The effect of autocorrelation on control charts performance and process capability indices calculation

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Abstract: Chemical processes as well as many non-industrial processes exhibit autocorrelation, for which the above-mentioned control procedures are not suitable. This paper considers the problem of monitoring a process in which the observations can be represented as a first-order autoregressive model following a heavy tailed distribution. It also presents practical usage of time series control charts on the chemical process of monitoring concentrations. The second part discusses the effect of autocorrelation on the process capability analysis when the observations are made by an autoregressive model of the first order. The process capability indices provide a measure of how a process fits within the specification limits. When calculating indices, it is usual to assume that the process data are independent. The simulation study in this part discusses the effect of a higher autocorrelation order on process capability indices.

Keywords: statistical process control; SPC; autocorrelation; control charts; process capability indices; process variability; time series modelling; CUSUM control chart; EWMA control chart; ARIMA control chart; Shewhart's control charts.

Reference to this paper should be made as follows: Kovářík, M. and Briš, P. (xxxx) 'The effect of autocorrelation on control charts performance and process capability indices calculation', *Int. J. Quality Engineering and Technology*, Vol. X, No. Y, pp.xxx–xxx.

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1 Introduction

An important role for the effective design of the control chart is whether the production system does not cause the effects of interdependence between the data obtained (i.e., whether the output data from the process is not autocorrelated). The phenomenon of data autocorrelation is mainly concerned with the production where measurements are carried out in processes automatically, there is frequent sampling, respectively, process selections and further under the conditions of certain specific processes. Typical representatives of processes with the autocorrelated output data are large-scale and mass production processes using automated control and continuous production processes in the chemical, metallurgical and food industries. In these sectors, so-called internal factors of production, such as reactors, recycling streams, or pumping of material from tanks, which can cause dependency in the output data, are often found. Another real-world examples of autocorrelated processes are for example weather prediction, economic time series modelling, e.g., index/stock prediction, signal processing or medical ultrasound imaging where autocorrelation is used to visualise blood flow.

A certain degree of autocorrelation is generally congenital for data, and some control charts such as cumulative sums (CUSUM) and exponentially weighted moving average (EWMA) chart use this property. However, there are high autocorrelation results in data distortion against the control limits of the diagram, leading to erroneous decisions about the process interference. The effect of strong autocorrelation is usually high incidence of points outside the control limits.

Conventional statistical process control (SPC) methods such as Shewhart's control charts and CUSUM control charts assume that the process data obtained is independent. However, this assumption has been challenged once it has been found that in many practical situations, data is serially correlated. The main effect of m autocorrelation in process data in SPCs, the control limits are calculated that are much narrower than required. This causes a significant increase in the average false signals and a reduction in the process change detection. As a result, the calculation of the average number of selections is wrong when the deviation in the process of the normalised size δ and the moment is revealed in the control diagram, which is referred to as ARL(δ). The performance of classical control diagrams in case of autocorrelation in data will deteriorate significantly.

This motivated the pioneering work of Alwan and Roberts (1988), who proposed monitoring the expected errors after estimating the appropriate time series model for the process. This method is intuitive, resp. once the autocorrelation is explained by the basic time series model, while the residues satisfy the conditions of independent random process errors. Traditional SPC methods can therefore be used to monitor residues. Subsequent work on the subject can be divided into roughly two topics; time series based models (Alwan and Roberts, 1988; Lu and Reynolds, 1999a, 1999b; Apley and Shi, 1999; Apley and Tsung, 2002; Testik, 2005) and model free approaches (Apley and Tsung, 2002; Krieger et al., 1992; Atienza et al., 2002; Dyer et al., 2003; Runger and Willemain, 1996; Sun and Xu, 2004; Balkin and Lin, 2001; Zhang, 2000; Alwan and Alwan, 1994; Young and Winistorfer, 2001; Runger, 1996). In the first case, three general approaches have been proposed: those that monitor residues (Alwan and Roberts, 1998; Wardell et al., 1994; Lu and Reynolds, 1999a, 1999b, 2001; Apley and Shi, 1999; Apley and Tsung, 2002; Testik, 2005; Montgomery and Mastrangelo, 1991; Mastrangelo and Brown, 2000; Alwan, 1992; Timmer et al., 1998; Atienza et al., 2002, 1998; English et al., 2000; Loredo et al., 2002; Alwan and Radson, 1992), those based on direct observations (Montgomery and Mastrangelo, 1991; Mastrangelo and Brown, 2000; Alwan, 1992; Timmer et al., 1998; Lu and Reynolds, 2001; Atienza et al., 2002; Bai, 1994) and those based on new statistical characteristics (Atienza et al., 1998). A brief overview of these approaches is provided below. Wardell et al. (1994) and Lu and Reynolds (1999a, 1999b) proposed the use of an exponential weighted moving average (EWMA) control chart for residue monitoring. Aunali et al. (2019) introduced a control chart using M/M/1 queueing discipline to study and maintain a process control in a regular interval. Gohel et al. (2018) introduced new improvements of CUSUM and EWMA control charts for ready-mixed concrete (RMC). Apley and Shi (1999) suggested a general likelihood ratio test (GLRT) approach to determine mean shift in the autocorrelated processes. Apley and Tsung (2002) have proposed an activation cumulative score (cuscore) diagram that is similar to GLRT but easier to implement. Castagliola and Tsung (2005) examined the effect of abnormality on residual control charts. They designed a modified Shewhart control chart for residues – the so-called special cause chart (SCC), which is in conditions of abnormality more robust. Testik (2005) considered the uncertainty in the time series model to estimate parameters and to monitor residues of the first order authoritative process AR(1) and suggested a solution in the form of wider EWMA control limits.

Johnson and Bagshaw (1974) and Bagshaw and Johnson (1975) have derived an approximate distribution of the number of selections leading to the signal under the condition that the process is governed by the AR(1) or MA(1) model for the CUSUM control chart. They also argue that incorrect conclusions can be drawn using the conventional CUSUM diagram if there is a correlation in the data. Harris and Ross (1991) discussed the effect of autocorrelation on CUSUM and EWMA charts and showed that the median and average number of selections leading to the signal in these diagrams are sensitive to the presence of autocorrelation in the data. Alwan (1991) discussed the effect of masking special causes using autocorrelated data and demonstrated that, even in the presence of a low degree of autocorrelation, points outside the control boundaries of the diagram do not necessarily imply a process change. Padgett et al. (1992) examined the Shewhart diagrams, with such a correlation process that can be described by the AR(1) model plus random errors, and found that this type of autocorrelation causes false signals. Alwan (1992) discussed the capability of Shewhart diagrams under the condition that observations reflect a general diagram of autoregressive moving averages, ARIMA(p, q). Alwan and Alwan (1994) discussed the influence of autocorrelation on frequently advocated complementary tests of non-random groupings. Schmid and Schone (1997) theoretically showed that the number of withdrawals leading to signal the autocorrelated processes is greater than in the case of independent variables provided that all autocovariance numbers are greater or equal to zero. Prybutok et al. (1997) found that undiagnosed, but estimated correlations in the data, will reduce the average time the signal increases as the degree of correlation. Boyles (2000) provided an estimation method for the first order autoregressive common cause model, which distinguished the variability of models of autocorrelated common causes from actual baseline observations.

Early detection of attributable causes ensures that the necessary corrective measures can be taken before a large number of non-compliant products are produced. If there is autocorrelation in the data, measures must be taken to prevent its impact on the correct implementation of SPC techniques. A simple idea that can 'dissolve' autocorrelation is less frequent sampling of process data. However, the inefficient use of the data available may lead to a reduction in the performance of control charts, as with the limited data, it may take much longer to reveal a real shift in the process mean value than when all observations are available. In addition to a simple approach using less frequent sampling, two general approaches have been developed for the design of control charts in the case of correlated processes.

