Polynomial Approximation of Quasipolynomials Based on Digital Filter Design Principles

Libor Pekař, Pavel Navrátil

Faculty of Applied Informatics, Tomas Bata University in Zlín, Czech Republic
{pekar,pnavratil}@fai.utb.cz

Abstract. This contribution is aimed at a possible procedure approximating quasipolynomials by polynomials. Quasipolynomials appear in linear time-delay systems description as a natural consequence of the use of the Laplace transform. Due to their infinite root spectra, control system analysis and synthesis based on such quasipolynomial models are usually mathematically heavy. In the light of this fact, there is a natural research endeavor to design a sufficiently accurate yet simple engineeringly acceptable method that approximates them by polynomials preserving basic spectral information. In this paper, such a procedure is presented based on some ideas of discrete-time (digital) filters designing without excessive math. Namely, the particular quasipolynomial is subjected to iterative discretization by means of the bilinear transformation first; consequently, linear and quadratic interpolations are applied to obtain integer powers of the approximating polynomial. Since dominant roots play a decisive role in the spectrum, interpolations are made in their very neighborhood. A simulation example proofs the algorithm efficiency.

Keywords: Approximation, bilinear transformation, digital filter, MATLAB, polynomials, pre-warping, quasipolynomials.

1 Introduction

Mainly due to that delay appears in many real-word systems, such as economical, biological, networked, mechanical, electrical etc. [1, 2], this phenomenon have been intensively studied during recent decades [3, 4]. The most specific feature of time delay systems (TDS) and models can be viewed in the fact that they own infinite spectra; thus, they are included in the family of infinite-dimensional systems. Linear time-invariant TDS, considered in this paper, can primarily be described by ordinary difference-differential (or shifted-argument) equations [5] which can be subjected to the Laplace transform [6]. As a consequence, the corresponding transfer function (matrix) is obtained in which its denominator quasipolynomial (QP) called also as the characteristic QP dominantly decides about dynamical and stability system properties [7].

QP roots (or zeros) agree with system poles, except for some special cases. Hence, the knowledge about the spectrum of QP roots is a crucial matter for TDS analysis.

Various tools and methods have been developed and designed for TDS spectrum computation or estimation, mainly in the state space domain, see e.g. [8]. These results represent a certain kind of TDS discretization as well. In the input-output Laplace space, the Quasi-Polynomial mapping Rootfinder (QPmR) has proved to be a very effective and practically usable when computing QP zeros within the determined region of the complex plane [9-11]. This method omits any QP simplification or approximation and a special software package was developed for its practical usability. TDS model reduction or rationalization represents another way how to cope with the problem [12-14]. However, expect for the well-known Padé approximation of exponential terms, these methods have been designed primarily for approximation of the complete TDS model, not only of the QP itself, and their applicability is mostly worsen due to a high mathematical knowledge level required out of the user. Delta models [15] represent an easy-to-handle rationalization and discretization methodology for practitioners, usable for TDS as well [16], which are closely related to the notion of the bilinear transformation [17] and give a polynomial discrete-time approximation representation of the system model.

The goal of this paper is to design a sufficiently simple, fast and practically usable technique for polynomial approximation of a QP without the necessity of advanced mathematical knowledge or using uncommon software tools. It is based on two main principles adopted from digital filter designing: As first, exponential terms in the QP are subjected to the innate time shifting and consequently to linear or quadratic interpolation such that the eventual shifts are integer multiplies of a basic time period. As second, s-powers representing derivatives are recursively put through the bilinear transformation the efficiency of which is further enhanced by pre-warping [17] that preserves the particular selected frequency under discretization. Since the dominant (i.e. the rightmost) pole (or the pair) has the decisive impact to system dynamics, all the interpolations and extrapolations are performed in the neighborhood of a close dominant QP root estimation.

The rest of the paper is organized as follows: Basic properties of zeros of a retarded QP and herein utilized techniques and tools are provided in the next, preliminary, section. Afterward, the reader is acquainted with the approximating procedure in details. A numerical example is given to illustrate the accuracy and efficiency of the technique; then the paper is concluded.

2 Preliminaries

Prior to the description of the approximation algorithm, retarded quasipolynomials and their spectral features ought to be introduced. This section, moreover, provides the reader with necessary mathematical tool and techniques that are utilized during the approximation procedure.

