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Disturbance rejection FOPID controller design in v-domain

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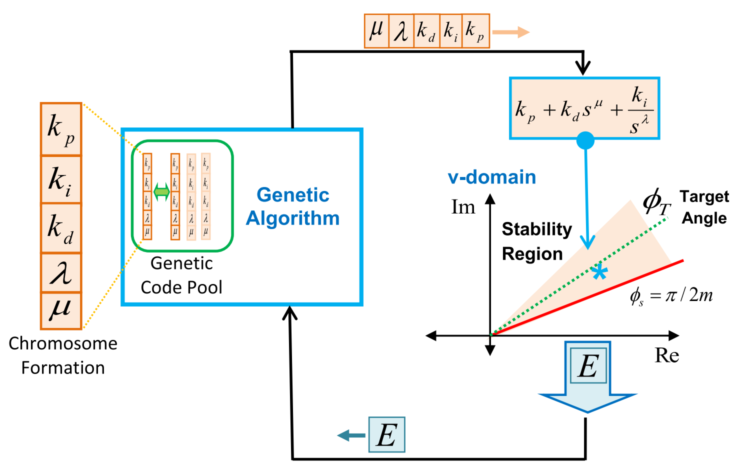
HIGHLIGHTS

- This study introduces a v-domain design scheme for optimal FOPID controllers.
- The purposed method is based on minimum angle system pole placement in v-plane.
- Multi-objective genetic algorithm is used for controller coefficient optimization.
- Disturbance rejection FOPID controller design examples are illustrated.

GRAPHICAL ABSTRACT



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ABSTRACT

Due to the adverse effects of unpredictable environmental disturbances on real control systems, robustness of control performance becomes a substantial asset for control system design. This study introduces a v-domain optimal design scheme for Fractional Order Proportional-Integral-Derivative (FOPID) controllers with adoption of Genetic Algorithm (GA) optimization. The proposed design scheme performs placement of system pole with minimum angle to the first Riemann sheet in order to obtain improved disturbance rejection control performance. In this manner, optimal placement of the minimum angle system pole is conducted by fulfilling a predefined reference to disturbance rate (RDR) design specification. For a computer-aided solution of this optimal design problem, a multi-objective controller design strategy is presented by adopting GA. Illustrative design examples are demonstrated to evaluate performance of designed FOPID controllers.

Introduction

Disturbance rejection performance is a major concern to achieve robust control performance in real control applications. Since unavoidable interference of unpredictable environmental disturbances on negative feedback control loops, control performance of real control systems can be severely deteriorated in practical control applications, even though simulation results of these systems indicate a satisfactory control performance. Therefore, system designers should ensure a necessary degree of disturbance rejection control performance in design tasks of practical controllers.

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Throughout the last decades, researchers have been revealed several assets of Fractional Order Control (FOC) systems. They have claimed that controllers with fractional order derivative and integral elements can enhance control performance robustness [1–6]. After a common agreement on the fact that non-integer order modeling provides more practical mathematical model of real systems compared to integer order counterparts, the control systems field has progressed towards this new concept. Correspondingly, fractional order system perspectives have gained more importance and utilized for enhancement of frequency domain representation of control systems to procure more rigorous design objectives in frequency domain design that can allow robust control performance.

In the literature, many research works have revealed that fractional order controllers can exhibit more performance robustness against parametric model perturbations compared to integer order controllers [2–6]. Enhancement of disturbance rejection performance by using FOC was highlighted in several studies [7–10]. Such improvements in the control performance are shown to the fact that fractional orders elements of FOPID controller can provide more tuning options that give more freedom to fulfill design objectives. This merit allows further improvement of control performance compared to conventional PID controllers [11–13].

System stability is an essential concern to be completed in for control system design tasks before consideration of controller performance concerns. For this reason, stabilization of non-integer order control systems has been deeply studied, and many works only addressed stability of such systems in several perspectives; stabilization of the systems according to system pole placements [14–23], closed loop system stabilization based on stability boundary locus (SBL) analyses [24], stabilization by means of zero exclusion principle and value set analysis [25,26]. Linear Matrix Inequalities (LMI) technique was proposed for stability checking of fractional order systems [27–29]. Also, graphical stabilization methods have been proposed for robust stabilization of plant models with interval uncertainty [30,31]. Stabilization of fractional order PID controllers for uncertain fractional order systems was shown by using robust D-stability method [32]. Some recent theoretical works address stability of more complicated system models such as stochastic nonlinear delay systems [33], impulsive stochastic delay differential systems [34] and semi-Markov switched stochastic systems [35].

In practical controller design, stability and performance objectives should be considered to reach optimal control performance that can meet requirements of real world control applications. Essentially, optimal tuning problems of controllers can be reduced to an attempt to determining the best controller coefficients in a set of stabilizing controller options. For this design strategy, the stabilization objective turns into a principal component of optimal controller design tasks. Optimal controller design efforts have widely intensified in two design domains: (i) frequency domain methods, for instance, phase margin objective of loop shaping techniques is used to ensure system stability [11–13] and (ii) time domain methods, for instance, sum of square of control error is used to obtain a stable system response [36,37]. Design opportunities of these two domains are largely exploited in numerous controller design works, and a fresh design domain can open up new design options. In this manner, the current study presents a design framework for exploitation of the first Riemann sheet of conformal mapping $s = v^m$, which is called v-domain, for optimal fractional order controller design efforts. This study illustrates a design framework for disturbance rejection FOPID controllers in v-domain. Previously, we have demonstrated the v-domain for optimal stabilization of control systems by placing the minimum angle system pole to a target region [15,21,22]. A recent study demonstrates a disturbance rejection FOPID controller design in v-domain by using particle swarm optimization [23]. The current study extends this approach to solve multi-objective optimal controller design problem by using a GA and aims improvement of disturbance reject control performance of resulting FOPID controller design. In this manner, a multi-objective optimization scheme is proposed to deal with positioning of the system pole with minimum angle inside the stability region of v-domain so as to enhance disturbance rejection capability of the stabilized FOPID controller.

Disturbance rejection objective can be implemented by using Reference to Disturbance Ratio (RDR). The RDR index essentially expresses ratio of spectral power density of the reference signal to spectral power density of disturbance signal at output of a closed loop control system. It was proposed to measure additive input disturbance rejection performance of closed loop control systems [38–40]. In several works, RDR index have been utilized for measuring the disturbance rejection capacity of FOPID systems [41] and disturbance rejection constraints of controller tuning tasks [42–44].