The first approach uses the standard control charts and explanation of autocorrelation and a method of estimating the variance of the process to estimate the actual process variance adjusting both control limits (see, e.g., Vasilopoulos and Stamboulis, 1978; Van Brackle and Reynolds, 1997; Schmid, 1995). The second approach approximates the process data with the time series model so that the prediction of each additional observation can be made using previous observations, and then the classical control charts are used for the residues, or some of their slightly modified versions (see, e.g., Alwan and Roberts, 1988; Harris and Ross, 1991; Montgomery and Mastrangelo, 1991; Mastrangelo and Montgomery, 1995; Lu and Reynolds (1999a, 199b). The reasonable use of residual diagrams is such that assuming the correct time series model is determined for the data, the residues will be independent random variables originating from the same distribution. Thus, all statistical assumptions will be met, and some of the traditional SPC methods can be used. Once a change in mean or variance of residues in the process is found, it is concluded that the mean or variance of the process itself has changed. Thus, the representation of residues in the control chart provides a mechanism for detecting the process change.

However, many people seem to agree that residual diagrams do not have the same characteristics as traditional ones, i.e., the original observation diagrams, and that the ability of a diagram to detect a mean shift depends on a suitable model to suitably approximate data. This was first demonstrated by Longnecker and Ryan (1991) and Ryan (1991), where the AR(1) model was used. In Longnecker and Ryan (1992), additional models were considered where residual diagram performance was assessed for individual values from AR(1), AR(2) and ARIMA(1, 1). They pointed out that a residual diagram for individual values may have poor ability to detect a mean shift in the process, and showed that the diagram has a high probability of detecting a mean shift when it occurs, but if this shift is not immediately recognisable, there is a low probability that the shift will be detected later, especially for the AR(1) positive autocorrelation process. Wardel et al. (1994) studied deriving the distribution of the number of selections leading to the signal in residual diagrams and a little later Zhang (1997), Lu and Reynolds (1999a, 1999b) and others. Harris and Ross (1991) studied the response of ARIMA(0, 1, 1) and AR(1) process residues to a process mean shift and concluded that the residual analysis is insensitive to median value in the case of positively autocorrelated processes and traditional control methods are recommended for residue monitoring.

Until now, it seems that small attention has been paid to the development of control charts for correlated attribute data. Deligonul and Mergen (1987), as well as Bhat and Lal (1990), suggested the existence of a two-state model of Markov chains for autocorrelation of attributive data. Harvey and Fernandes (1989) argue that correlated computing data can be modelled and EWMA accessed. Wisnowski and Keats (1999) also came to the same conclusion. Stimson and Mastrangelo (1996) studied the tracking of serially correlated processes with attribute data obtained from multi-stage production. Lai et al. (1998) tested control procedures based on the number of selections leading to the signal of matching (matching) units of applications to almost no defective processes in the presence of serial correlation. Lai et al. (2000) studied the problem of process monitoring where the process is of high quality and the measured values have some serial dependence. Tang and Cheong (2006) have designed a control apparatus that is effective in detecting a change in proportion, unsatisfactory for high yield correlation processes within each control group. Finally, Shepherd et al. (2006) suggested two apparatuses of control charts. These control charts are based on a sequence of random variables that are used to classify a unit as satisfactory or unsatisfactory according to a stationary Markov chain model and 100% sequential sampling.

The influential works of the current review of the discussed issues were the ones of Papaleonida (2002) and Papaleonida and Psarakis (2002). There is an extensive overview of procedures and approaches in monitoring autocorrelated processes. Later, it was Knoth and Schmid (2004) who provided a detailed overview of their work in this issue, including a comparative study of some adjusted and residual EWMA and CUSUM apparatuses. Development SPC techniques for monitoring autocorrelated processes reached considerable attention in the literature concerning the quality of engineering. The focus is mainly on the detection of process mean shift as well as on changes in the variability (volatility) and autocorrelation structure of the time series, which are also important indicators of the presence of process changes (it must then influence the system due to a definable cause). It is often overlooked.

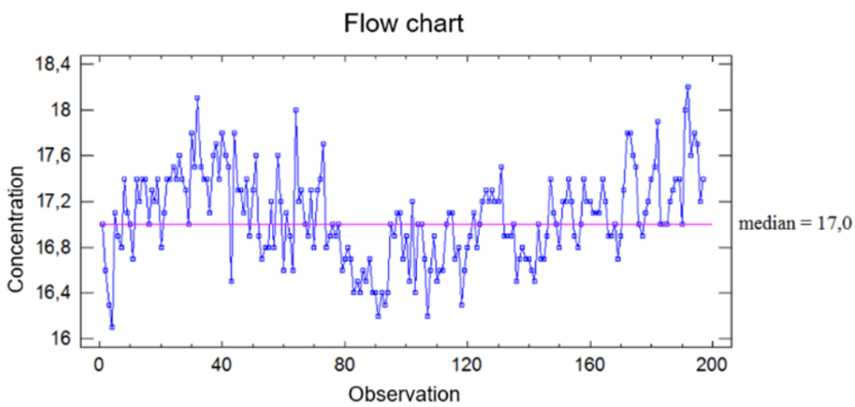
Dooley and Kapoor (1990) discussed monitoring changes in the mean value and variability autocorrelation structure (MVAS) of process observations using simultaneous engineering CUSUM charts, χ^2 and autocorrelation diagram of residues estimated model of the process data mentioned. Yourstone and Montgomery (1989) pointed out that the autocorrelation function (ACF) "Will detect changes in the structure of the autocorrelation as well as shifts in the mean and variability of data quality process in real time". They designed a real-time monitoring of residual autocorrelation of the first m n calculated for the most recent measurement process. Such selective is called a graph diagram of autocorrelation sample autocorrelation chart (SACC). On the basis of the mean time of the occurrence of an unsatisfactory *d* value (ARL), Atienza et al. (1997)

examined properties in the diagram SACC. They pointed out that while SACC may detect changes in the mean and variability of the range, it is far from as good as the Shewhart control chart (SCC) used on residues. The only advantage of SACC is that it can detect changes in the autocorrelation structure of the series better than SCC. So, for a simultaneous monitoring of changes in MVAS series, relying on the SACC diagram only.

2 The application of control charts for the autocorrelated data from chemical industry

The following practical examples will be examined by the autocorrelated typical set of measurements where the model constructed such time series, which will then be used to construct the ARIMA control chart to monitor these autocorrelated data. The following analysis relates to 197 measurements of NH_3 (g/m^3) concentrations in chemical process. Measurements were taken every two hours. The concentration process should be maintained at 17.1 ± 1.2. The first step in analysing any process is its graphical diagnostics. For example, a flow chart may be very informative for sequentially ordered data.

