. The example provides the reader with particular numerical results of Algorithm 1.

4.1 Simple Model Parameter Identification

Let $\delta = 0.05$, $B = 100$ (note that practical issues of this setting are concisely discussed in subsection 4.4). The relay-feedback test yields the sustained oscillations with the following ultimate data: $A = 0.963, T_u = 363.4\,\text{s}$, i.e., $\omega_u = 1.73 \cdot 10^{-2}$ rad/s. Then, $k_u = 132.04$ according to (4). It can be estimated from Fig. 2 that $\tau = \tau_m \approx 136.5\,\text{s}$ and the computation of (6) gives $k = k_m = b_{0D,m} / a_{0D,m} = 3.23 \cdot 10^{-2}$.

The attempt to calculate (8) results in $k_u k_m \cos(\omega_u \tau_m) = -3.013$, which (unfortunately) unable to get real-valued SFPM parameters, see Remark 1. Therefore, the LMA is used to determine $a_{0D,m}, \vartheta_m$.

Let $\lambda^{(0)} = 1$ and the initial parameters' estimation be $a_{0D,m} = \omega_u = 1.73 \cdot 10^{-2}$, $\vartheta_m = 0.25\pi / a_{0D,m} = 45.43$. It comes from the assumption that $1/a_{0D,m}$ is close to the process (natural) time constant (for the delay-free system), and the selected value of ϑ_m means the "center of stability region". Factor λ is multiplied or divided by 5 in every single iteration step, according to the change of the residual sum (10). The use of the LMA then results in

$$a_{0D,m} = 4.776 \cdot 10^{-3}, \vartheta_m = 6.828\,\mathrm{s} \tag{12}$$

and (10) for the nonlinear equation set (5) (with $G(s) = G_{SFPM}(s)$) eventually has the error value of 0.2392.

4.2 Thermal Process Model Characteristic Quasipolynomial Estimation

The SFPM rightmost poles read

$$\begin{aligned} &s_1 = -0.00494, s_2 = -0.73822, s_{3,4} = -0.82529 \pm 1.05290\mathrm{j}, \\ &s_{5,6} = -0.89818 \pm 2.00889\mathrm{j}, s_{7,8} = -0.94808 \pm 2.94507\mathrm{j} \end{aligned} \tag{13}$$

The leftmost two real poles and the first conjugate pair are set as poles of (2) by solving the set of equations $A(s_i) = 0, i = 1,2$, and $\operatorname{Re} A(s_3) = \operatorname{Im} A(s_3) = 0$, where $A(s)$ is the denominator (i.e., the characteristic quasipolynomial) of (2). The set is solved by the LMA again.

By comparing denominators of (2) and (7), the ideal setting reads $a_2 = a_0 \to 0, a_1 \to \infty, a_{0D} \to a_1 \cdot a_{0D,m}, \vartheta = \vartheta_m$ or $a_0 = a_2 \to 0, a_1 \to a_{0D} / a_{0D,m}$, $a_{0D} \to \infty, \vartheta = \vartheta_m$, which is, however, non-feasible. Among various other initial settings, two interesting results with a low minimum of (10) have been obtained

$$\begin{aligned} &1: \quad a_2 = 65.495, a_1 = 2093.747, a_0 = -6.854, a_{0D} = 10, \vartheta = 6.797 \\ &2: \quad a_2 = 0.3363, a_1 = 0.9625, a_0 = 2.386 \cdot 10^{-2}, a_{0D} = 2.267 \cdot 10^{-2}, \vartheta = 8.146 \end{aligned} \tag{14}$$

for initial settings $a_2 = 5, a_1 = 10/a_{0D,m}, a_0 = 5, a_{0D} = 10, \vartheta = \vartheta_m$ and $a_2 = 0.1, a_1 = 0.1, a_0 = 0.1, a_{0D} = 0.1, \vartheta = \vartheta_m$, respectively. Let data vector 1 in (14) be denoted as the denominator of the HPM 1, while the latter parameters set in (14) be that of the HPM 2.

Poles of the HPM 1 are almost identical to (13), while those of the HPM 2 read

$$s_1 = -0.00494, s_2 = -0.73822, s_{3,4} = -0.16760 \pm 0.96169\mathrm{j}, s_{5,6} = -0.82529 \pm 1.0529\mathrm{j} \tag{15}$$

As can be seen, especially the real part of $s_{3,4}$ and the imaginary part of $s_{5,6}$ are pretty far from the required roots (13); however, the HPM 2 provides interesting time and frequency responses (see subsection 4.4).

4.3 Remaining Thermal Process Model Parameter Identification

The remaining parameters of HPM 1 and HPM 2, i.e., $b_{0,} b_{0D}, \tau_0$, can be obtained from the substitution according to step 9 of Algorithm 1 and estimated from the relay test data again.

