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# Robust PI Control of Interval Plants With Gain and Phase Margin Specifications: Application to a Continuous Stirred Tank Reactor

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**ABSTRACT** The paper is focused on robust Proportional-Integral (PI) control of interval plants with gain and phase margin specifications and on the application of this approach to a Continuous Stirred Tank Reactor (CSTR). More specifically, the work aims at the determination of PI controller parameter regions, for which not only robust stability but also some level of robust performance of the closed-loop control system is guaranteed, and this robust performance is represented by the required gain and phase margin that has to be ensured for all potential members of the interval family of controlled plants, even for the worst case. The applied technique is based on the combination of the previously published generalization of stability boundary locus method (for specified gain and phase margin under the assumption of fixed-parameter plants) with the sixteen plant theorem. This extension enables the direct application of the method to design the robustly performing PI controllers for interval plants. The effectiveness of the improved method is demonstrated on a CSTR, modeled as the interval plant, for which the robust stability and robust performance regions are obtained.

**INDEX TERMS** Robust control, PI controllers, continuous stirred tank reactor, robust stability, robust performance, gain margin, phase margin, interval plant.

## I. INTRODUCTION

The controlled plants that are burden with nonlinear, high-order, or other complex behavior, imprecise knowledge of their physical properties, or changeability of parameters are commonly modeled in a simplified way as the linear time-invariant (LTI) systems with interval uncertainty, in short, as the interval plants. It means the plants are described by the linear differential equations or corresponding transfer functions with coefficients that can vary (''slowly'' in time) within given bounds (intervals), where the coefficients change independently on each other. The principal advantage of this approach is that all the mentioned aspects are supposed to be covered by the interval plant family and that the popular techniques of linear robust control can be utilized for analysis and synthesis of the relevant control systems.

According to an unpublished Honeywell survey from 2000, 97% of control systems in the chemicals, refining, and pulp and paper industries utilize a Proportional-Integral-Derivative (PID) algorithm [1]. The control community expects that the rate of PID controllers and their special cases (such as PI and PD) in industrial control applications exceeds 90% even nowadays [2]. Thus, it is still worth researching on the tuning of PI(D) controllers, especially for systems under uncertainty. Obviously, the application of robustly stabilizing or robustly performing PI(D) controllers for interval plants, or more generally, for systems with parametric uncertainty, represents an attractive research field [3], [4], because these relatively simple LTI models with uncertainty may cover a wide spectrum of real-world controlled systems.

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Great attention has been paid to the development of the methods for obtaining sets of parameters of (nominally) stabilizing PI(D) controllers for various plants. For example, D-decomposition method, which was revisited in [5], the parameter space approach [6], the singular frequency method [7], the Hermite-Biehler approach [4], or stability boundary locus-based method [8], [9] belong among the most popular techniques. However, there are many other alternative methods available in the literature. Several of them will be mentioned in the following sentences. The computation of all stabilizing PI(D) controllers by using the Kronecker summation method was presented in [10]. A Nyquist plot-based technique for calculation of KP-stable regions was developed in [11] (for time delay and PI controller parameters) and [12] (for PID controller parameters). A Lyapunov equation-based stability mapping approach can be found in [13], [14]. A combined approach to determine robustly stabilizing parameter spaces was proposed in [15]. Naturally, a range of results is focused on the stabilization of time-delay plants. In addition to some of the abovementioned works that are also applicable to time-delay systems, it is worth mentioning [16], [17]. Moreover, PI(D) controller design based on generalized stability boundary locus for various types of time-delay plants was introduced in [18], [19], [20]. Furthermore, there are also overview papers available, such as [21], which compared the parameter space approach and the Hermite-Biehler theorem-based approach.

Many of the abovementioned methods may be extended in order to be applicable for some kind of systems with parametric uncertainty, frequently for the case of interval plants. Thus, the robust stability is requested instead of the (nominal) stability. A typical tool that is employed in this regard is the sixteen plant theorem [22], [23]. This extreme point-based result allows the determination of robustly stabilizing controllers through the simultaneous stabilization of sixteen Kharitonov plants. However, it is valid only for the first-order controllers (such as PI controllers). If the PID controller is used, some more general non-extremal results have to be applied, such as Kharitonov segments (see, e.g., [24]), which are known from the generalized Kharitonov theorem [25] and similar thirty-two edge theorem [23], [25].