The Rest of the paper is organized as follows: Section 2 provides brief theoretical background on stability analysis of FOC systems, brief introductions of RDR index and GA. Section 3 presents problem formulations to adopt GA for optimal tuning of FOPID control systems. The following section illustrates example designs to demonstrate effectiveness of the method by comparing results of proposed approach with results of other design methods.

Mathematical background and preliminary knowledge

Stability of fractional order systems via minimum angle root

Continuing developments in fractional calculus [1,2] allow better mathematical representation of real systems. Therefore, fractional order system models have been widely preferred to improve real-world performance of control systems. A linear time invariant (LTI) system models are widely used for modeling control systems and these models are written by constant coefficient fractional order differential equations in a general form [1] of

$$a_m D^{\alpha_m} y + a_{m-1} D^{\alpha_{m-1}} y + \dots + a_2 D^{\alpha_2} y + a_1 D^{\alpha_1} y + a_0 D^{\alpha_0} y = b_n D^{\phi_n} u + b_{n-1} D^{\phi_{n-1}} u + \dots + b_2 D^{\phi_2} u + b_1 D^{\phi_1} u + b_0 D^{\phi_0} u, \tag{1}$$

By applying Laplace transform to both side of Eq. (1), fractional order LTI systems can be commonly represented in transfer function form [45,46] as

$$T(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{i=0}^{m} b_i s^{\phi_i}}{\sum_{i=0}^{n} a_i s^{\alpha_i}}, \tag{2}$$

where $a_i \in R$ and $b_i \in R$ are the coefficients of the denominator and the numerator polynomials, respectively. Parameters $\alpha_i \in R$ and $\phi_i \in R$ represent fractional orders of transfer function of systems. For stability analysis, roots of the characteristic polynomial are calculated. The characteristic polynomial of Eq. (2) is written by

$$\Delta(s) = \sum_{i=0}^{n} a_i s^{\alpha_i}. \tag{3}$$

To facilitate root calculations, conformal mapping $s = v^m$ are commonly utilized to obtain Expanded Degree Integer Order Characteristic Polynomials (EDIOCPs) [16–18] and stability checking of the non-integer order models is conducted by considering root locus of EDIOCPs within the first Riemann sheet. The EDIOCPs are expressed by applying $s = v^m$ to Eq. (3) as

$$\Delta_m(v) = \sum_{i=0}^{n} a_i v^{(m\alpha_i)}. \tag{4}$$

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In this method, roots within the first Riemann sheet, which are called the principle characteristic roots [47], are used for checking stability of the fractional order systems [14–18,20,46,47]. The conformal mapping $s = v^m$ maps stability region of complex s-plane, which is the left half plane (LHP), to a slice of v-plane that is confined by the angular range of $[\pi/2m, \pi/m]$. Fig. 1 depicts the deformation of stability region toward to right half plane of v-plane by conformal mapping $s = v^m$. According to the figure, FOC system is stable when all principle characteristic roots [47] reside in the stability region $[\pi/2m, \pi/m]$. For fractional order dynamic systems, validity of LHP stability has been demonstrated by introducing principle characteristic equation in v-domain [47]. The stability boundary with the angle of $\theta_s = \pi/2$ in s-plain is mapped to the angle of $\varphi_2 = \pi/2m$ in v-plane.

For the stability analysis, the principle root set of EDIOCPs $\Delta_m(v)$ is written by [18,20,47]

$$R = \{v_r \ : \ \Delta_m(v_r) = 0 \wedge |\arg(v_r)| < \frac{\pi}{m}, \ \ r = 1, 2, 3, .. \}. \tag{5}$$

Therefore, $\Delta_m(v)$ is robustly stable, if the following condition is hold. [14,17,19,21,46]

$$\min\{Arg(R)\} > \pi/2m \tag{6}$$

RDR indices for disturbance reject FOPID controllers

Similar to SNR measure to express signal transmission quality in communication channels, the RDR was proposed for closed loop control systems [38–40] and it expresses a measure for assessment of additive input disturbance rejection performance of closed loop control systems. Analysis on closed loop control systems demonstrated that RDR index of negative feedback closed loop control system is equal to spectral power density of controller function [39] and RDR spectrum was expressed in the form of [39,40]

$$RDR(\omega) = |C(j\omega)|^2 \tag{7}$$

where, the frequency domain representation of controller transfer function is denoted by $C(j\omega)$ and it is obtained by using $s = j\omega$. Higher RDR values increase disturbance rejection control performance of closed loop control systems. The RDR spectrum is expressed in decibel (dB),

$$RDR_{dB}(\omega) = 20\log|C(j\omega)|. \tag{8}$$

Conventional FOPID controller is generally expressed with the following transfer function.

$$C(s) = k_p + k_d s^{\mu} + \frac{k_i}{s^{\lambda}}, \tag{9}$$

In Eq. (9), k_p, k_d and k_i are controller coefficients, and λ and μ are real valued derivative and integral orders of FOPID controllers. The

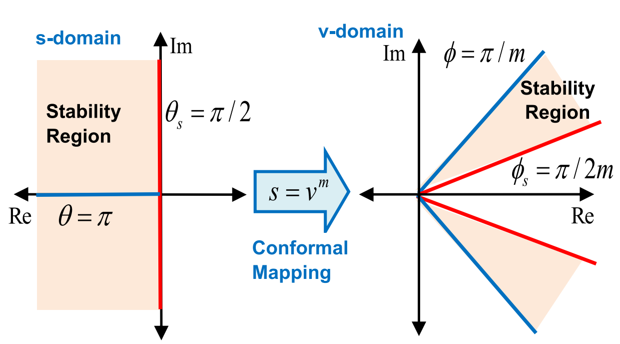

Fig. 1. Effect of $s = v^m$ conformal mapping on LHP stability region of s-plain [14,18–19,21].