Figure 1 Flow chart for data from chemical concentration processes (see online version for colours)



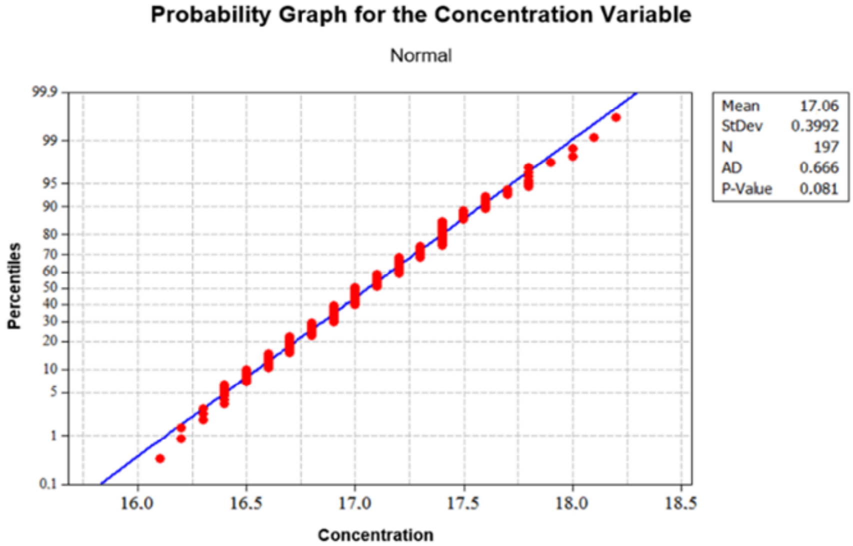
Source: Custom processing in Statgraphics

The previous graph shows strong fluctuations around the mean. The following numerical output shows the results of several test runs that are used to determine whether the observed values constitute a sequence of independent observations.

The previous figure shows a probabilistic chart that, even by the *p*-value of the Anderson-Darling test, does not reject the assumption of data normality.

The low *p*-value in the last column corresponds to the grouping test (too few waveforms above and below the median). This test compares the number of waveforms above and below the median (44) with the expected assumption that the observations given were randomly selected from any population (88.08). To reject the hypothesis of independence between consecutive observations at the 5% level of significance, *p* must be lower than 0.05. If we did not pay enough attention to both independence and data, creating a control chart for individual values would cause the following.

Figure 2 Verification of normality of measured concentrations (see online version for colours)



Source: Custom processing in Statgraphics

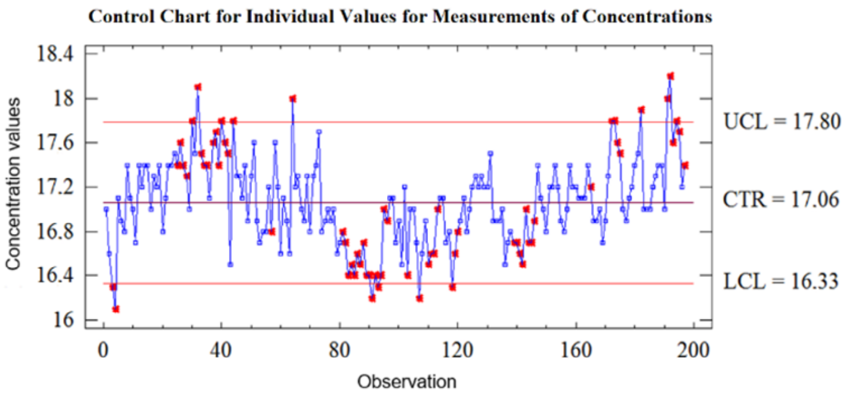
Table 1 Numerical diagnostics of the flow diagram

Test	*Observed*	*Awaited*	*Longest*	*P(>=)*	*P(<=)*
Runs above and below the median	44	88.0795	23	1.0	1.388E-11
Runs up and down	110	115.0	5	0.8406	0.207331

Note: Variable: concentration; 197 values ranging from 16.1 to 18.2; Median = 17.0).

Source: Custom processing in Statgraphics

Figure 3 Control chart for individual chemical process concentration measurement values (see online version for colours)



Source: Custom processing in Statgraphics

The resulting control chart shows many points outside the 3σ control limits and many violated tests of non-random groupings. Alarms are signalled at the top and bottom of each cycle. In addition, there are also long point patterns either above or below the mean. The flow chart is of no use for monitoring this process in case of such fluctuations of the mean value. Since fluctuations are an integral part of the process dynamics, it does not mean that the process is 'out of control', in a statistically uncontrolled state. Now, we move on to constructing a parametric model of time series. In order to monitor the process, we must first understand the nature of its dynamics. For stationary process (a process with constant long-term mean value and Scattering), there is a very useful class of ARIMA models (autoregressive, integrated, moving average). The general model that we observed is the concentration of Y_t at time period t, such as linear combination concentrations measured in the last p periods, the random effect on the system in the current time period and the t and the random effects that occurred in the previous q time periods.

$$\begin{aligned} Y_t &= \theta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + a_t \\ &\quad -\theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_p a_{t-q} \end{aligned} \tag{1}$$

The parameters that define this model are:

θ_0 constant

$\phi_1, \phi_2, \ldots, \phi_p$ autoregressive parameters

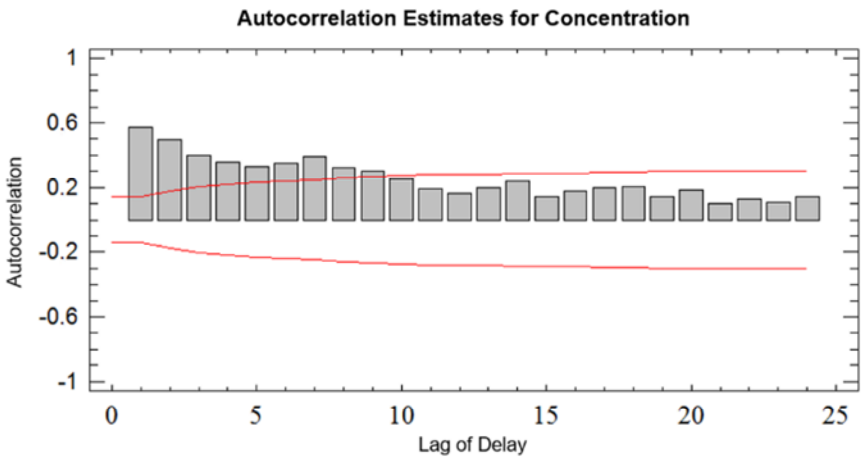
$\theta_1, \theta_2, \ldots, \theta_q$ moving average parameters.

Such a model is capable of representing different modes of dynamic behaviour and has been widely applied to many different types of systems. In fact, it can show the fact that if the data is selected and from a system that follows the p^{th} order of the difference equation, it should reflect a model with p autoregressive parameters and $q = p - 1$ parameters of moving averages. From a statistical point of view, the general problem is to determine an order of the model (to set the p and q values) and to estimate the model parameters. In the following section, we will consider solving this problem by limiting the models in which the $q = p - 1$. If we want to find out what type of ARIMA model to use for a particular dataset, we will rely on two major graphical tools: the ACF and the partial ACF.

The ACF displays estimates of the correlation coefficient between observations separated by k time points where k is called the delay order. This chart shows very well how the effect of the random effect entering the system interferes with the future. Imagine the situation when the pendulum is deflected from our j equilibrium. This will return to its original position over time. Either it returns to its original position over time, possibly as an exponential (first-order system) or possibly as an override showing damped oscillations until it returns to its equilibrium (second-order system).