Despite the fact that robust stability is the fundamental requirement, a certain level of performance is usually also demanded in practical control applications. If the performance needs to be ensured even for the worst case from the family of controlled plants, one speaks about robust performance. The classical measures of control system performance are, among others, well-known gain margin and phase margin. Their utilization in the robust performance context was studied, e.g., in [26]–[30]. Stabilization of a fixed-parameter plant for specified gain and phase margins through the stability boundary locus method was presented in [9], [31]. Some other approaches to mapping the performance requirements into the parameter space can be found in [32]–[34].

Chemical reactors can be seen as the center of all chemical process industries [35]. Their control represents a nontrivial task since they usually suffer from complicated behavior and possible safety problems. The classical idealized types of reactors include batch reactor, Continuous Stirred Tank Reactor (CSTR), and plug flow reactor. A CSTR contains a vessel surrounded by a jacket for heating or cooling, an agitator for (perfect) mixing inside the vessel, feed lines that enter, and a liquid product stream that exits the vessel. A composition and a temperature of the product stream are the same as the contents of the liquid throughout the vessel. For more details, see [36].

Control of CSTRs is the essential and deeply studied discipline, and robust control techniques provide the promising results in this regard. For example, the application of robust static output feedback control to a CSTR was presented in [37]. The same CSTR, described by a model with interval uncertainty, was robustly stabilized by means of PI controller either via stability boundary locus method in [38] or via Kronecker summation method in [39]. Possible approaches to designing robust PID controllers for CSTRs can be found, e.g., in [40], [41]. Furthermore, the paper [42] provides a comparison of three robust control methods for use in CSTR control. An example of an alternative approach, represented by the linear matrix inequalities-based robust model predictive control, was addressed in [43].

This paper deals with robust control of interval plants using PI controllers with gain and phase margin specifications. Thus, it focuses on finding the regions of PI controller parameters, where not only closed-loop robust stability but also some level of closed-loop robust performance, represented by the required gain and phase margin, is ensured for all potential members from the interval family of controlled plants. In fact, this work intends to extend the interesting results published previously in the papers [9], [31], where the generalization of stability boundary locus method for specified gain and phase margins is introduced and further discussed, but where the presented applications are limited to the controlled plants with fixed parameters. The current article combines the mentioned technique with the sixteen plant theorem, which allows its direct utilization for the interval plants. The application of the improved method is demonstrated on a CSTR, which is described by the linearized mathematical model with interval uncertainty, i.e., the robust performance region of PI controller parameters for given gain and phase margin specifications is obtained. Note that the same interval plant model was robustly stabilized by PI controllers using various methods in [38], [39], but the present paper addresses not only robust stability but also robust performance problem.

The article is structured as follows. The existing stability boundary locus method for the robust stabilization of interval plants through PI controllers is reminded in Section 2. Then, Section 3 adopts the generalization of this stabilization approach, which allows the additional gain and phase margin specifications, and extends the idea for interval plants.

The extensive Section 4 applies the modified method to a CSTR. Namely, the mathematical model is presented, and the relevant robust stability and robust performance regions are determined in this Section. The final Section 5 brings some concluding remarks.

## II. ROBUST STABILIZATION OF INTERVAL PLANTS VIA PI CONTROLLERS

Assume the fundamental closed-loop control system with an interval plant, which is shown in Fig. 1.