RDR of closed loop FOPID control system were derived regarding the design parameters of FOPID controller as [39]

$$RDR(\omega) = \left(k_p + k_i\cos(\tfrac{\pi}{2}\lambda)\omega^{-\lambda} + k_d\cos(\tfrac{\pi}{2}\mu)\omega^{\mu}\right)^2 + \left(k_d\sin(\tfrac{\pi}{2}\mu)\omega^{\mu} - k_i\sin(\tfrac{\pi}{2}\lambda)\omega^{-\lambda}\right)^2. \tag{10}$$

For the purpose of designing a disturbance reject FOPID controller, the following RDR constraint is used to determine the lower bound of disturbance rejection capacity in a given operating frequency range of $\omega \in [\omega_{\min}, \omega_{\max}]$:

$$\min\{RDR_{dB}(\omega)\} \geqslant M \text{ for } \omega \in [\omega_{\min}, \omega_{\max}], \tag{11}$$

The disturbance rejection design specification $M \in R$ defines a lower boundary for minimum RDR performance of resulting control system [40]. Accordingly, Eq. (11) ensures RDR performance to be equal or greater than M in the frequency range of $\omega \in [\omega_{\min}, \omega_{\max}]$.

Optimization by genetic algorithms

Metaheuristic search methods such as GA have foremost importance in computational intelligence practice because of their advantages of providing straightforward computational scheme for solution of very sophisticated optimization problems [50–52]. The stochastic search nature of GA allows searching a better solution at each run of the algorithms and multi-running of these algorithms is considered a way to deal with local-minima problems of multimodal optimization problems [53]. Therefore, GA have become a fundamental optimization tools for computer-aided system design and optimization problems.

Due to difficulties in obtaining analytical solution of arbitrary order differential equations, metaheuristic optimization methods have been widely preferred to solve optimization problems associated with fractional order system design [21–23,36,37,48,49]. In the current study, due to its proven search capability and wide utilization, we preferred GA to solve optimization problem associated with the minimum angle pole placement in v-plane.

The GA is fundamentally categorized as population-based search methods, and it essentially mimics mechanisms in genetic science. [50–53] The GA exploits genetic and evolutionary mechanisms in order to search for the best solution of an optimization problem in a predefined search space. Chromosomes represent candidate solutions of optimization problem and they form a genetic pool. The fitness value of chromosomes is evaluated according to the objective function of optimization problem. The best fitting chromosomes to a solution of the problem have a higher chance of surviving in genetic pool. New chromosomes of the pool, which represent new candidate solutions of the optimization problem, are generated from the survival chromosomes by means of reproduction, crossover, mutation, inversion processes. Evaluation process of genetic algorithm determines fitness value of new chromosomes [52]. GA improves candidate solutions at each generation by survivals of fitting chromosomes in genetic pool.

In the current study, Matlab GA toolbox was implemented for searching FOPID controller coefficients. The following section explains formulation of this controller optimization problem in more detail.

Multi-Objective design of disturbance reject FOPID control System: Minimum angle pole placement

The current study aims to combine v-domain controller stabilization scheme with disturbance rejection objectives to design an optimal FOPID controller that can achieve robust control performance requirements. Essentially, the proposed a multi-objective design scheme chooses a disturbance rejection FOPID controller from a set of stabilizing controllers that are in reservoir of the first Riemann sheet. Hence, multi-objective design scheme has two objectives: The first objective aims to stabilize the system w.r.t. minimum angle system pole placement strategy, and the second objective improves disturbance rejection performance according to a predefined minimum RDR specification. This multi-objective design scheme ensures design of an optimal FOPID controller that can exhibit enhanced disturbance rejection control performance.

Fig. 2 shows a block diagram of a closed loop FOPID control system involving additive input disturbance model. The parameter d stands for unknown additive input disturbance.

Let assume a standard time-delayed one pole fractional order plant model, which is expressed in the form of

$$G(s) = \frac{a_0}{b_1 s^{\alpha} + b_0} e^{-Ls} \tag{12}$$

where time delay parameter is denoted by the parameter L and the parameter α represents the fractional order of the controller system.

Then, the following equation expresses the closed loop transfer function of this FOC system.

$$T(s) = \frac{Q(s)}{R(s)} = \frac{a_0 k_d e^{-Ls} s^{(\lambda+\mu)} + k_p a_0 e^{-Ls} s^{\lambda} + k_i a_0 e^{-Ls}}{a_0 k_d e^{-Ls} s^{(\lambda+\mu)} + b_1 s^{(\lambda+\alpha)} + (b_0 + k_p a_0 e^{-Ls}) s^{\lambda} + k_i a_0 e^{-Ls}} \tag{13}$$

In order to obtain a constant coefficient polynomial characteristic equation, Pâde approximation $\left(e^{-Ls} \approx \frac{(1-\theta s)}{(1+\theta s)}, \quad \theta = L/2\right)$ is employed for the terms of e^{-Ls} . Thus, characteristic polynomial of FOPID control system is written for this system model by

$$\begin{aligned}\Delta(s) = {} & b_1 \theta s^{(\lambda+\alpha+1)} + b_1 s^{(\lambda+\alpha)} - a_0 k_d \theta s^{(\lambda+\mu+1)} + a_0 k_d s^{(\lambda+\mu)} \\ & + (b_0 - k_p a_0)\theta s^{(\lambda+1)} + (b_0 + k_p a_0) s^{\lambda} - a_0 k_i \theta s + a_0 k_i.\end{aligned} \tag{14}$$

One applies $s = v^m$ mapping and expresses $\Delta_m(v)$ as

$$\begin{aligned}\Delta_m(v) = {} & b_1 \theta v^{(\lambda+\alpha+1)m} + b_1 v^{(\lambda+\alpha)m} - a_0 k_d \theta v^{(\lambda+\mu+1)m} \\ & + a_0 k_d v^{(\lambda+\mu)m} + (b_0 - k_p a_0)\theta v^{(\lambda+1)m} \\ & + (b_0 + k_p a_0) v^{\lambda m} - a_0 k_i \theta v^m + a_0 k_i.\end{aligned} \tag{15}$$

By considering FOPID controller coefficients, characteristic roots of the system in the first Riemann sheet is written in v-domain by

$$R_i = \left\{ v_r \ : \ \Delta_m(p, v_r, k_p, k_i, k_d, \lambda, \mu) = 0 \wedge 0 \leqslant \arg(v_r) < \frac{\pi}{m}, \ p \in A, \ r = 1, 2, 3, \dots \right\}. \tag{16}$$

Let us devise a multi-objective optimization problem that allows design of disturbance reject and stable controller design. Firstly, a stabilization objective should adjust the position of minimum angle system pole for a target angle φ_T, this objective can be expressed in the squared error form as [21,22]

$$\varepsilon_s = \left(\min_{v_r \in R_i} |\arg(v_r)| - \varphi_T\right)^2. \tag{17}$$

The term $\min|\arg(v_r)|$ refers to the minimum angle root in the set R_i. Specification of the target angle can be performed by slicing of a target angle line with the angle of $\varphi_T = \frac{(d+1)\pi}{2m}$, where $d \in [0, 1]$ is a partitioning factor of the stability region.