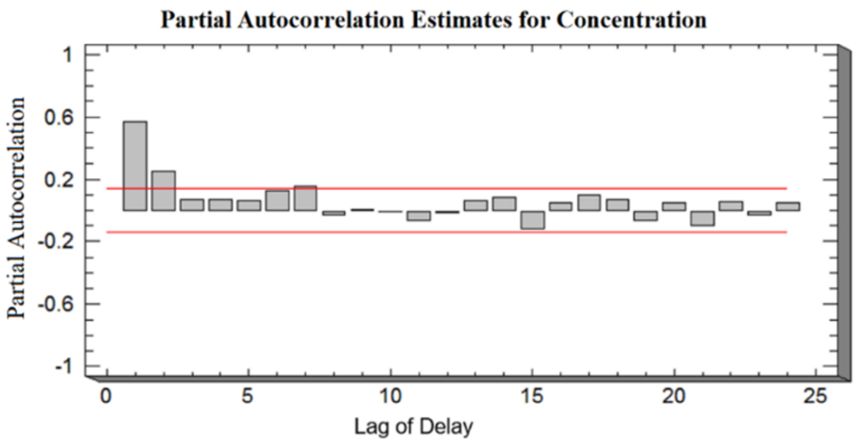
The shape of the ACF is like a fingerprint, identifying the types of dynamics that are present in the process. For autoregressive models, another function called the partial ACF will be useful. It draws partial autocorrelation coefficients as a function of k delay.

Figure 4 Chart of the selective ACF (see online version for colours)



Source: Custom processing in Statgraphics

Figure 5 Graph of the partial ACF (see online version for colours)



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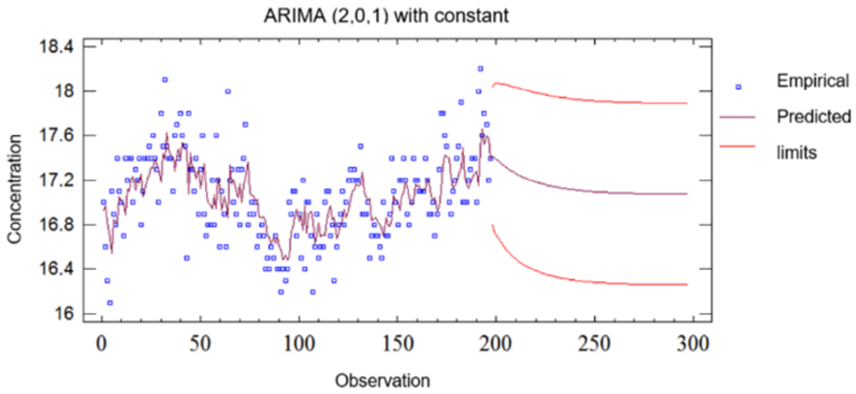
All coefficients that exceed the 95% probability limits would require the autoregressive parameter of this order. Thus, the above graph shows significant partial autocorrelations at lags 1 and 2, suggesting that the ARIMA model with the parameter $p = 2$ is likely for this data.

Subsequently, the most appropriate ARIMA model will be identified for this data.

The best approximating model is $p = 2$ and $q = 1$. In the previous graph, the measured values, including the most optimal interleaving model with the 95% prediction intervals for 100 predictions, are shown.

Process mean estimates $\hat{\mu} = 17.07$ and standard deviations of random influences $\hat{\sigma}_Y = 0.3142$ are very important characteristics here. Note that this is not equal to the process standard deviation $\hat{\sigma}_Y$, that is a function $\hat{\sigma}_a$ and parameters of the estimated model.

Figure 6 Predictive functions for the selected model (see online version for colours)



Source: Custom processing in Statgraphics

Figure 7 Numerical diagnostics of best fitted ARIMA model

Automatic Forecasting Diagnosis - Concentration (Statgraphics Program)
Variable: Concentration
Observations = 197
Starting index = 1.0
Sampling interval = 1.0

Predictive diagnostics
Selected prediction model: ARIMA (2,0,1) with constant
Number of extrapolation values: 100

Diagnostics for ARIMA model

Parameter	*Estimate*	*Standard deviation error*	*t statistics*	*P value*
AR(1)	1.12018	0.141195	7.93353	0.000000
AR(2)	-0.162049	0.11433	-1.41738	0.157984
MA(1)	0.74416	0.116662	6.37879	0.000000
Mean value	17.0722	0.126207	135.271	0.000000
Constant	0.714836			

White noise variance estimate = 0.0987145 with 193 degrees of freedom

White noise standard deviation estimate = 0.314189

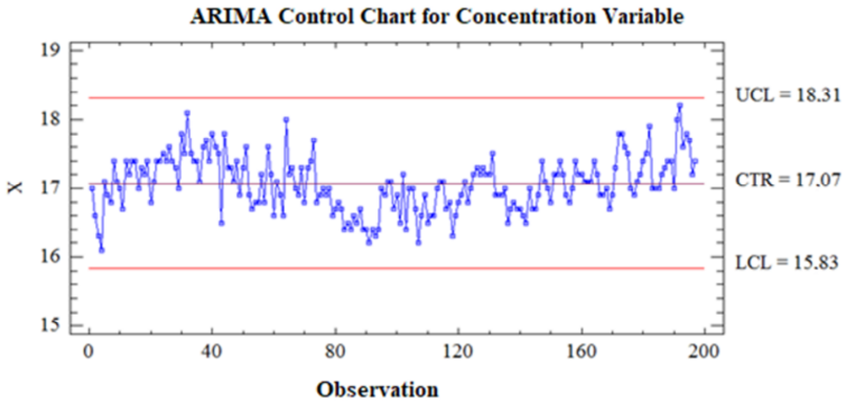
Iterations: 10

Source: Custom processing in Statgraphics

Then, the ARIMA control chart is configured. We selected the 2nd order of the AR parameter and the 1st order of the MA parameter. We calculated the standard deviation of the model based on the MSE characteristic. It follows the graphical output in the form of the ARIMA control chart.

In this control diagram, the central line is estimated by the process mean value $\hat{\mu}$ = 17.07 with control limits plus minus process standard deviation $\hat{\sigma}_Y$. The process standard deviation estimate is $\hat{\sigma}_Y$ = 0.4130. This results in wider control limits (following Figure 8), which allows for autocorrelation in data. It is important to note that the above chart monitors the long-term behaviour of the process. Individual points in the graph are not independent, so the standard runtime rules cannot be used. The graph also does not monitor the effects of entering the system, CI occurs in each time period.

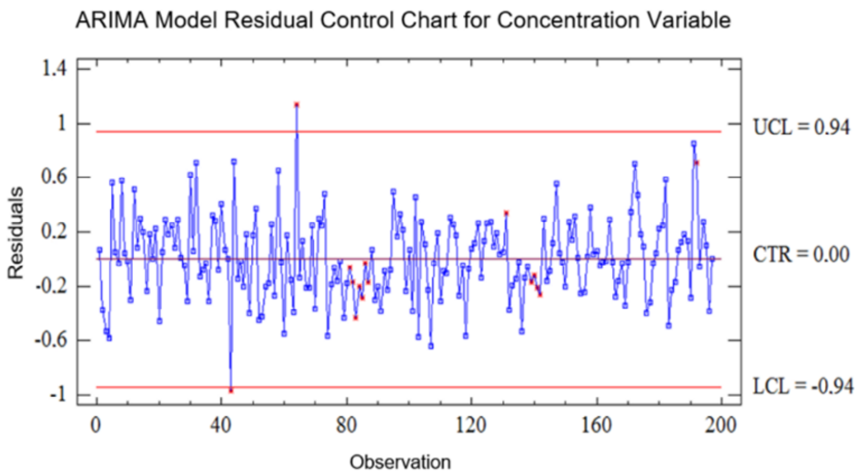
Figure 8 ARIMA control chart with extended control limits (see online version for colours)



Source: Custom processing in Statgraphics

In particular, its purpose is to find out when the process deviates from the long-term average more than expected, given the process dynamics. To monitor the separated random effects, we will use the residual control diagram of the estimated ARIMA model.