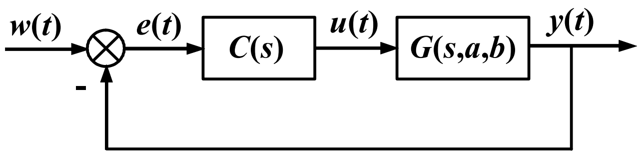


**FIGURE 1. Feedback control loop with an interval plant.**

In accordance with the conventions, the signals $w(t)$, $e(t)$, $u(t)$ and $y(t)$ from Fig. 1 represent the reference signal, control (tracking) error, control (actuating) signal and controlled (output) signal, respectively. The block $C(s)$ symbolizes an ordinary PI controller in the form:

$$C(s) = P + \frac{I}{s} = \frac{Ps + I}{s} \tag{1}$$

and the block $G(s, a, b)$ is a strictly proper interval plant:

$$G(s, b, a) = \frac{B(s, b)}{A(s, a)} \tag{2}$$

with the numerator and denominator polynomials:

$$B(s, b) = \sum_{i=0}^{m} \left[b_i^-, b_i^+\right] s^i \tag{3}$$

$$A(s, a) = s^n + \sum_{i=0}^{n-1} \left[a_i^-, a_i^+\right] s^i \tag{4}$$

where superscripts "−" and "+" denote the lower and upper bounds of the relevant parameters, respectively, and where $m < n$.

The robust stabilization of interval plant (2) by means of PI controller (1) can be solved using simple but elegant extreme-based result known as the sixteen plant theorem [22], [23]. According to this principle, a first-order controller (in most practical cases PI controller (1)) robustly stabilizes the interval plant (2) if and only if this controller stabilizes each of the sixteen Kharitonov plants. Note that the Kharitonov plants:

$$G_{k,l}(s) = \frac{B_k(s)}{A_l(s)}, \quad k, l \in \{1, 2, 3, 4\} \tag{5}$$

can be easily assembled by taking the Kharitonov polynomials of the numerator ($B_1$ to $B_4$) and denominator ($A_1$ to $A_4$) polynomials. All combinations (four polynomials in the numerator, and four in the denominator) lead to the mentioned sixteen Kharitonov plants. Remind that the Kharitonov polynomials for the numerator polynomial (3) are constructed as follows [44]:

$$\begin{aligned} B_1(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots \\ B_2(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots \\ B_3(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots \\ B_4(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots \end{aligned} \tag{6}$$

and, naturally, the scheme is analogous for the denominator polynomial (4) as well.

Thus, the problem of robust stabilization of an interval plant may be transformed into the task of simultaneous stabilization of sixteen (or less in special cases) "ordinary" plants with fixed parameters. Now, the attention will be turned to this partial stabilization.

A relatively simple but effective graphical approach for the determination of the stabilizing PI controller based on obtaining the stability boundary locus [8], [9] will be utilized.

Consider a fixed-parameter plant given by a standard transfer function (e.g., consider one of the Kharitonov plants, but the subscripts $k$ and $l$ will be omitted for simplicity in the following equations), in which the numerator and denominator polynomials are decomposed into their even and odd parts, and the complex variable $s$ is substituted by $j\omega$:

$$G(j\omega) = \frac{B_E(-\omega^2) + j\omega B_O(-\omega^2)}{A_E(-\omega^2) + j\omega A_O(-\omega^2)} \tag{7}$$

The subscripts "$E$" and "$O$" symbolize even and odd parts of the numerator or denominator polynomials, respectively.

In further steps, the closed-loop characteristic polynomial of the feedback system with controller (1) and the decomposed plant (7) is calculated, and its real and imaginary parts are equaled to zero (for more details see [8], [9]). This results in the equations for the proportional and integral parameters:

$$\begin{aligned} P(\omega) &= \frac{X_5(\omega)X_4(\omega) - X_6(\omega)X_2(\omega)}{X_1(\omega)X_4(\omega) - X_2(\omega)X_3(\omega)} \\ I(\omega) &= \frac{X_6(\omega)X_1(\omega) - X_5(\omega)X_3(\omega)}{X_1(\omega)X_4(\omega) - X_2(\omega)X_3(\omega)} \end{aligned} \tag{8}$$

where

$$\begin{aligned} X_1(\omega) &= -\omega^2 B_O(-\omega^2) \\ X_2(\omega) &= B_E(-\omega^2) \\ X_3(\omega) &= \omega B_E(-\omega^2) \\ X_4(\omega) &= \omega B_O(-\omega^2) \\ X_5(\omega) &= \omega^2 A_O(-\omega^2) \\ X_6(\omega) &= -\omega A_E(-\omega^2) \end{aligned} \tag{9}$$