Secondly, a disturbance rejection objective is added to ensure the resulting control system exhibit a predefined RDR performance. By considering Eq. (11), the RDR objective can be expressed in the squared error form as

$$\varepsilon_r = \left(\min_{\omega \in [\omega_{\min}, \omega_{\max}]} \{RDR_{dB}(\omega)\} - M\right)^2. \tag{18}$$

The term $\min_{\omega \in [\omega_{\min}, \omega_{\max}]} \{RDR_{dB}(\omega)\}$ refers to the minimum RDR value of the system in a given operating frequency range $\omega \in [\omega_{\min}, \omega_{\max}]$. Parameter $M \in R$ in decibel is a target minimum RDR that the designed FOPID control system should satisfy in this frequency range.

Multi-objective optimization problem to stabilize a FOPID controller with a desired disturbance rejection performance can be expressed with the weighted sum of both objectives as follows,

$$\min E(k_p, k_i, k_d, \lambda, \mu) = W\gamma\varepsilon_s + (1-\gamma)\varepsilon_r, \tag{19}$$

where the parameter $\gamma \in [0, 1]$ is the weight coefficient that can be used to adjust priority of optimization between two objectives. When setting $\gamma = 0.5$, the optimization gives an equal priority for the both objectives. The coefficient W is the magnitude normalization that is used to compensate negative impacts of large magnitude differences between two objectives. It can be set according to

$$W = \frac{|\varepsilon_{r,\max}|}{|\varepsilon_{s,\max}|} \tag{20}$$

where $\varepsilon_{r,\max}$ and $\varepsilon_{s,\max}$ are maximum magnitudes of the objective ε_r and ε_s. In this study, W is set to 4240 for $\varepsilon_{r,\max} = 10^3$ and $\varepsilon_{s,\max} = 3\pi/40$.

This problem is solved by implementing GA according to the flow chart in Fig. 3, and the results obtained are illustrated in the next section.

Illustrative examples

Illustrative examples are presented in this section to demonstrate application of the proposed design scheme for disturbance rejection FOPID controller design. For initial configuration of GA, the search ranges of FOPID controller coefficients were set to $k_p \in [0, 50]$, $k_i \in [0, 50]$, $k_d \in [0, 50]$, $\lambda \in [0.3, 2]$ and $\mu \in [0.3, 2]$. Plant models from published work [54,55] have been considered in illustrative examples, and performances of controller designs in these works are compared with the performance of the proposed design method. Matlab GA toolbox was implemented to perform multi-objective optimization. After designing optimal FOPID controller, to carry out transient analysis according to step and disturbance responses of the controllers, FOC system simulations were performed in Matlab/Simulink by using FOTF toolbox [56]. Simulink simulation model of the control system is shown in Fig. 4.

Example 1. Let us design FOPID controller for a time-delay plant function [54], given by

$$G(s) = \frac{1.0003}{0.8864 s^{1.0212} + 1} e^{-0.4274s}. \tag{21}$$

Design specifications are $d = 0.5$ for target angle line partitioning factor and $M = 15$ dB for lower boundary of RDR index in the operating frequency range of $[0, 100]$ rad/sec.

It is known that there is a performance tradeoff between set-point control and disturbance rejection control of closed loop control systems [40]. For this reason, a set-point filter at reference input is commonly used to improve set-point performance while

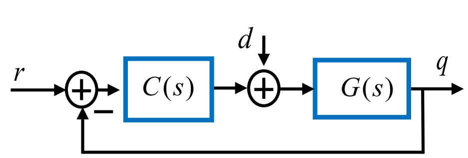

Fig. 2. Block diagram of closed loop FOPID control system and additive input disturbance rejection model.

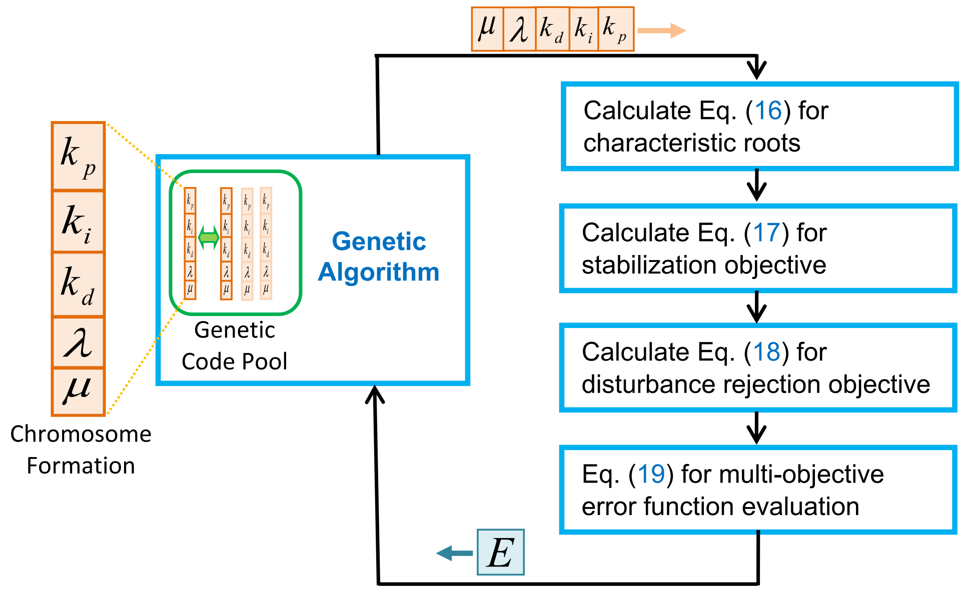

Fig. 3. Block diagram describing application of GA for optimal stabilization of FOPID controller for disturbance rejection control.