2.1 Retarded Quasipolynomial and Its Spectrum

A retarded QP can be expressed as

$$X(s,\mathbf{x},\boldsymbol{\tau}) = s^n + \sum_{i=0}^{n-1}\sum_{j=1}^{h_i} x_{ij} s^i \exp\left(-s\sum_{k=1}^{L}\lambda_{ij,k}\tau_k\right) \tag{1}$$

where $\boldsymbol{\tau} = [\tau_1,..\tau_L] \in \mathbb{R}_+^L, \tau_i > 0$ represents independent delays, $\lambda_{ij,k} \in \mathbb{N}_0$, $\mathbf{x} = [x_{01},..x_{n-1,h_{n-1}}] \in \mathbb{R}^{\sum_{i=0}^{n-1} h_i} \neq 0$.

Let $\Sigma := \{s_i\}$ be the spectrum of roots (zeros) of (1).

Property 1 [2], [5]. For (1) it holds that

1. If there exist nonzero $x_{ij}, \lambda_{ij,k}$ for some positive τ_k and some *i*, *j*, *k*, then the number of QP zeros is infinite.
2. For any fixed real $\beta > -\infty$, the number of roots with $\operatorname{Re} s_i > \beta$ is finite.
3. Isolated roots behave continuously and smoothly with respect to $\boldsymbol{\tau}$ on $\mathbb{C}$.

Definition 1. The *spectral abscissa*, $\alpha(\boldsymbol{\tau})$, is the function

$$\alpha(\boldsymbol{\tau}) := \boldsymbol{\tau} \mapsto \sup \operatorname{Re} \Sigma \tag{2}$$

Property 2 [18]. For function $\alpha(\boldsymbol{\tau})$, it holds that:

1. It may be nonsmooth, and hence not differentiable, e.g. in points with more than one real root or conjugate pairs with the same maximum real part.
2. It is non-Lipschitz, for instance, at points where the maximum real part has multiplicity greater than one.

To sum up main findings from Property 1 and Property 2, although the rightmost part of the spectrum contains isolated roots, the position of which is continuously changed with $\boldsymbol{\tau}$, the abscissa might evince abrupt changes in its value.

Definition 2. The *leading* (dominant) *root*, s_L, or pair $\{s_L, \bar{s}_L\}$ satisfies

$$\alpha(\boldsymbol{\tau}) = \operatorname{Re} s_L = 0 \tag{3}$$

i.e. it represents the rightmost root or the pair from Σ.

2.2 Discretization Techniques and Tools

As mentioned above, the approximation algorithm is designed to be practically implementable by means of the most standard programs without the necessity of the use of special software and the knowledge of advanced math.

Derivatives in (1) are expressed by s-powers. The idea of the derivative approximation is based on the iterative digital-filter-like discretization of the QP depending on the current leading root estimation via the bilinear (or Tustin) transformation

$$s \to \frac{2}{T}\frac{1-q}{1+q} \tag{4}$$

where q means the shifting operator that agrees with z^{-1} in the z-transform and T is the sampling period.

Let $X(s,\mathbf{x},\boldsymbol{\tau})$ be the characteristic quasipolynomial of a system. Since, however, transformation (4) does not preserve frequencies (namely, the system eigenfrequency), it is desirable to find another mapping that keeps "continuous" frequencies (i.e. those in the s-plane) and "discrete" frequencies (i.e. those in the z-plane) identical. It can be derived [17] that this requirement is satisfied if the following modified mapping is used

$$s \to \frac{\omega}{\tan(\omega T/2)}\frac{1-q}{1+q} \tag{5}$$

where ω stands for the desired frequency. Note that the operation of the frequency preservation is called *pre-warping*. Because of the decisive role of leading roots, we have set $\omega = \operatorname{Im} s_L$.

From the point of view of derivative discretization or delta models, the value of T in (4) or (5) should be sufficiently small; however, the lower T is, the higher resulting approximating polynomial degree is obtained. In the contrary, the z-transform demands significantly lower values; for instance, in [19] the following recommendation for periodic systems is given

$$T = [0.2/\omega_0, 0.5/\omega_0] \tag{6}$$

where ω_0 expresses the frequency of undumped oscillations. Note that $\omega_0 \approx |s_L|$.

Regarding terms expressing delays in (1), i.e. the exponentials, they can be subjected to inherent shifting

$$\exp(-\vartheta s)X(s) \mathrel{\hat{=}} x(t-\vartheta) \mathrel{\hat{=}} q^{\vartheta/T}x(k) \mathrel{\hat{=}} z^{-\vartheta/T}X(z) \tag{7}$$

Nevertheless, in general, delay value ϑ might not be an integer multiple of *T*; hence, term $z^{-\vartheta/T}$ should be interpolated by a linear combination of integer powers of z . The following lemma gives two possible solutions of this task.