The contemporaneous solution of the parametric relations (8) for an appropriate set of frequencies $\omega$ defines the stability boundary locus in the P-I plane. The resulting parametric curve and the line $I = 0$ divide the P-I plane into the regions of stability or instability. The stable areas can be ascertained simply by choosing a test point within each region. Remind that the final stability region need not be a convex set. Although the technique is quite fast and effective, proper frequency gridding may represent an important issue. Nevertheless, the range of inspected frequencies can be reduced by means of the Nyquist plot-based method from [45], which is utilized in [8], [9], according to which the change of stability to instability and vice versa may occur only at frequencies that fulfill:

$$Im\left[G(j\omega)\right] = 0 \tag{10}$$

To sum up, the sixteen stability regions obtained via the above-described procedure, one for each of the sixteen Kharitonov plants (5), need to be plotted in the P-I plane, and their mutual intersection defines the final robust stability region, that is the region of parameters of PI controllers (1) that robustly stabilize the interval plant (2).

## III. ADDITIONAL GAIN AND PHASE MARGIN SPECIFICATIONS

Stabilization of a plant model with fixed parameters for specified gain and phase margins has been presented in [9], [31]. The technique uses the feedback control loop that is enriched by a virtual gain-phase margin tester $Me^{-j\theta}$ [26], [27], [30]. Such a feedback loop with an interval plant is shown in Fig. 2.

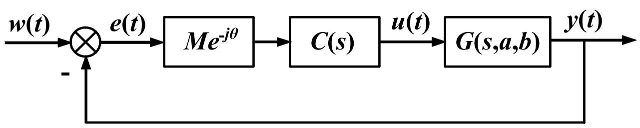


**FIGURE 2.** Feedback control loop with an interval plant and gain-phase margin tester.

First, assume a controlled plant in the form of standard transfer function with fixed parameters, i.e., assume that $G(s, a, b)$ in Fig. 2 is replaced by $G(s)$ and consequently rewritten to (7). Following the analogous procedure as outlined in Section 2, the stability boundary locus (or rather performance boundary locus in this case) for PI controllers (1) can be computed by solving the parametric equations (8) simultaneously, but now a set of relations (9) is modified to [9]:

$$\begin{aligned}
X_1(\omega) &= M\left(\omega B_E(-\omega^2)\sin\theta - \omega^2 B_O(-\omega^2)\cos\theta\right)\\
X_2(\omega) &= M\left(B_E(-\omega^2)\cos\theta + \omega B_O(-\omega^2)\sin\theta\right)\\
X_3(\omega) &= M\left(\omega B_E(-\omega^2)\cos\theta + \omega^2 B_O(-\omega^2)\sin\theta\right)\\
X_4(\omega) &= M\left(\omega B_O(-\omega^2)\cos\theta - B_E(-\omega^2)\sin\theta\right)\\
X_5(\omega) &= \omega^2 A_O(-\omega^2)\\
X_6(\omega) &= -\omega A_E(-\omega^2)
\end{aligned} \tag{11}$$

The performance boundary locus for a selected gain margin $M$ can be obtained by equaling $\theta$ to zero in (11), whereas assuming that $M = 0$ results in the performance boundary locus for a chosen phase margin $\theta$. The intersection of two relevant performance regions (the first one for guaranteed minimum gain margin, and the second one for guaranteed minimum phase margin) leads to the region that ensures the given minimum gain margin and phase margin at the same time.

The papers [9] and [31] used this approach for the "stabilization" of a fixed-parameter plant. Nevertheless, this work extends the idea and applies the technique to an interval plant. It was proved that the sixteen plant theorem is valid not "only" for the robust stability, but the performance specifications (such as gain margin and phase margin) are also satisfied [46], [4], [23]. For example, it was shown that the worst-case $H_\infty$ norm is related to one of the sixteen Kharitonov plants [47], [48], [23]. Moreover, it was presented that the outer boundary of the Nyquist envelope of a strictly proper stable interval plant is covered by the Nyquist plots of the sixteen Kharitonov plants [49], [50]. All in all, the worst-case gain margin and phase margin of the feedback control system with a first-order controller (typically PI controller (1)) and a strictly proper interval plant (2) can be determined on the basis of gain margins and phase margins of the related sixteen Kharitonov plants. A combination of this fact with the stability boundary locus-based method [9] will be applied to designing a robust PI controller for a CSTR in the following Section 4.