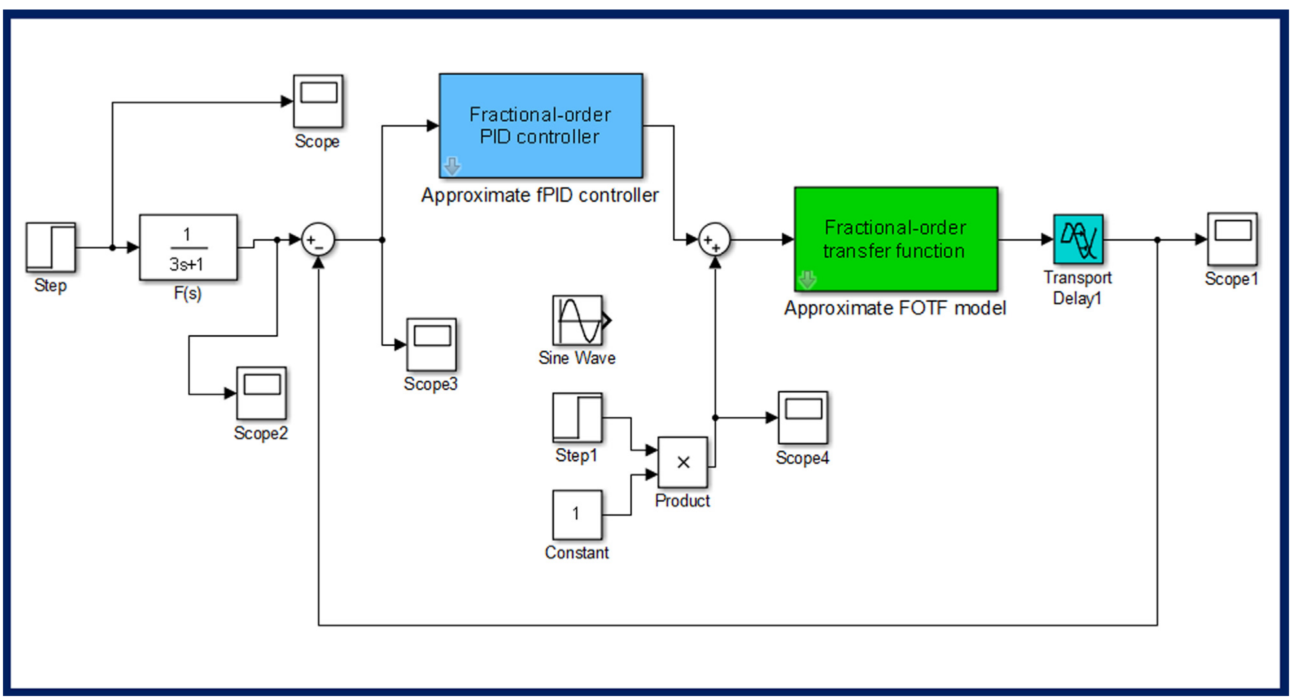

Fig. 4. Simulink simulation model of designed control system.

exhibiting high disturbance rejection control performance. Therefore, a set-point filter ($F(s) = \frac{1}{3s+1}$) was implemented to improve set-point performance of the disturbance rejection FOPID control systems.

In the optimization process, we used $s = v^{10}$ mapping by configuring $m = 10$. For stabilization of FOPID controller, the partitioning factor d is set to $1/2$ for placement of minimum angle system pole over the middle of the stability region. The corresponding target angle for pole placement is calculated by $\varphi_T = \frac{(d+1)\pi}{2m} = \frac{3\pi}{40}$ and configured in optimization process.

Following equation is the characteristic polynomial of the system.

$$\begin{aligned}\Delta(s) = {} & 0.1894\, s^{(\lambda+1.0212+1)} + 0.8864\, s^{(\lambda+1.0212)} - 0.2138 k_d s^{(\lambda+\mu+1)} \\ & + 1.0003 k_d\, s^{(\lambda+\mu)} + (0.2137 - k_p 0.2138) s^{(\lambda+1)} \\ & + (1 + k_p 1.0003) s^{\lambda} - 0.2138 k_i\, s + 1.0003 k_i.\end{aligned} \tag{22}$$

By applying $s = v^{10}$ mapping to the fractional order characteristic equation, $\Delta_m(v)$ is expressed in v-domain as

$$\begin{aligned}\Delta_{10}(v) = {} & 0.1894\, v^{(\lambda+1.0212+1)10} + 0.8864\, v^{(\lambda+1.0212)10} \\ & - 0.2138 k_d\, v^{(\lambda+\mu+1)10} + 1.0003 k_d\, v^{(\lambda+\mu)10} \\ & + (0.2137 - k_p 0.2138)\, v^{(\lambda+1)10} \\ & + (1 + k_p 1.0003)\, v^{\lambda 10} - 0.2138 k_i v^{10} + 1.0003 k_i.\end{aligned} \tag{23}$$

By taking $\gamma = 0.5$, an equal priority for the system stability objective and disturbance rejection objective were assigned and, the multi-objective GA was performed. After completion of the optimization task, the GA yielded optimal FOPID controller as

$$C(s) = 0.7493 + 0.9997s^{0.5808} + \frac{3.4928}{s^{0.7479}}. \quad (24)$$

Fig. 5(a) shows the position of the minimum angle system pole within the first Riemann Sheet. The figure reveals that the minimum angle pole shown by red asterisks, places on the target angle that was indicated by the green line within the stability region. This figure confirms that the optimized FOPID control system is stabilized for a target angle $\varphi_T = \frac{3\pi}{40}$.

Fig. 5(b) shows the RDR spectrum of the optimized FOPID control system. It illustrates the rates of disturbance rejection of the control system for each frequency component in operating frequency range of $[0, 100]$. Figure validates that minimum RDR objective was achieved to have the lowest RDR over the lower RDR bound $M = 15$ in this operating frequency range. The RDR spectrum reveals that the lowest RDR performance is obtained about 16 dB at the frequency 2.5 rad/sec, and this is the worst-case disturbance rejection performance of the system in the operating frequency range $[0, 100]$. In order to observe time response for the lowest and highest disturbance rejection rates of RDR spectrum (See Fig. 6), a periodical disturbance in the sinusoidal waveform was applied at 40 th sec simulation time, and time responses of the designed system are shown for the worst case (it is the lowest RDR value at the frequency of $\omega = 2.5$ rad/sec) and the best-case (it is the highest RDR value at frequency of $\omega = 100$ rad/sec) in Fig. 6. This figure shows that additive sinusoidal input disturbances with the amplitude of 1 can be suppressed in a correspondence with RDR spectrum in Fig. 5(b). This correspondence validates tuning of disturbance rejection capacity of the closed loop control system by considering RDR spectrum.