Lemma 1. Consider a term $z^{-(\lfloor\vartheta\rfloor+\overline{\vartheta})}$, $\lfloor\vartheta\rfloor\in\mathbb{N}_0$, $0<\overline{\vartheta}<1$. In the vicinity of $z_0\in\mathbb{C}$, the term can be interpolated linearly as (8) or quadratically as (9):

$$\left(1-\lfloor\vartheta\rfloor\right)z_0^{-\lfloor\vartheta\rfloor}z^{-\overline{\vartheta}}+\lfloor\vartheta\rfloor z_0^{-\lfloor\vartheta\rfloor+1}z^{-(\overline{\vartheta}+1)} \tag{8}$$

$$\begin{aligned}&0.5\left(2-\lfloor\vartheta\rfloor\right)\left(1-\lfloor\vartheta\rfloor\right)z_0^{-\lfloor\vartheta\rfloor}z^{-\overline{\vartheta}}+\lfloor\vartheta\rfloor\left(2-\lfloor\vartheta\rfloor\right)z_0^{-\lfloor\vartheta\rfloor+1}z^{-(\overline{\vartheta}+1)}\\&+0.5\lfloor\vartheta\rfloor\left(\lfloor\vartheta\rfloor-1\right)z_0^{-\lfloor\vartheta\rfloor+2}z^{-(\overline{\vartheta}+2)}\end{aligned} \tag{9}$$

The proof is omitted due to the limited space. In this study, the value of z_0 is selected as

$$z_0=z_L=\exp\left(Ts_L\right) \tag{10}$$

which agrees with the z-transform and is closely related to mapping (5), and it is consistent with the idea of the leading root importance.

3 Polynomial Approximation Algorithm

Two versions of the algorithm are presented. The former one can be utilized whenever the leading root of a QP sufficiently close to the studied QP. The latter can be used even if no leading root estimation is known and, naturally, it is expected to give less accurate results and is computationally more complex.

Algorithm 1.

Input: The QP $X(s,\mathbf{x},\boldsymbol{\tau})$ to be approximated.

Step 1: Consider that there exists a QP $X(s,\mathbf{x}_0,\boldsymbol{\tau}_0)$ with $\left\|X(s,\mathbf{x},\boldsymbol{\tau})-X(s,\mathbf{x}_0,\boldsymbol{\tau}_0)\right\|<\Delta$, for a sufficiently small $\Delta>0$, the leading root of which, $s_0=\hat{s}_0$, is known exactly. Set $\varepsilon>0$.

Step 2: Compute polynomial $P\left(z^{-1}\mid\hat{s}_0\right)$ according to (4)-(9). Define and compute

$$\hat{s}_1:=\left\{\arg\min\left|s-\hat{s}_0\right|:s=T^{-1}\log(z),P\left(z^{-1}\mid\hat{s}_0\right)=0\right\} \tag{11}$$

Step 3. While $\left|\hat{s}_1-\hat{s}_0\right|\geq\varepsilon$, set $\hat{s}_0:=\hat{s}_1$ and go to Step 5.

Output: $P\left(z^{-1}\right)$ and its roots.

Remark 1. The norm in Step 1 of Algorithm 1 can simply be computed as a point norm in s_0, i.e. $\|X(s_0, \mathbf{x}, \boldsymbol{\tau})\|$. The problem may appear if $|s_1 - s_0| > \delta$ for some $\delta > 0$ and any value of $\|X(s_0, \mathbf{x}, \boldsymbol{\tau})\|$ due to Property 2.

Algorithm 2.
Input: The QP $X(s, \mathbf{x}, \boldsymbol{\tau})$ to be approximated.
Step 1: Define the mesh grid $\tau_{k,j+1} = \tau_{k,j} + \Delta\tau_{k,j}$, $\tau_{k,0} = 0$, $\boldsymbol{\tau} = [\tau_{1,N}, \tau_{2N}, \ldots \tau_{L,N}]$, $k = 1 \ldots L$, $j = 0 \ldots N-1$, and set $\varepsilon > 0$.
Step 2: Compute

$$\hat{s}_{0,\ldots,0} = s_{0,\ldots,0} := \{\arg\max \operatorname{Re} s : X(s, \mathbf{x}, \mathbf{0}) = 0\} \tag{12}$$