## IV. APPLICATION TO A CONTINUOUS STIRRED TANK REACTOR

### A. MATHEMATICAL MODEL OF A CONTINUOUS STIRRED TANK REACTOR

Propylene glycol is an organic compound that is industrially produced by the hydrolysis of propylene oxide with an excess of water [51]. This hydrolysis in a CSTR has been selected as a controlled process [37], [52], [38], [39]. The corresponding chemical reaction is described by the equation:

$$C_3H_6O + H_2O \rightarrow C_3H_8O_2 \tag{12}$$

Not only the reactants (propylene glycol and water) but also methanol is fed in a CSTR, in order to increase the propylene oxide solubility in water. The surplus water assures higher selectiveness to propylene glycol and removes consecutive reactions of propylene glycol with propylene oxide. The reaction has first-order kinetics with regard to propylene oxide as a crucial component. The dependence of the reaction rate constant on the temperature can be expressed by the Arrhenius law:

$$k = k_\infty e^{-\frac{E}{RT_r}} \tag{13}$$

where $k$ is the reaction rate constant, $k_\infty$ represents the pre-exponential factor, $E$ stands for the activation energy, $R$ means the universal gas constant, and $T_r$ signifies the temperature of the reaction mixture [37], [52], [38], [39].

Suppose that a CSTR is ideally mixed, reacting volume is constant, and the volumetric flow rate of the inlet stream equals to the volumetric flow rate of the outlet stream. Then, the mass balance for any species of the system is given as:

$$V_r \frac{dc_n}{dt} = q_r\left(c_{n0} - c_n\right) + V_r \nu_n r, \quad n = 1, 2, 3 \tag{14}$$

where $V_r$ represents the reacting volume, $c_n$ stands for the molar concentration of the $n$th component, $c_{n0}$ means the feed molar concentration of the $n$th component, $q_r$ signifies the volumetric flow rate of the reaction mixture, $\nu_n$ is the stoichiometric coefficient of the $n$th component, and $r = kc_{C_3H_6O}$ determines the molar ratio of the chemical reaction [37], [52], [38], [39].

Moreover, it is supposed that the particular heat capacities, densities, and volumetric flow rates are independent of temperature or mixture composition. Furthermore, the mixing volume and the heat of mixing are assumed to be neglected. Thus, the simplified enthalpy balances of the reaction mixture and the cooling medium are, respectively [53], [37], [38], [39]:

$$V_r \rho_r c_{pr} \frac{dT_r}{dt} = q_r \rho_r c_{pr} (T_{r0} - T_r) - UA (T_r - T_c) + V_r \left(-\Delta_r H^o\right) r \quad (15)$$

$$V_c \rho_c c_{pc} \frac{dT_c}{dt} = q_c \rho_c c_{pc} (T_{c0} - T_c) + UA (T_r - T_c) \quad (16)$$

where $T$ stands for the temperature, $\rho$ means the density, $c_p$ represents the specific heat capacity, $\Delta_r H^o$ is the reaction enthalpy, $U$ signifies the overall heat transfer coefficient, and $A$ stands for the heat exchange area. Moreover, the subscript symbols have the following meaning: 0 represents the feed, $c$ is the cooling medium, and $r$ means the reaction mixture.

The values of steady-state inputs and constant parameters can be found in [37], [38] (see Table 1 in these papers). Besides, there are three physical parameters, namely reaction enthalpy, pre-exponential factor, and overall heat transfer coefficient, that may vary within the intervals. Their minimum, maximum, and mean values are presented again in works [37], [38] (see Table 2 in them).