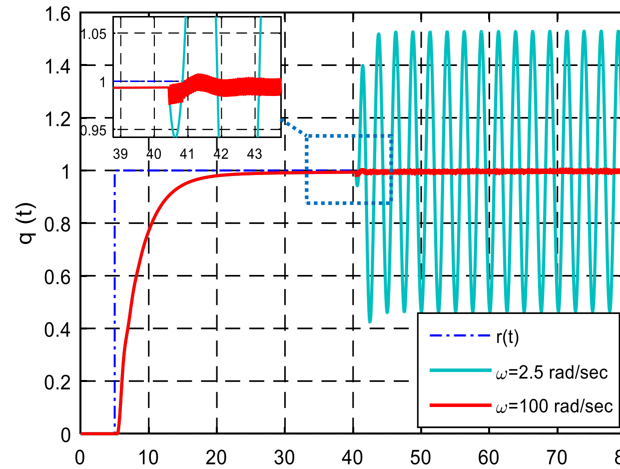

Fig. 6. Sinusoidal disturbance responses of system at $\omega = 2.5$ rad/sec (for the worst case) and $\omega = 100$ rad/sec (for the best case) according to RDR spectrum in Fig. 4.

Fig. 7(a) shows step responses of the control system for two different configurations of the optimization task. The figure apparently demonstrates effects of disturbance rejection objectives on the controller design process. In the case of $\gamma = 1$, it discards the disturbance rejection objective in optimization task and performs only the stabilization objective as a single objective. In case of $\gamma = 0.5$, it performs an optimization task for both objectives with equal priority. The figure clearly shows that disturbance rejection objective improves disturbance rejection performance of the resulting FOPID controller for the additive step disturbance at 40 sec simulation time. Fig. 7(b) shows the step disturbance responses of two methods for comparison purposes. Both methods can yield stabilizing FOPID controllers. However, the proposed design method can further improve disturbance rejection performance of the control system by using the disturbance rejection objective ε_r.

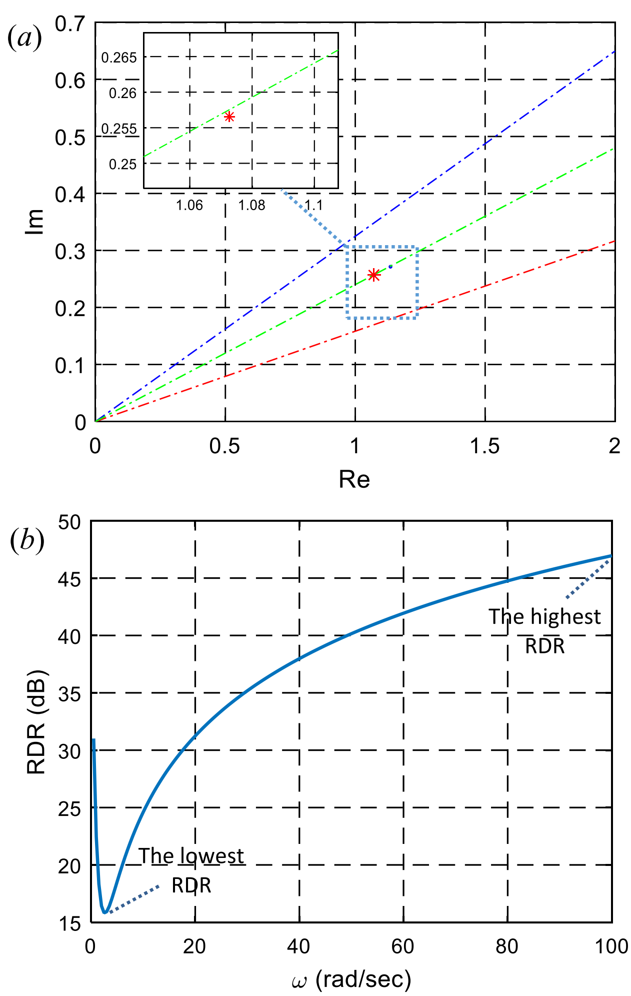

Fig. 5. (a) Placement of the minimum angle pole in the first Riemann sheet of v-domain. (b) RDR spectrum of the corresponding closed loop FOPID control system.

Example 2. Let us design consider a time-delay plant function, given by [55]

$$G(s) = \frac{1}{s+1}e^{-0.5s}. \quad (25)$$

Design specifications are given as: The target angle partitioning factor is $d = 0.6$ and the RDR minimum boundary is $M = 10$ dB in the operating frequency range of $[0, 100]$ rad/sec. We used a set-point filter $F(s) = \frac{1}{3s+1}$ so that it can improve set-point performance of the resulting disturbance rejection FOPID control system [40,43].

We used $s = v^{10}$ mapping by setting $m = 10$, and the target angle for placement of minimum angle system pole is obtained $\varphi_T = \frac{(d+1)\pi}{2m} = \frac{8\pi}{40}$ for $d = 0.6$.

The following equation shows the characteristic polynomial of the closed loop FOPID control system.

$$\Delta(s) = 0.2500\, s^{(\lambda+1+1)} + s^{(\lambda+1)} - 0.2500k_d s^{(\lambda+\mu+1)} + k_d\, s^{(\lambda+\mu)} \\ +(0.2500 - k_p 0.2500)\, s^{(\lambda+1)} + (1+k_p)s^{\lambda} - 0.2500k_i s + k_i. \quad (26)$$