The initial setting follows the idea that $b = b_{0D}$ and τ_0 is "a small positive number". Hence, since $b_0 + b_{0D} = (a_0 + a_{0D})k$, we do let set $b_{0D} = 0.5(a_0 + a_{0D})k$ and $\tau_0 = 0.1$. Then, the LMA with the identical control parameters set to that in the preceding subsection is applied to solve (5) with $G(s) = G_{HPM}(s)$. The eventual results for HPM 1 and HPM 2 are, respectively

$$\begin{array}{ll} 1: & b_0 = 0.1728,\ b_{0D} = 0.1530,\ \tau_0 = 1027.311 \\ 2: & b_0 = 7.420,\ b_{0D} = \text{-}7.419,\ \tau_0 = 4.95 \cdot 10^{-4} \end{array} \tag{16}$$

The corresponding residual errors (10) for the HPM 1 are 0.2970 and 6.1736, where the former value holds for the identity of real and imaginary parts in (5), while the latter holds for the identity of the gain and the phase. The errors for the HPM 2 are 0.2998 and 6.1717, respectively.

4.4 Results Evaluation

Let us display step responses and frequency responses (namely, Nyquist plots) of the original (exact) process model and the eventual models SFPM, HPM 1, and HPM 2. The time-domain responses are provided to the reader in Fig. 5. Note that the input step change is $\Delta u = 100$ W and the output data unit is °C. The HPM 1 and HPM 2 responses include non-smooth and high-oscillatory sections, respectively; see details in Fig. 6.

Errors of time-domain responses measured by the Integral Absolute Error (IAE) and the Integral Time Absolute Error (ITAE) from the original response are given in Table 1. As can be seen, the HPM 1 offers the best response in the time domain. However, the non-smooth section indicates high-frequency modes in the model dynamics.

Nyquist plots are displayed in Fig. 7. Note that the frequency range is selected as $\omega \in [0, 0.1]$ rad/s. The corresponding errors measured by the Root Mean Squares (RMS) and the 2-norm of the differences from the original data are given in Table 2. These results require a detailed analysis.

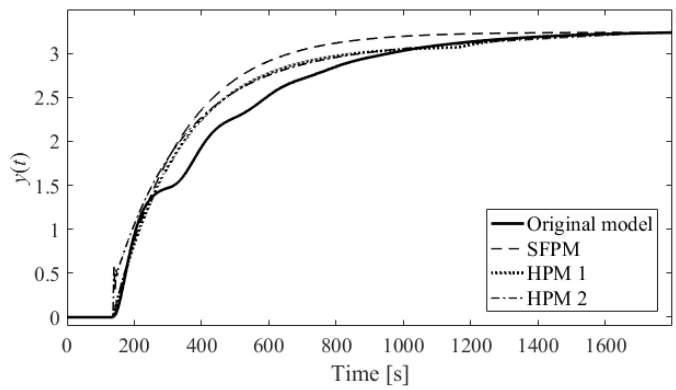

Fig. 5. Step responses of the original process, the SFPM, HPM 1, and HPM 2.

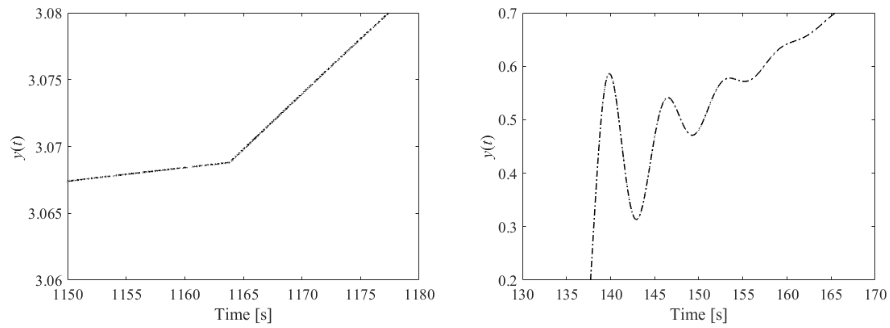

Fig. 6. Selected detail views on HPM 1 (left) and HPM 2 (right) time responses.

Table 1. Error measures of step responses.

Model	IAE	ITAE
SFPM	248.707	$1.533 \cdot 10^5$
HPM 1	164.270	$9.030 \cdot 10^4$
HPM 2	185.553	$9.380 \cdot 10^4$

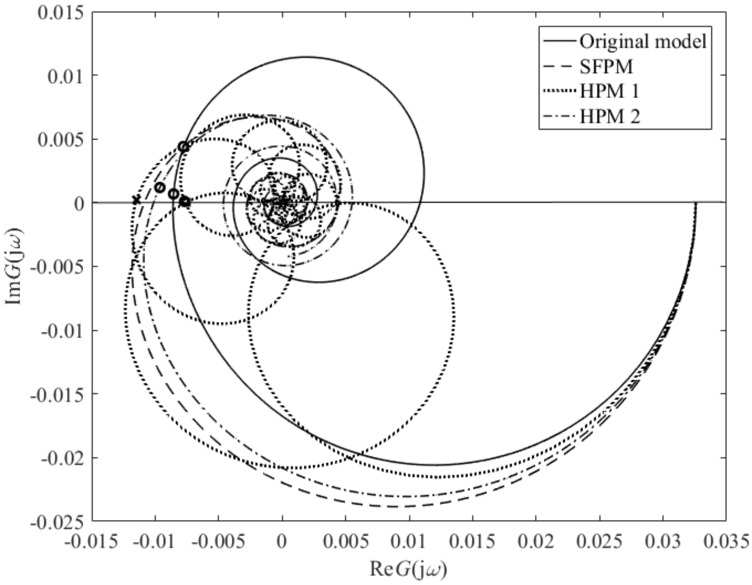

Fig. 7. Nyquist plots of the original process, the SFPM, HPM 1, and HPM 2.