The linearized single-input single-output model of the CSTR, with the reaction mixture flow rate $q_r$ $\left[\mathrm{m^3 min^{-1}}\right]$ as the control input (the other inputs are constant) and the reaction mixture temperature $T_r$ [K] as the controlled output, was introduced in [38] and studied in [39] as well. The model is given by the fourth-order transfer function:

$$G(s, b, a) = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (17)$$

whose parameters are supposed to vary within the following bounds:

$$\begin{aligned} b_2 &\in [-0.0291,\ -0.0245] \\ b_1 &\in [-0.0199,\ -0.0127] \\ b_0 &\in [-0.0005740,\ -0.0003549] \\ a_3 &\in [0.5801,\ 0.9030] \\ a_2 &\in [0.1002,\ 0.2299] \\ a_1 &\in [0.0062,\ 0.0142] \\ a_0 &\in [0.0001094,\ 0.0002412] \end{aligned} \quad (18)$$

To summarize, the utilized mathematical model of a controlled process in the CSTR is given by the family of plants with interval uncertainty (17), (18), i.e., by the interval plant for short.

### B. ROBUST STABILITY

First, one of the Kharitonov plants (5) of the interval plant (17) with the uncertain coefficients (18) will be chosen, and the region of stabilizing PI controllers will be calculated and plotted.

Consider, e.g., the Kharitonov plant $G_{4,2}(s)$:

$$G_{4,2}(s) = \frac{B_4(s)}{A_2(s)} = \frac{b_2^+ s^2 + b_1^+ s + b_0^-}{s^4 + a_3^- s^3 + a_2^- s^2 + a_1^+ s + a_0^+} = \frac{-0.0245s^2 - 0.0127s - 0.000574}{s^4 + 0.5801s^3 + 0.1002s^2 + 0.0142s + 0.0002412} \quad (19)$$

The even and odd parts of the polynomials in numerator or denominator of the transfer function in the form (7) are as follows:

$$\begin{aligned} B_E(-\omega^2) &= 0.0245\omega^2 - 0.000574 \\ B_O(-\omega^2) &= -0.0127 \\ A_E(-\omega^2) &= \omega^4 - 0.1002\omega^2 + 0.0002412 \\ A_O(-\omega^2) &= -0.5801\omega^2 + 0.0142 \end{aligned} \quad (20)$$

Substituting (20) into (9) and (8) leads to the parametric equations for the proportional and integral gain of the PI controller. Their simultaneous solution for a range of nonnegative frequencies $\omega$ results in the stability boundary locus. The attained curve together with $I = 0$ line splits the P-I plane into the stability and instability regions that are depicted in the P-I plane in Fig. 3.

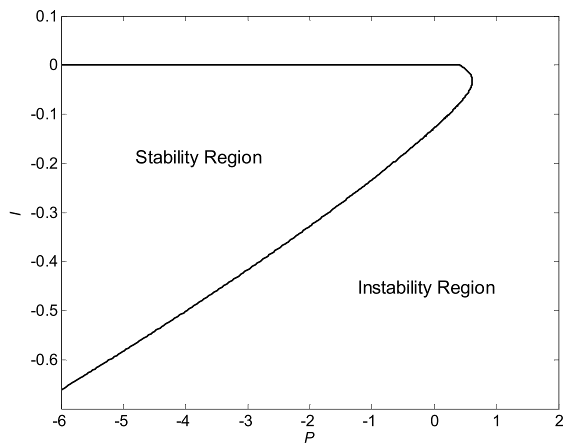


**FIGURE 3.** Stability region for the Kharitonov plant (19).

In order to obtain the robust stability region for the original interval plant (17), (18), the previous procedure has to be repeated, and the partial stability regions need to be calculated for all remaining fifteen Kharitonov plants. Fig. 4 shows the resulting set of sixteen stability regions.