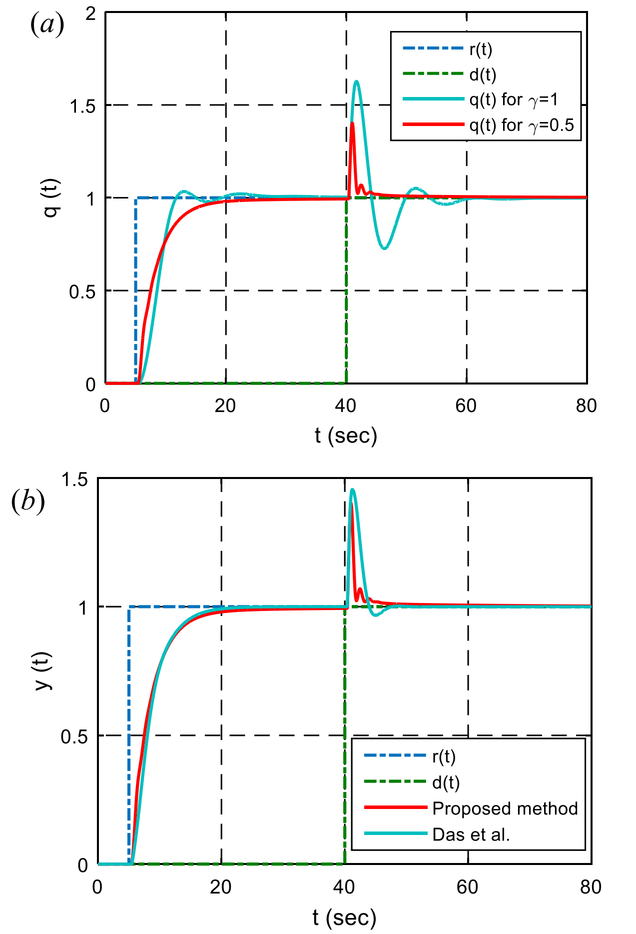

Fig. 7. (a) Step responses of FOPID controlled systems for $\gamma = 1$ (only stabilization objective) and $\gamma = 0.5$(both stabilization and disturbance rejection objective). (b) Step responses of controlled systems obtained with the proposed method and the method proposed by Das et al. [54].

By applying $s = v^{10}$ mapping, $\Delta_m(v)$ is obtained in v-domain as

$$\Delta_{10}(v) = 0.2500\, v^{(\lambda+1+1)10} + v^{(\lambda+1)10} - 0.2500 k_d\, v^{(\lambda+\mu+1)10} + k_d\, v^{(\lambda+\mu)10} + (0.2500 - k_p 0.2500)\, v^{(\lambda+1)10} + (1 + k_p)\, v^{\lambda 10} - 0.2500 k_i v^{10} + k_i. \quad (27)$$

By taking $\gamma = 0.5$, an equal priority of system stability and disturbance rejection objective is assigned for the multi-objective GA optimization process. After completion of the multi-objective optimization process, the GA founds optimal FOPID controller coefficients as

$$C(s) = 0.7991 + 0.8466 s^{0.9421} + \frac{2.4455}{s^{0.6525}}. \quad (28)$$

Fig. 8(a) shows placement of the minimum angle system pole in the first Riemann sheet. The figure reveals that the minimum argument root, which is indicated by red asterisks, approximates to the target angle that was indicated by the green line inside the stability region. This result also validates the stabilization of the optimized FOPID control system.

RDR spectrum of the optimized FOPID control system is illustrated in Fig. 8(b). The distribution of disturbance rejection rates indicates existence of a dip characteristic for RDR spectrum within frequency range [0, 100] rad/sec and it confirms that the minimum RDR objective is greater than the boundary $M = 10$ dB. This spectrum reveals bounds of the rejection capacity of the designed system for the periodical disturbance in the sinusoidal waveform.

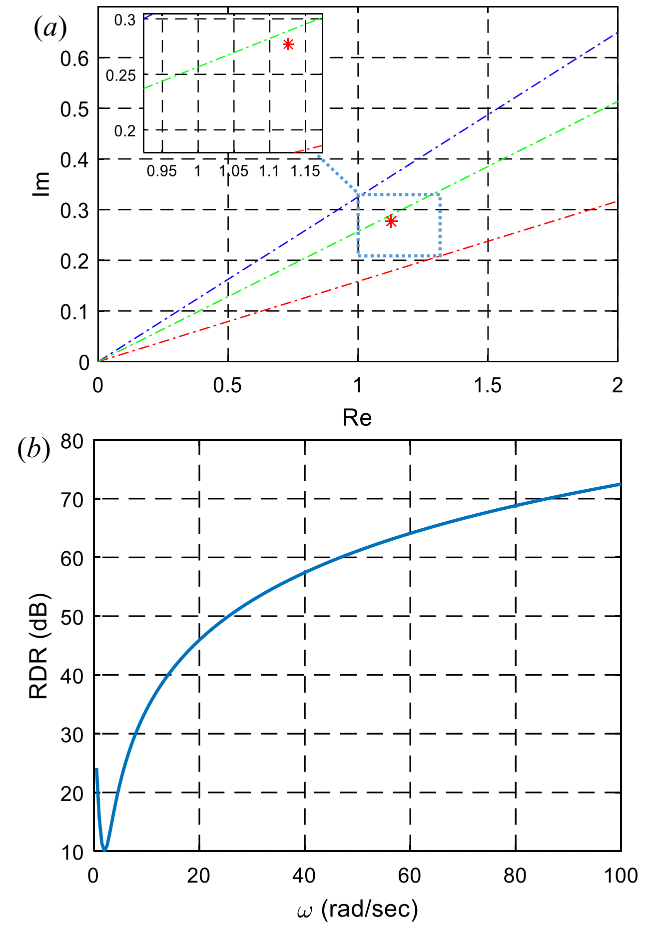

Fig. 8. (a) Placement of the pole with minimum angle in the first Riemann sheet of v-domain. (b) RDR spectrum of the corresponding closed loop FOPID control system.

Disturbance responses of the system at the lowest RDR rate (worst-case) and the highest RDR rate (best-case) of the RDR spectrum are shown in Fig. 9. The designed FOPID controller can better suppress high frequency disturbances because RDR spectrum increases at high frequencies.

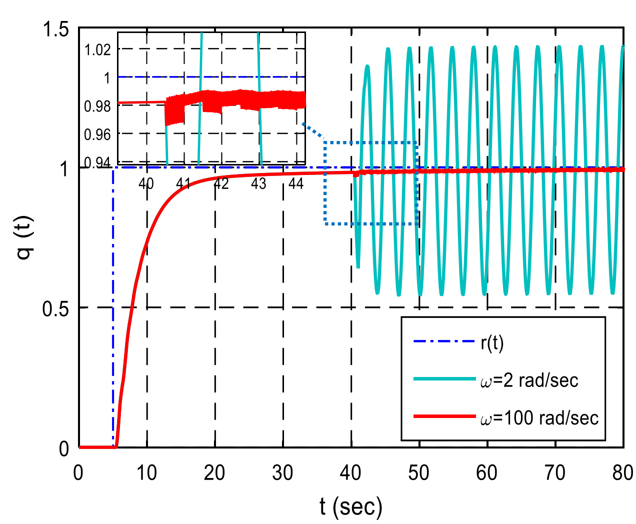

Fig. 9. Sinusoidal disturbance responses of system at $\omega = 2.0$ rad/sec (for the worst case) and $\omega = 100$ rad/sec (for the best case) according to RDR spectrum in Fig. 9.