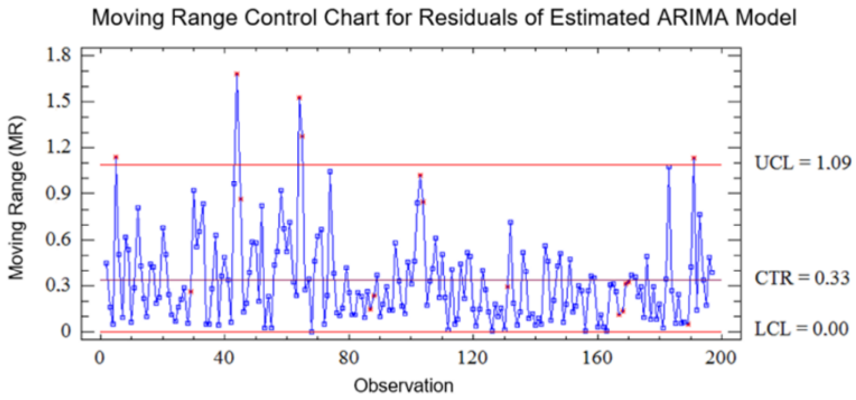
Figure 9 Control chart for residuals estimated ARIMA model (see online version for colours)



Source: Custom processing in Statgraphics

The ARIMA control chart illustrates observation estimates and *t*, which are influences affecting the system at each time point and time. In the previous graph, two random influences above 3σ of control limits are shown, which means that there was a sample during sampling with unusually big mistakes. There are also two positions in the graph where random influences showed a significant course of negative values. For a more detailed graphical analysis, a control chart moving range can also be used.

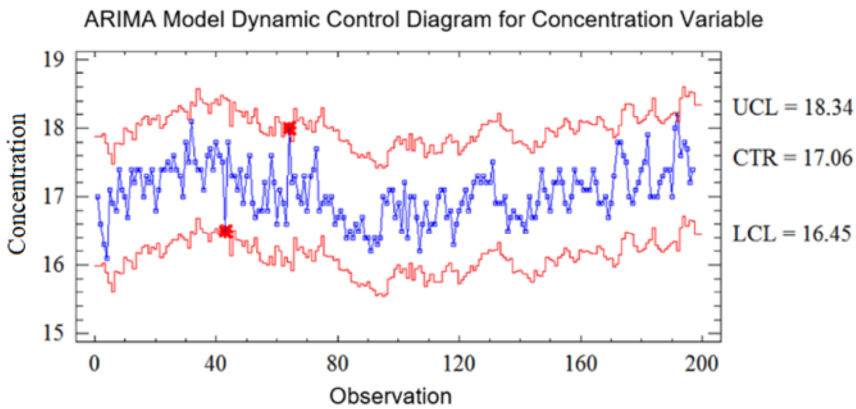
Figure 10 Moving range control chart for residuals of the estimated model (see online version for colours)



Source: Custom processing in Statgraphics

In practice, however, a dynamic control diagram is used much more often as shown in the following figure.

Figure 11 Dynamic ARIMA control chart (see online version for colours)



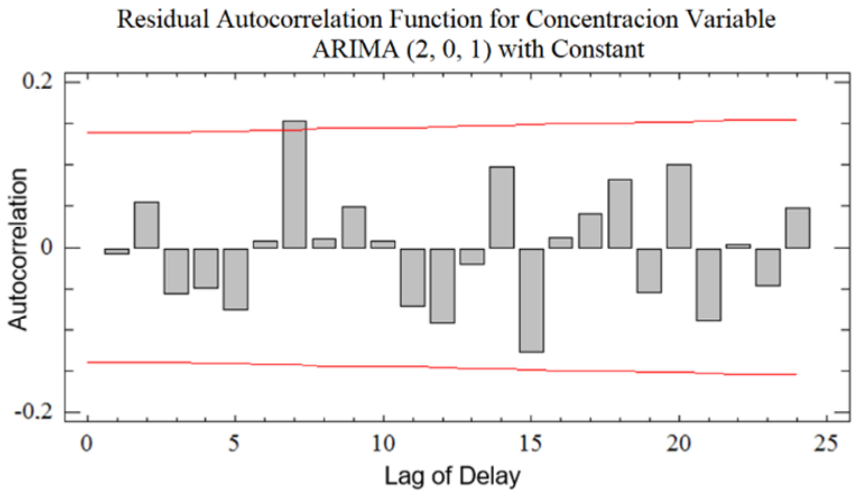
Source: Custom processing in Statgraphics

Diagrams represent the original data with gliding control limits. At each point in time *t*, the control limits are centred around the predicted value for Y_t created at time $t - 1$, $\pm 3\hat{\sigma}_a$. The graph gives the same signals with respect to the control limits being exceeded, as well as the control diagram for the residual model residues.

If you use the ARIMA control diagram, it is necessary to confirm the correctness of this model using the residual auto-tracking function. This consists in verifying the non-autocorrelation of residues.

From the previous graph, we see that all columns are within (or very close to) 95% of the probability limits, showing non-correlated residuals of the estimated ARIMA model. To determine capability, we need to add an average concentration value of 17.0722 to the white noise values.

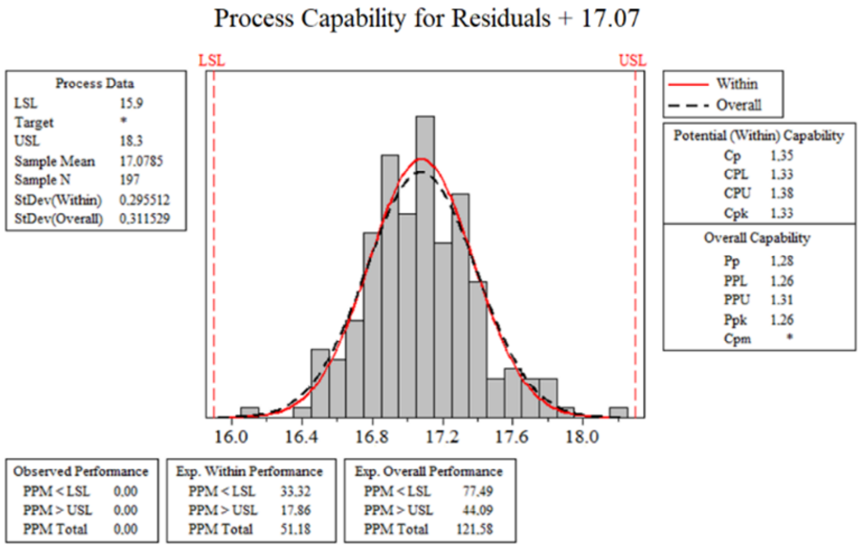
Figure 12 Residual ACF (see online version for colours)



Source: Custom processing in Statgraphics

The non-random cause signalled in the control chart was identified and performed the measures that the situation has not been repeated. The control of concentration in the chemical process is capable with a coefficient of $C_p = 1.35$ and $C_{pk} = 1.33$.

Figure 13 Process capability calculated for residuals of the estimated model (see online version for colours)



Source: Custom processing in Minitab 16

In the case of data dependency, standard control charts should not be used. The control charts should be used when taking into account the dynamics of the process, for example, the ARIMA control charts. In the case of data independence, the classical control diagram would have a lot of false signals and its practical implementation would be risky.

The ARIMA control chart requires a time series model identification before using it. If possible, models can be limited to ARIMA models with $q = p - 1$, reducing the problem only to determining the correct autoregressive model of the order p.