Table 2. Error measures of Nyquist plots.

Model	RMS	2-norm
SFPM	$2.641 \cdot 10^{-2}$	0.118
HPM 1	$4.416 \cdot 10^{-2}$	0.198
HPM 2	$4.218 \cdot 10^{-2}$	0.189

By comparing data in Fig. 7 and Table 2, it is surprising that the HPM 2 has worse performance than the SFPM as the course of its Nyquist plot is closer to that of the original model. The justification arises from a frequency warping of the HPM 2 so that the corresponding points in Fig. 7 (for a particular frequency value) are farther from the original data than those of the SFPM. The same holds for the HPM 1, where – in addition – an extraordinary shape appears. Its gain and phase vary fast so that these functions are not monotonous and oscillate. However, the course of its Nyquist plot envelops that of the SFPM.

Another exciting issue is positions of $\operatorname{Re} G(\mathrm{j}\omega_u)$ and $\operatorname{Im} G(\mathrm{j}\omega_u)$ (recall that $\omega_u = 1.73 \cdot 10^{-2}$). From the relay test, the theoretical values read $\operatorname{Re} G(\mathrm{j}\omega_u) = -1/k_u = -0.0076$, $\operatorname{Im} G(\mathrm{j}\omega_u) = 0$ (denoted by the square in Fig. 7).

This position almost coincides with $\left[\operatorname{Re}G(\mathrm{j}\omega_u),\operatorname{Im}G(\mathrm{j}\omega_u)\right]=\left[-0.0077,9.88\cdot10^{-5}\right]$ for the HPM 1 (a circle symbol). However, this is not the true critical (ultimate) point. The same Nyquist plot position is already reached for $\omega_u = 0.86\cdot10^{-2}$. Moreover, the point for which every change of $\operatorname{Re}G(\mathrm{j}\omega)$ implies the transition from stability to instability (and vice versa) of the feedback loop (i.e., the crossing of -1 for the open-loop frequency response) agrees with the leftmost crossing point of the real axis. This true critical point is denoted by the cross symbol and appears at $\omega = 1.27\cdot10^{-2}$. Note that $\left[\operatorname{Re}G(\mathrm{j}\omega_u),\operatorname{Im}G(\mathrm{j}\omega_u)\right]$ for the original model, the SFPM, and the HPM 2 are $\left[-0.0085,6.91\cdot10^{-4}\right]$, $\left[-0.0070,0.0044\right]$, $\left[-0.0096,0.0012\right]$, respectively (circles).

To sum up, both the HPMs yield better performance than the SFPM in the time domain, while the opposite result holds in the frequency domain. The poor results for HPMs are given mainly by the hybrid nature of Algorithm 1. That is, the pole assignment attempts to get the HPM as close to the SFPM as possible. However, the subsequent LMA optimization is designed to utilize relay-feedback test data to the HPM. Hence, the eventual HPMs represent a trade-off between the original process dynamics (i.e., its ultimate frequency data) and the already obtained SFPM.

Finally, let us make a concise note on the practical implementation. The used relay has no hysteresis. However, it is better to set a nonzero dead-zone of the relay static characteristics due to signal noise. We also use a single asymmetric test. It is suitable for the estimation of the process static gain, yet it causes an error in the ultimate data determination. Therefore, it is better to use a double test under one experiment. First, to apply an asymmetry, then to use the ideal relay for more precise ultimate data.

5 Conclusion

An approach to a single-run relay-feedback identification experiment for time-delay models with multiple parameters has been proposed. The method has been based on using a simple model, the parameters of which are estimated first using the standard relay experiment. Then, the eventual dominant simple-model poles have been assigned to the pole loci of the complex infinite-dimensional model of a thermal circuit process. Finally, the remaining parameters of the thermal process model have been determined from the known ultimate data of the original relay experiment. The well-established Levenberg-Marquardt algorithm has been used to solve the particular sets of nonlinear algebraic equations arising in some steps of the algorithm.

The obtained numerical results yield a necessity to enhance the algorithm regarding eventual parameters' estimation since the step responses and Nyquist plots of the models have revealed some unpleasant properties. A multiple relay test (yet with a reduced number of requisite experiments) or more powerful (yet more complex) numerical tools for the solution of given equations have to be used in the future.

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