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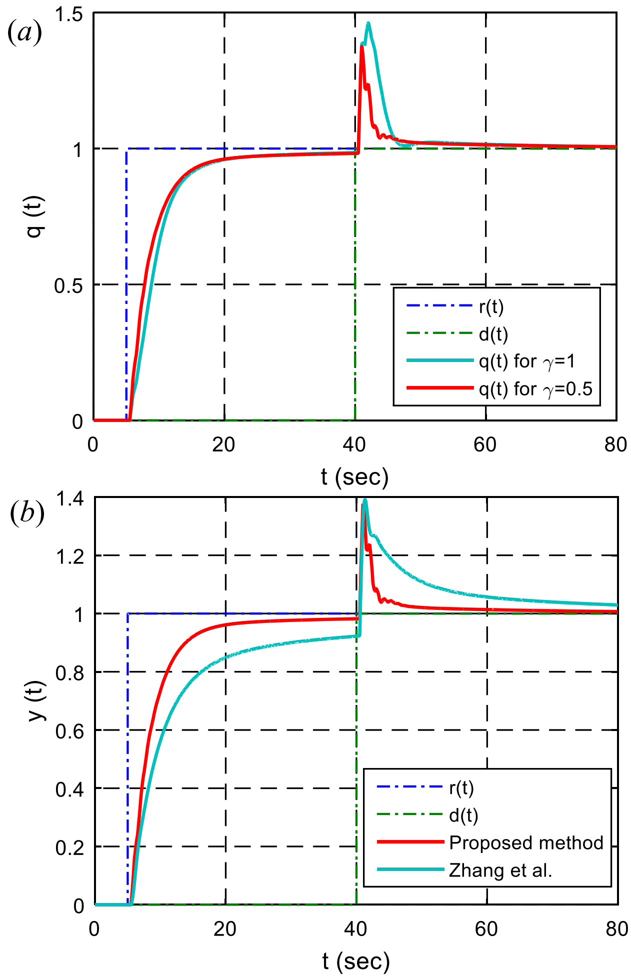

Fig. 10. (a) Step responses of FOPID controlled systems for $\gamma = 1$ (only stabilization objective) and $\gamma = 0.5$(both stabilization and disturbance rejection objective). (b) Step responses of FOPID controlled systems that are designed according to the proposed method and the design method proposed by Zhang et al. [55].

Step responses of the control system for two different configurations of multi-objective optimization are shown in Fig. 10(a). The figure reveals contributions of disturbance rejection objective to the design process. In the case of $\gamma = 1$, it discards the disturbance rejection objective and performs only stabilization objective in a single objective optimization manner. For $\gamma = 0.5$, it performs optimization for both objectives with an equal weight. The figure clearly shows that disturbance rejection objective considerably improves disturbance rejection performance of the FOPID controller. Fig. 10(b) shows a comparison of step disturbance response of the proposed method with an optimal FOPID controller that was designed by Zhang et al. The simulation results show that the proposed design method can improve set-point and disturbance rejection performances by performing multi-objective optimization in v-domain.

Discussions and conclusions

This study demonstrates a disturbance rejection FOPID controller design scheme that is based on minimum angle system pole placement strategy, and RDR spectrum shaping by using multi-objective GA optimization. This approach utilizes an alternative design domain, namely v-domain, which distinguishes the purposed method from other design approaches that are performed in t-domain (time domain), s-domain and frequency domain. Primarily, system stabilization task can be conducted in the first Riemann sheet by placing the system pole with minimum angle to the desired point within the stable region [22]. However, it is obvious that controller stabilization effort does not guarantee satisfactory control performance. To achieve performance requirements, additional control performance objectives and constraints can be applied in a multi-objective optimization manner. This study considers the design of disturbance rejecting FOPID controller by adopting multi-objective GA.

Some remarks can be summarized as,

* The v-domain provides a fresh design domain for design of FOC system. Previous studies mainly considered v-domain for stability analysis [16,18,19] and controller stabilization [15,21,22] problems. This study demonstrates a multi-objective optimal controller design in v-domain. We anticipate that v-domain design can open a fresh path for optimal fractional order controller design works.

* Placing the minimum angle pole in the stability region of v-plane is a straightforward solution to guaranty stabilization of FOC system designs. Bounds of stability region are definite and stability theorem is well established in v-domain [16,18,19]. The current study demonstrated that additional control performance objectives could be easily incorporated with this stability objective in order to obtain optimal FOPID controller performance in the stability region of v-domain. In time and frequency domain optimal design tasks of FOPID controllers, guarantying stability of resulting optimal controller design is not a straightforward task because determination of stability ranges is not an easy problem in these domains. On the other hand, some promising solutions have been proposed recently for robust stabilization of systems in time domain [57–59]. The adoption of these methods for stabilization of fractional order system may yield useful time domain solutions for the improvement of robust stabilization performance of fractional order systems.

* In previous works, RDR spectrum is used for assessment and enhancement disturbance rejection control performance for additive input disturbance model [39,41–43]. This study reveals that RDR objective can be easily incorporated with v-domain optimal FOPID controller design scheme. Simulation results of the purposed v-domain design scheme reveal that RDR spectrum can be an effective tool for enhancement disturbance rejection control performance.

An advantage of optimal FOPID controller design in v-domain comes from the asset that fractional order optimal controller stabilization task is more straightforward and reliable in v-domain than those in the frequency domain. Frequency domain loop shaping design approaches are widely applied for optimal FOPID controller design in frequency domain [11]. To stabilize the optimal controller in the frequency domain, the phase margin and gain margin constraints should be satisfied for the whole frequency range according to Bode diagram [11–13]. This requirement may complicate stabilization process or reduces reliability of the results. For stabilization of optimal fractional order controllers in v-domain, placement of minimum angle system pole into the stability region is sufficient to ensure stability of optimal FOPID controller for the whole frequency range. The computation task requires only finding roots of the expanded order characteristic polynomial. Therefore, the optimal fractional order controller stabilization in v-domain is rather straightforward [21] and reliable. [16,18,47].

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

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Declaration of Competing Interest

The authors have declared no conflict of interest.

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