3 The effect of autocorrelation on calculation of process capability indices

In the following, we use simulation study, where we assume that a certain inherent quality attribute has a normal distribution with mean of 40 and standard deviation of 7. Specification limits USL = 61 and LSL = 19. Further contemplated are the various target values: 40, 41, 42, 45 and 50. Next, two processes will be compared. The process with independent observations and the process with observations controlled by AR(1) $X_t = X_{t-1} + e_t$, where $\{e_t\}$ is a series of uncorrelated errors $e_t \sim N(0, \sigma_e^2)$ and $\sigma_e = 7$.

The process capability indices C_p, C_{pk}, C_{pm} and C_{pmk}, mean value are calculated for each process as well as standard deviation. C_{pk} and C_{pmk} will not be given because $C_p = C_{pk}$ and $C_{pm} = C_{pmk}$, see Table 2.

Table 2 shows that the higher the autocorrelation value, the lower the process capability index.

Table 2 Mean value (μ), standard deviation (σ), C_p and C_{pm} of the non-correlated process controlled by the AR(1) model

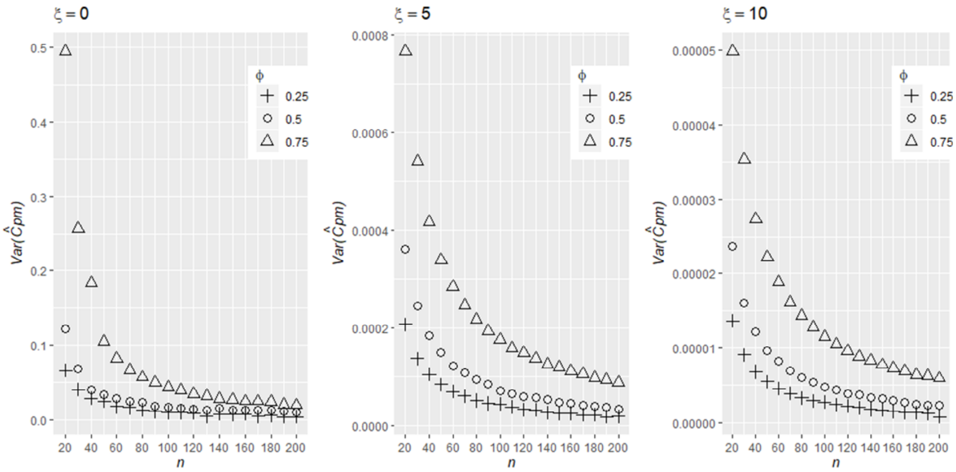
$\lvert\phi\rvert$	μ	σ	C_p	C_{pm}			
				$d = 0$	$d = 1$	$d = 3$	$d = 5$
Does not exist	40	7.00	1.00	1.00	0.99	0.919	0.814
0.25	40	7.23	0.968	0.968	0.959	0.894	0.796
0.5	40	8.05	0.866	0.866	0.859	0.812	0.737
0.75	40	10.6	0.661	0.661	0.659	0.636	0.598

Note: Note: $d = \mu - T$.

Source: Custom processing

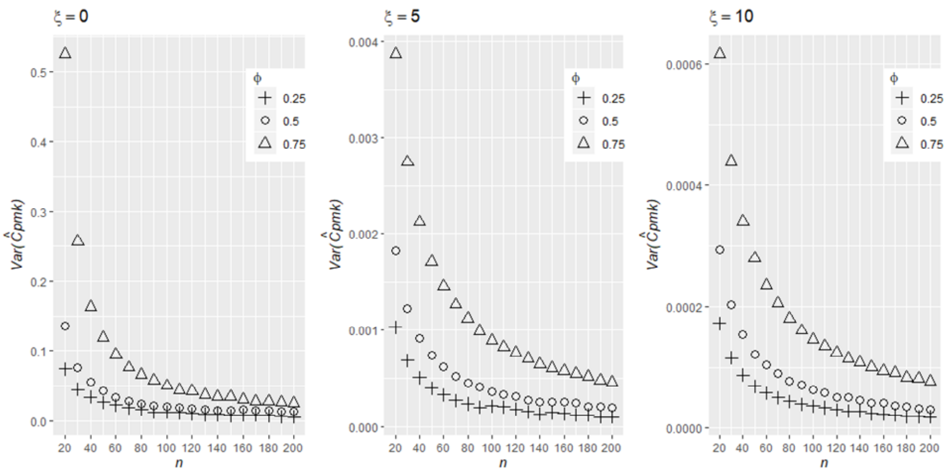
To compare differences in estimates $\hat{C}_{pm}$ and $\hat{C}_{pmk}$, a simulation study was conducted for the first order stationary autoregressive process with parameter ϕ. For $C_p = 1.33$, and $\xi = 0, 5, 10$, where $\xi = (\mu - T) / \sigma$, $\phi = 0.25, 0.5, 0.75$ and $n = 10, 20, \ldots, 200$, Figure 14 shows a large estimate variability for $n < 100$ great variability in estimation $Var(\hat{C}_{pm})$. Leaving C_p and n constant and increasing ξ as $|\phi|$, then $Var(\hat{C}_{pm})$ will grow. Similar results are obtained for estimation $Var(\hat{C}_{pmk})$ by substituting C_{pm} for C_{pmk} and C_p for C_{pk} (see Figure 15). Now, if we keep constant values ϕ and ξ and n will grow, the estimates $Var(\hat{C}_{pm})$ and $Var(\hat{C}_{pmk})$ will decrease. If we leave ϕ and n constant values, assuming ξ will grow, there will be a situation where the target value will be far from the process mean and estimates $Var(\hat{C}_{pm})$ and $Var(\hat{C}_{pmk})$ will fall.

Figure 14 Capability index estimation variability $\hat{C}_{pm}$ is a function of the selection range with $C_p = 1.33$, $\xi = 0, 5, 10$ and $\phi = 0.25, 0.5, 0.75$



Source: Custom processing in R language

Figure 15 Capability index estimation variability $\hat{C}_{pmk}$ is a function of the selection range with $C_{pk} = 1.33$, $\xi = 0, 5, 10$ and $\phi = 0.25, 0.5, 0.75$