exactly.
Step 3: For ($j_1 = 0 \ldots N-1$, for ($j_2 = 0 \ldots N-1$, ... (for $j_L = 0 \ldots N-1$ do: If $\exists j_l \neq 0, l = 1 \ldots L$ do Steps 4-6))) (nested loops).
Step 4: Define $M := \max\{k : j_k \neq 0\}$ and set $\overline{\boldsymbol{\tau}} = [\tau_{1,j_1}, \tau_{2,j_2} \ldots, \tau_{L,j_L}]$, $\hat{s}_0 = \hat{s}_{j_1,\ldots,j_{M-1},j_M-1,0\ldots0}$.
Step 5: Compute polynomial $P(z^{-1} \mid \overline{\boldsymbol{\tau}}, \hat{s}_0)$ according to (4)-(9) and $\hat{s}_1$ by means of (11).
Step 6. While $|\hat{s}_1 - \hat{s}_0| \geq \varepsilon$, set $\hat{s}_0 := \hat{s}_1$ and go to Step 5.
Output: $P(z^{-1})$ and its roots.

4 Numerical Example

Consider a skater on the remotely swaying bow sketched in Fig. 1. The skater controls the power input, $P(t)$, to the servo giving rise to the angle deviation, $u(t)$, from the horizontal position and, consequently, the angle between the skater and the bow symmetry axis, $y(t)$, emerges. If the friction is neglected but the skater's reaction time and servo latency included, the following particular transfer function can be written [20]

$$G(s) = \frac{0.2\exp(-(\tau_1 + \tau_2)s)}{s^2(s^2 - \exp(-\tau_2 s))} \tag{13}$$

A particular generalized (third-order) finite-dimensional linear proportional-integrative-derivative controller stabilizes system (13) with $\boldsymbol{\tau} = [\tau_1, \tau_2] = [0.08, 0.08]$, which yields the feedback characteristic quasipolynomial (14), see [21].

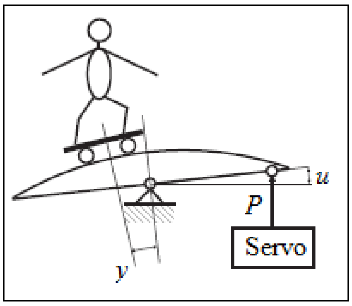

Fig. 1. A skater on the remotely controlled swaying bow

$$X(s,\cdot,\boldsymbol{\tau}) = s^2\left(s^2 - \exp(-0.08s)\right)\left(s^3 + 469418.6s^2 + 640264.6s + 10560107\right) + 0.2\exp(-0.16s)\left(82226506s^3 + 106523134s^2 + 26247749s + 5617613\right) \quad (14)$$

the roots of which decide about control system stability.

Let us test Algorithm 1 for various values of T first. With respect to the upper bound of condition, the sampling period can be written as $T = \left(k_T|s_L|\right)^{-1}, k_T \geq 2$, the value of which is updated in every iteration step (see Steps 2 and 3 in Algorithm 1). Assume that the leading pair, $s_0 = -9.9669\text{e}-3 \pm 3.9672704\text{i}$, of $X(s, \mathbf{x}_0 = \mathbf{x}, [0.08, 0.07])$ is known exactly and set $\varepsilon = \text{e}-6$. Selected algorithm results are summarized in Table 1. Due to limited space, some observations are further commented rather than being included in the table.

The exact leading pair of roots of $X(s, \mathbf{x}, \boldsymbol{\tau})$ found by the QPmR reads $s_L = -0.0294116 \pm 3.9281699\text{i}$. However, in fact, leading roots of the approximating polynomial do not constitute a conjugate pair because of its complex coefficients. We have observed from the test that higher values of k_T (limited by an upper bound of approx. $k_T \approx 30$) give less number of iterations within Steps 2 and 3, better approaching of leading roots and higher leading root estimation accuracy. The beneficial impact of pre-warping (5) can be seen in significantly better leading root estimation (mainly in the imaginary part of the root); however, such an improvement is not confirmed in the case of less dominant roots. The advantage of the quadratic interpolation (9) compared to the linear one (8) can also be observed. It distinctively improves less-dominant pairs mutual approaching and has a slight impact to their loci estimation and the approaching of the leading pair. In the contrary, it has no effect on the leading poles estimation.

However, the discretization procedure gives rise to “parasitic” high-frequency roots not included in the original QP. These polynomial roots are located very close to the imaginary axis with a high imaginary part value. Such an observation has been made by Vyhlídal and Zítek in [16] as well; they have utilized a delta models in their work.