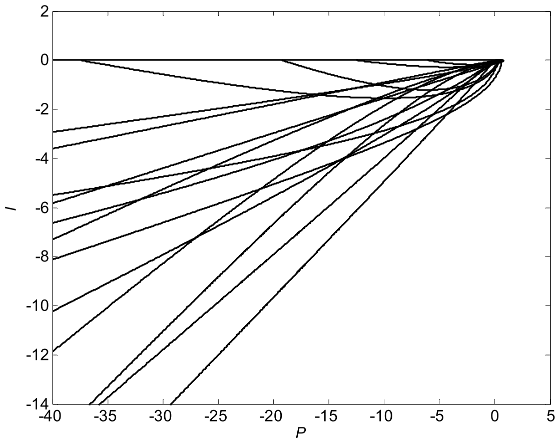


**FIGURE 4.** Stability regions for all sixteen Kharitonov plants of (17).

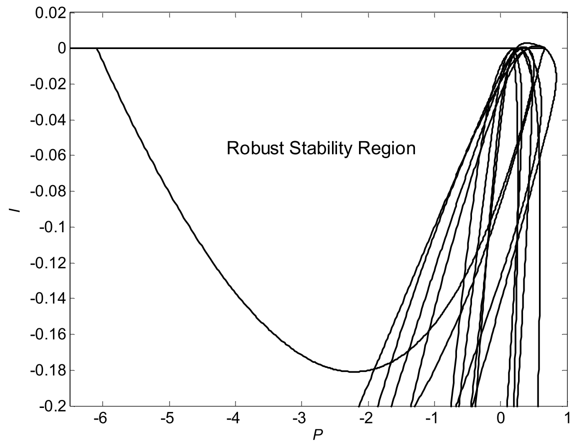


**FIGURE 5.** Robust stability region for the interval plant (17), (18).

The zoomed version of Fig. 4 can be seen in Fig. 5. It offers a closer look at the intersection of the sixteen stability regions that defines the robust stability region.

So, all PI controllers (1) with the parameters $P$ and $I$ lying inside the robust stability region shown in Fig. 5 guarantee that the interval plant (17), describing the CSTR, is stabilized by the feedback loop (Fig. 1) for all possible parameters (18).

### C. ROBUST PERFORMANCE

In the next step, the aim is to find a set of PI controllers that are not only robustly stabilizing, but that also ensure the required level of robust performance, expressed by the minimum gain margin and phase margin for the worst-case member of the interval plant family.

For the initial demonstration, assume the same Kharitonov plant $G_{4,2}(s)$ with the transfer function (19) as in the previous sub-section 4.2. Thus, the even and odd parts of the numerator and denominator of (7) are again in the form (20). Then, the parts (20) are used in the modified set of auxiliary variables (11), where $M$ and $\theta$ are the chosen gain margin and phase margin, respectively, and subsequently, (11) is put into the parametric equations (8).

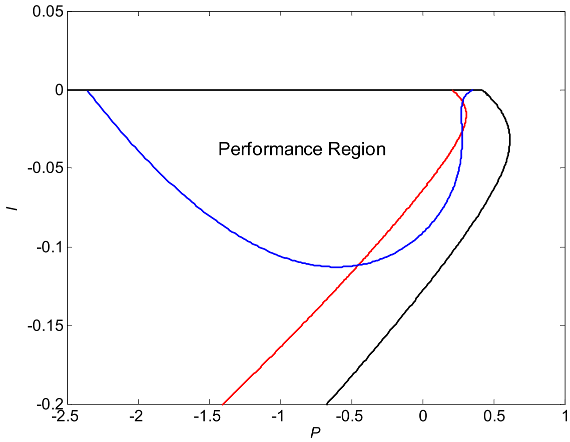


**FIGURE 6.** Performance region for $M > 2$ ($\approx$ 6 dB) and $\theta > 30°$ for the Kharitonov plant (19).

Assume the gain margin $M = 2(\approx 6\text{dB})$ and phase margin $\theta = 30°$.

First, both cases will be examined separately. The performance boundary locus for $M = 2$ is drawn in Fig. 6 by the red curve. The left-hand side area from this red curve represents the region in which the gain margin is guaranteed to be greater than 2. Moreover, the performance boundary locus for $\theta = 30°$ is depicted in Fig. 6 by using the blue curve. The interior of the blue shape (with the upper limit given by the line $I = 0$) defines the region with the phase margin greater than 30°.