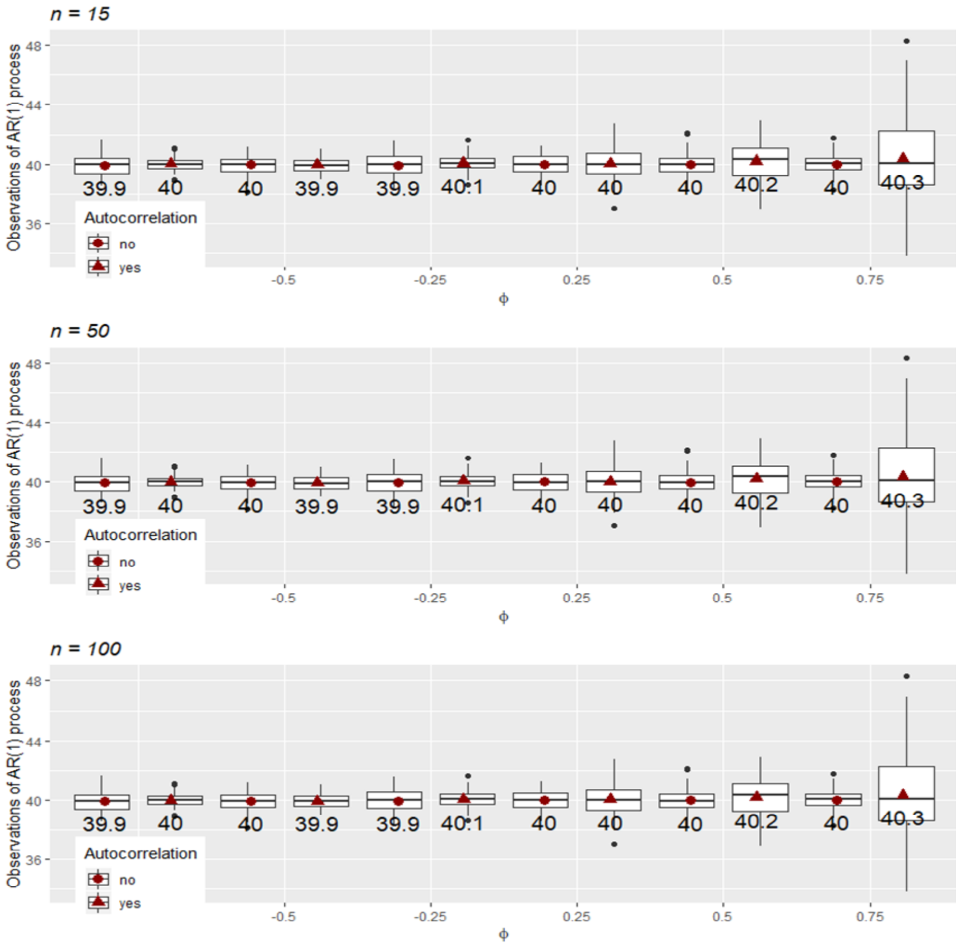


Source: Custom processing in R language

Using a simulation study, we will analyse the effect of autocorrelation on expected sample mean values and expected standard error values. We generate 1,000 selections from the non-automated model and 1,000 selections from the AR(1) model for the following cases: n = 15, 50, 100, 200; T = 40, 41, 42, 43, 44, 45 and ϕ = –0.75, –0.5, –0.25, 0.25, 0.5, 0.75. Figure 16 shows that the autocorrelation does not affect the expected value of the sample mean, but another situation occurs with the standard error. Recall, that $Var(X_t) = \frac{\sigma_e^2}{1-\phi^2}$, where σ_e^2 is white noise. For example, in Figure 17, for

n = 15 and ϕ = –0.25, in the case of an autocorrelated process, an estimate of the expected standard error is 6.9, for ϕ = –0.5 it is 7.3 and for ϕ = –0.75 it is 10.5. For independent observations, the values are 6.8, 6.2, and 6.7. As n increases, the estimated standard error value of autocorrelated data slightly increases. For example, for ϕ = 0.25 estimates of expected values for n = 15, 50, 100 are 7.21, 7.25, and 7.31.

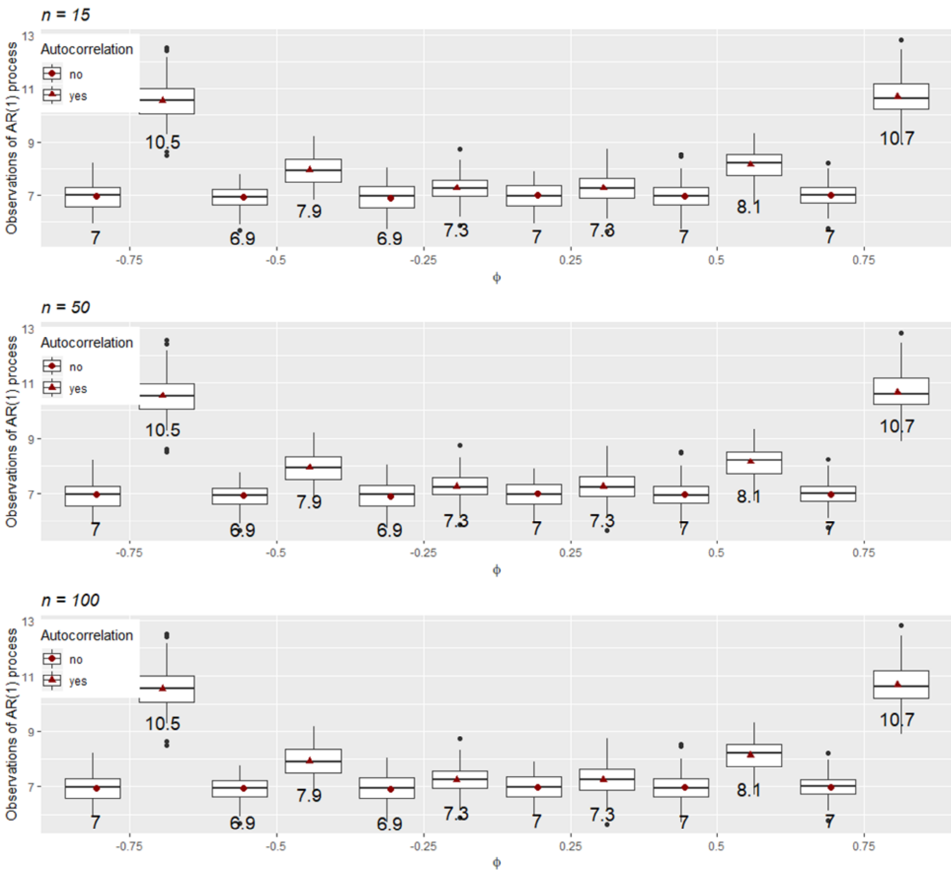
Figure 16 Expected values/standard errors of sample mean for non-correlated process and AR(1) process (see online version for colours)



Source: Custom processing in R language

The simulation study in Tables 3 and 4 captures the performance of capability index estimates. Again, a comparison of the estimated expected values of capability indices is shown in Tables 3 and 4 with the theoretical values that are shown in table 2 that estimates are slightly deflected for autocorrelated processes. The deviation with increasing n decreases. For example, for ϕ = –0.75 and n = 15, 50, 100, the expected values are $\hat{C}_p$ 0.703, 0.673 a 0.668, while the actual value is 0.661. For ϕ = 0.25 and n = 15, 50, 100, the expected values are $\hat{C}_{pk}$ 0.937, 0.935 and 0.938, while the actual value is 0.968. For n = 15 and ϕ = 0.5, the expected values are $\hat{C}_{pm}$ 0.84, 0.84, 0.83 a 0.81, when $\mu - T$ = 0, 1, 2 and 3, while the actual values are 0.866, 0.859, 0.841 and 0.812.

Figure 17 Expected values/standard errors of sample standard deviation for non-correlated process and AR(1) process (see online version for colours)



Source: Custom processing in R language

Previously, a simulation study has shown that a higher autocorrelation results in lower capability index values. Furthermore, it has been shown that estimates are slightly deflected for the autocorrelated processes and this deflection decreases with an increasing n. The autocorrelation does not affect the expected average of the sample average estimates of capability indices, but it affects the estimated expected standard error value, which increases slightly for the autocorrelated data with an increasing n.