Table 1. Results of Algorithm 1

Method	k_T	Dominant pair of roots		Num. of iter.
(4), (8)	2	–2.11963e–2 + 3.8592406i	–9.1702e–2 – 3.917348i	5
	5	–2.80361e–2 + 3.9172242i	–3.60554e–2 – 3.9223709i	4
	10	–2.90655e–2 + 3.9254363i	–3.30879e–2 – 3.9277296i	3
	20	–2.9325e–2 + 3.927487i	–3.05583e–2 – 3.9281274i	3
	30	–3.8027e–3 + 3.9947106i	–3.504e–3 – 3.9948482i	8
(5), (8)	2	–2.83932e–2 + 3.9282875i	–0.1062955 – 3.9922307i	5
	5	–2.92479e–2 + 3.9281888i	–3.78099e–2 – 3.9336952i	4
	10	–2.93707e–2 + 3.9281745i	–3.34359e–2 – 3.9304912i	3
	20	–2.94013e–2 + 3.92817i	–3.06333e–2 – 3.9288096i	3
	30	–3.77434e–3 + 3.9950239i	–3.47358e–3 – 3.9951627i	14
(4), (9)	2	–2.11963e–2 + 3.8592406i	–6.70344e–2 – 3.8332474i	5
	5	–2.80361e–2 + 3.9172242i	–2.96717e–2 – 3.9154286i	4
	10	–2.90655e–2 + 3.9254364i	–2.93613e–2 – 3.924999i	3
	20	–2.93249e–2 + 3.9274854i	–2.93571e–2 – 3.9274282i	3
	30	–1.29309e–2 + 3.9607762i	–1.29307e–2 – 3.960776i	13
(5), (9)	2	–2.83932e–2 + 3.9282875i	–7.80977e–2 – 3.8996772i	5
	5	–2.92479e–2 + 3.9281888i	–3.09832e–2 – 3.9262835i	4
	10	–2.93707e–2 + 3.9281746i	–2.96686e–2 – 3.9277336i	3
	20	–2.94014e–2 + 3.9281712i	–2.94335e–2 – 3.9281141i	3
	30	–1.29634e–3 + 3.9610884i	–1.29632e–3 – 3.961088i	14

Algorithm 2 applied to (14) starts with $s_{0,..,0} = 0.1214757 \pm 4.5573833\mathrm{i}$, see (12), and let $\Delta\tau_{k,j} = 0.01$. Eventual values of dominant pairs for very selected values of k_T (to be concise) are displayed in Table 2. Apparently, the results are very close to those in Table 1, which implies that the polynomial approximation based on the information about the dominant (leading) root is sufficiently robust with respect to successive procedure of delay values shifting introduced in Algorithm 2.

Method	k_T	Dominant pair of roots		Num. of iter.
(4), (8)	10	–2.90655e–2 + 3.9254363i	–3.30879e–2 – 3.9277296i	3
	20	–2.93249e–2 + 3.927486i	–3.05583e–2 – 3.9281274i	3.05
(5), (8)	10	–2.93707e–2 + 3.9281745i	–3.34359e–2 – 3.9304912i	3
	20	–2.94013e–2 + 3.9281699i	–3.06333e–2 – 3.9288096i	3.01
(4), (9)	10	–2.90655e–2 + 3.9254363i	–2.93613e–2 – 3.924999i	3
	20	–2.9325e–2 + 3.927487i	–2.93572e–2 – 3.9274298i	3
(5), (9)	10	–2.93707e–2 + 3.9281745i	–2.96686e–2 – 3.9277336i	3
	20	–2.94014e–2 + 3.9281706i	–2.94334e–2 – 3.9281135i	3.05

5 Conclusion

The presented paper has been aimed at the possible polynomial approximation of a quasipolynomial by means of tools and techniques used for digital filters design; namely, the bilinear transformation with/without pre-warping and a specific linear and quadratic interpolation for the acquisition of commensurate delays. This activity is useful mainly for stability and dynamical analysis of time delay systems when the characteristic quasipolynomial, as the transfer function denominator, is analyzed. Since the decisive information is contained in a small number of the rightmost, i.e. leading, quasipolynomial zeros, the approximating polynomial has been found iteratively based on the leading root estimation. A rather tricky step is a proper choice of the discretization step and the sampling period.

The presented simulation example has indicated the beneficial impact of the use of pre-warping and more complex, i.e. quadratic, interpolation to the leading root estimation accuracy and the approaching of both dominant roots in the complex conjugate pair, respectively. Besides the leading pair, a small number of less-dominant pairs must also be observed. This feature might play a leading role in the process of the determination of our future research in this field.

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