The intersection of the red gain margin region and the blue phase margin region from Fig. 6 delimits the region in which the minimum gain margin 2 and minimum phase margin 30° are guaranteed at the same time. By way of illustration, Fig. 6 contains the stability region from Fig. 3 as well (black curve).

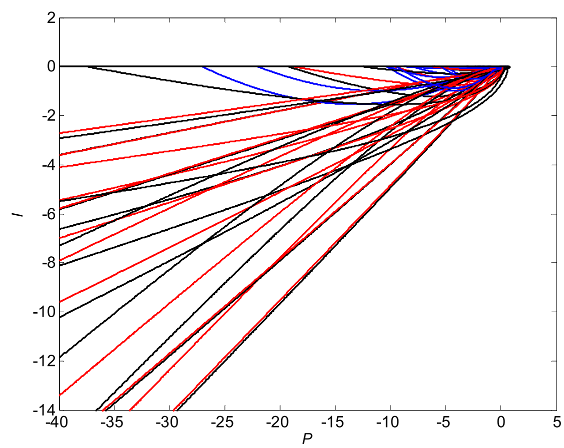


**FIGURE 7.** Gain margin regions (red curves), phase margin regions (blue curves), and stability regions (black curves) for all sixteen Kharitonov plants of (17).

The location of the robust performance region for the original interval plant (17) with the uncertain coefficients (18) demands the repetition of this procedure for the remaining fifteen Kharitonov plants. The Fig. 7 depicts the set of sixteen gain margin regions (red curves), phase margin regions (blue curves), and stability regions (black curves – for the complete idea).

The intersection of all thirty-two gain and phase margin regions leads to the final robust performance region, which is zoomed in Fig. 8.

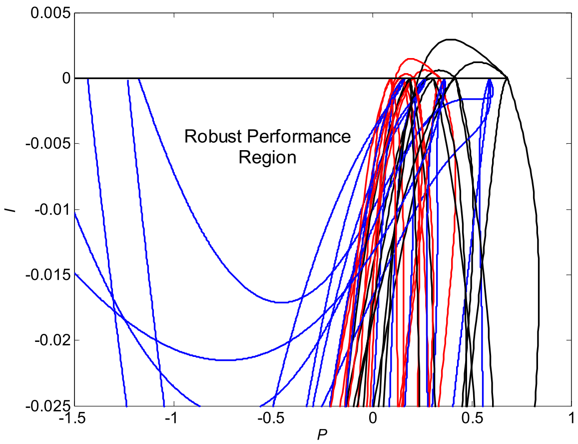


**FIGURE 8. Robust performance region for the interval plant (17), (18).**

Thus, all PI controllers (1) with the parameters $P$ and $I$ located inside the robust performance region from Fig. 8 ensure not only robust stabilization of a CSTR modeled by the interval plant (17), (18), but furthermore, it guarantees the minimum gain margin 2 and the minimum phase margin 30° even for the worst-case member of the interval plant family.

## V. CONCLUSION

This paper was aimed at the computation of robustly performing PI controllers with prescribed gain and phase margin specifications for interval plants. The roots of the presented method lie in the existing principle, known as the stability boundary locus method, which was extended in order to generate robust performance regions of PI controller parameters for interval plants. Subsequently, this approach was applied to a mathematical model of a CSTR with interval uncertainty. In the examples, this model was not only robustly stabilized but also robust performance region with the minimum gain margin 2 and the minimum phase margin 30° for all members of the interval plant family was found.

Note that the proposed method does not provide one specific optimal PI controller, but it gives the whole set of the robustly performing PI controllers that fulfill given specifications (if such controllers exist), and so the final choice depends on other user preferences. The method itself does not suffer from conservatism, and the obtained robust performance regions are valid exactly for the interval plants, but a conservatism may be presented due to potentially circumspect modeling of a real-world controlled system (such as nonlinearly behaving CSTR) by an LTI system with interval uncertainty.

Potential future research may be oriented to further progress in the area of robust performance of the feedback control loops with interval plants and PID controllers.

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