Table 3 The effect of autocorrelation on expected standard error values of capability indices for processes generated by AR(1) – situation without autocorrelation

n	ϕ	$\bar{x}$ / s	C_p	C_{pk}	C_{pm}			
					$d = 0$	$d = 1$	$d = 3$	$d = 5$
15	–0.75	$\bar{x}$	1.057	0.987	1.02	1.01	0.94	0.83
		s	0.2	0.206	0.19	0.19	0.17	0.14
	–0.5	$\bar{x}$	1.059	0.986	1.02	1.01	0.95	0.84
		s	0.2	0.206	0.19	0.19	0.17	0.14
	–0.25	$\bar{x}$	1.06	0.985	1.02	1.01	0.95	0.84
		s	0.2	0.206	0.19	0.19	0.17	0.15
	0.25	$\bar{x}$	1.074	1.00	1.03	1.02	0.95	0.84
		s	0.203	0.208	0.19	0.19	0.17	0.14
	0.5	$\bar{x}$	1.048	0.976	1.01	1.00	0.93	0.83
		s	0.198	0.204	0.19	0.19	0.17	0.14
	0.75	$\bar{x}$	1.07	0.995	1.03	1.02	0.95	0.84
		s	0.202	0.207	0.19	0.19	0.17	0.14
50	–0.75	$\bar{x}$	1.016	0.978	1.01	1.00	0.92	0.82
		s	0.103	0.11	0.1	0.1	0.09	0.08
	–0.5	$\bar{x}$	1.009	0.972	1.00	0.99	0.92	0.82
		s	0.102	0.109	0.1	0.1	0.09	0.08
	–0.25	$\bar{x}$	1.015	0.978	1.00	1.00	0.93	0.82
		s	0.102	0.11	0.1	0.1	0.09	0.08
	0.25	$\bar{x}$	1.013	0.975	1.00	0.99	0.92	0.82
		s	0.102	0.109	0.1	0.1	0.09	0.08
	0.5	$\bar{x}$	1.012	0.973	1.00	0.99	0.92	0.82
		s	0.102	0.109	0.1	0.1	0.09	0.08
	0.75	$\bar{x}$	1.018	0.979	1.01	1.00	0.93	0.82
		s	0.103	0.11	0.1	0.1	0.09	0.08
100	–0.75	$\bar{x}$	1.004	0.978	1.00	0.99	0.92	0.81
		s	0.071	0.077	0.07	0.07	0.06	0.05
	–0.5	$\bar{x}$	1.009	0.982	1.00	0.99	0.92	0.82
		s	0.072	0.077	0.07	0.07	0.06	0.05
	–0.25	$\bar{x}$	1.009	0.982	1.00	0.99	0.92	0.82
		s	0.072	0.077	0.07	0.07	0.06	0.05
	0.25	$\bar{x}$	1.007	0.98	1.00	0.99	0.92	0.82
		s	0.072	0.077	0.07	0.07	0.06	0.05
	0.5	$\bar{x}$	1.009	0.982	1.00	0.99	0.92	0.82
		s	0.072	0.077	0.07	0.07	0.06	0.05
	0.75	$\bar{x}$	1.009	0.983	1.00	0.99	0.92	0.82
		s	0.072	0.077	0.07	0.07	0.06	0.05

Notes: Mean value ($\bar{x}$); Standard error (s). $d = \mu - T$.

Source: Custom processing

Table 4 The effect of autocorrelation on expected standard error values of capability indices for processes generated by AR(1) – situation with autocorrelation

n	ϕ	$\bar{x}$ / s	C_p	C_{pk}	C_{pm}			
					$d = 0$	$d = 1$	$d = 3$	$d = 5$
15	–0.8	$\bar{x}$	0.703	0.673	0.7	0.69	0.67	0.62
		s	0.213	0.206	0.21	0.21	0.18	0.15
	–0.5	$\bar{x}$	0.916	0.873	0.9	0.9	0.84	0.76
		S	0.236	0.228	0.23	0.22	0.19	0.15
	–0.3	$\bar{x}$	1.019	0.963	1.00	0.99	0.92	0.81
		S	0.241	0.232	0.23	0.22	0.19	0.14
	0.25	$\bar{x}$	1.031	0.937	0.98	0.97	0.91	0.82
		S	0.245	0.23	0.22	0.21	0.19	0.15
	0.5	$\bar{x}$	0.915	0.794	0.84	0.84	0.81	0.75
		S	0.237	0.218	0.2	0.2	0.18	0.16
	0.75	$\bar{x}$	0.69	0.526	0.61	0.6	0.6	0.58
		S	0.204	0.18	0.16	0.16	0.15	0.14
50	–0.8	$\bar{x}$	0.673	0.658	0.67	0.67	0.65	0.6
		S	0.138	0.135	0.14	0.14	0.12	0.1
	–0.5	$\bar{x}$	0.885	0.863	0.88	0.88	0.83	0.75
		S	0.137	0.135	0.14	0.13	0.11	0.09
	–0.3	$\bar{x}$	0.985	0.955	0.98	0.97	0.9	0.8
		S	0.132	0.13	0.13	0.13	0.1	0.08
	0.25	$\bar{x}$	0.985	0.935	0.97	0.96	0.9	0.8
		S	0.131	0.129	0.13	0.12	0.11	0.08
	0.5	$\bar{x}$	0.876	0.812	0.85	0.85	0.8	0.74
		S	0.137	0.134	0.13	0.13	0.11	0.09
	0.75	$\bar{x}$	0.674	0.578	0.64	0.64	0.62	0.59
		s	0.137	0.132	0.12	0.12	0.12	0.11
100	–0.8	$\bar{x}$	0.668	0.658	0.67	0.66	0.64	0.6
		s	0.104	0.103	0.1	0.1	0.09	0.08
	–0.5	$\bar{x}$	0.877	0.86	0.88	0.87	0.82	0.74
		s	0.1	0.098	0.1	0.1	0.08	0.06
	–0.3	$\bar{x}$	0.973	0.952	0.97	0.96	0.9	0.8
		s	0.094	0.093	0.09	0.09	0.08	0.05
	0.25	$\bar{x}$	0.973	0.938	0.97	0.96	0.89	0.8
		s	0.093	0.093	0.09	0.09	0.08	0.06
	0.5	$\bar{x}$	0.873	0.829	0.86	0.86	0.81	0.74
		s	0.098	0.099	0.1	0.09	0.08	0.07
	0.75	$\bar{x}$	0.666	0.597	0.65	0.64	0.62	0.59
		s	0.103	0.103	0.1	0.1	0.09	0.08

Notes: Mean value ($\bar{x}$); standard error (s). $d = \mu - T$.

Source: Custom processing

4 Conclusions

Most of the traditional control chart procedures are based on the assumptions that process measurements that are monitored are independent and come from the same distribution. With the trend of high-speed data collection systems, the assumption of independence is usually violated. This means that the autocorrelation between measurements becomes an inherent characteristic of a stable process. The autocorrelation causes a significant deterioration in the performance of control charts in the process control. Several procedures have been proposed to address the problem of autocorrelated processes. The most popular procedure utilises either the Shewhart, CUSUM or EWMA control chart for residues of an appropriately estimated ARIMA model. However, procedures of this type are of a low sensitivity, especially if the processes are positively autocorrelated. As an alternative, we have examined the use of statistics used in time series procedures to monitor outliers and process mean shifts.

In the paper, we focused on practical applications of the use of control charts on real data that come from the autocorrelated processes. In all these cases, a suitable model was found describing the real process dynamics and a subsequent construction of the control chart on the residuals of this model. Furthermore, we dealt with the effect of autocorrelation on process capability indices, where we concluded that the higher the value of autocorrelation, the lower the process capability index. Furthermore, it has been shown that estimates are slightly deflected for the autocorrelated processes and this deflection decreases with an increasing n. The autocorrelation does not affect the expected mean of the sample mean estimates of capability indices, but it affects the estimated expected standard error value, which increases slightly for the autocorrelated data with an increasing n.

The observation of the autocorrelated process results primarily from the automatic data collection system. These data collection systems are usually controlled by software that can be upgraded with SPC functions for data processing. Under such an integrated system, the usefulness of the proposed procedure will be optimised.

Acknowledgements

This research was carried out with support of the project: RO/2018/10 'Identification of risks and their impact on sector and firm innovation and financial performance in the digital economy'.

The paper was also supported by the RO project VaV-IF-RO/2018/03: Development of comprehensive methodology for process planning, management and implementation organisation for INDUSTRY 4.0 concept utilisation.

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