Annals of DAAAM for 2009 & Proceedings of the 20th International DAAAM Symposium, Volume 20, No. 1, ISSN 1726-9679
ISBN 978-3-901509-70-4, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2009
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Annals of DAAAM International

SYSTEMS WITH NONLINEAR UNCERTAINTY STRUCTURE: ROBUST STABILITY ANALYSIS

MATUSU, R[adek] & PROKOP, R[oman]

Abstract: *Systems with parametric uncertainty have known structure but their parameters can vary within given or supposed intervals. This type of uncertainty may appear during modelling and description of real systems as a consequence of inaccurate measurement, identification, due to ambient conditions, etc. This contribution is focused on description of systems with nonlinear parametric uncertainty structure and on demonstration of possible graphical robust stability testing for this class of systems with special emphasis on utilization of The Polynomial Toolbox for Matlab.*

Key words: *robustness, stability analysis, parametric uncertainty, polynomial toolbox*

1. INTRODUCTION

The common effort of control researchers or engineers to create the mathematical model of controlled process as simple as possible almost always leads to the origin of uncertainty. Their emergence often consists in neglect of “less important properties”, especially from the realms of fast dynamics, nonlinearities or time-variant behaviours of the system. Nevertheless, the presence of uncertainty can not be excluded even if the processes are in essence linear, because the physical parameters are never known exactly, possible they can vary according to operating conditions. So, the principal problem is if the controller designed for nominal system will keep some properties of the feedback control loop also for system really controlled, which falls into certain neighbourhood, in other words, if the controller will keep these properties not only for one nominal system, but for the whole family of systems (Kučera, 2001).

The uncertainty in constructed mathematical model and thus the size of neighbourhood which should the controller cope with can be taken into consideration and described in the two main ways – as parametric or nonparametric uncertainty. The former, nonparametric description of uncertainty lies in restriction of area of possible appearance of frequency characteristic. It is associated with unmodelled dynamics, truncation of high frequency modes, nonlinearities, randomness in the systems, etc. The latter, parametric approach then represents known structure but uncertain knowledge of actual physical parameters of the controlled system. Their possible values are usually bounded by intervals.

The paper deals with robust stability analysis of systems with nonlinear uncertainty structure. In such cases, analytical methods are either quite complex or even do not exist. On that account, the graphical test is utilized in this work. The investigation of robust stability is accomplished via the value set concept and the zero exclusion condition, here practically with the assistance of The Polynomial Toolbox for Matlab.

2. STRUCTURES OF UNCERTAINTY

From the control theory point of view, the frequent subject of investigation is the uncertain characteristic polynomial of the closed-loop control system. It can be described by:

$$p(s,q) = \sum_{i=0}^{n} \rho_i(q) s^i \qquad (1)$$

In robustness problems, the vector of uncertain parameters q is often supposed to be confined by the uncertainty bounding set Q, which is usually given a priory, e.g. directly by user requirements.

The most important problem consists in ensurance of stability and hence the control engineers are very often interested in a robust stability. It is familiarly known, that the continuous-time polynomial $p(s)$ is stable if and only if all its roots are located in left complex half plane. Generally, the family of polynomials:

$$P = \{ p(\cdot,q) : q \in Q \} \qquad (2)$$

is robustly stable, if $p(\cdot,q)$ is stable for all $q \in Q$.
The uncertainty enters into the polynomial (2) through the coefficient functions $\rho_i(q)$. Nevertheless, the very significant is the way how the uncertain parameters enter into the coefficients of this polynomial. In accordance with this, several basic structures of uncertainty with increasing generality are distinguished:

- Independent uncertainty structure
- Affine linear uncertainty structure
- Multilinear uncertainty structure
- Nonlinear uncertainty structure (polynomial, general)

Moreover, the single parameter uncertainty is usually considered as a special case. This paper deals with nonlinear uncertainty structure and assumes Q to be a box.

3. TOOLS FOR ROBUST STABILITY ANALYSIS

Most of the analytical techniques are highly specialized and their applicability is restricted to particular type of uncertainty structure. The most known methods include Bialas eigenvalue criterion, the Kharitonov theorem, the edge theorem, the mapping theorem, etc. On the contrary, there is a very universal tool – the combination of the value set concept and the zero exclusion condition.

Assume a family of polynomials $P = \{ p(\cdot,q) : q \in Q \}$. The value set at frequency $\omega \in \mathbf{R}$ is given by:

$$p(j\omega,Q) = \{ p(j\omega,q) : q \in Q \} \qquad (3)$$

In other words, $p(j\omega,Q)$ is the image of Q under $p(j\omega,\cdot)$ – e.g. substitute s for $j\omega$ in a family $P = \{ p(s,q) : q \in Q \}$, fix ω and let the vector of uncertain parameters q range over the set Q.

The zero exclusion condition for Hurwitz stability of polynomial family $P = \{ p(s,q) : q \in Q \}$ says: Suppose invariant degree of polynomials in the family, pathwise connected uncertainty bounding set *Q*, continuous coefficient functions $a_i(q)$ for $i = 0,1,2,\dots,n$ and at least one stable member $p(s,q^0)$. Then the family *P* is robustly stable if and only if the complex plane origin is excluded from the value set $p(j\omega,Q)$ at all frequencies $\omega \geq 0$, i.e. *P* is robustly stable if and only if:

$$0 \notin p(j\omega,Q) \quad \forall \omega \geq 0 \tag{4}$$

Lots of further information can be found e.g. in (Ackerman, *et al.*, 1993; Barmish, 1994; Bhattacharyya, *et al.*, 1995; Matušů & Prokop, 2008; Sánchez-Peña & Sznaier, 1998).

4. EXAMPLE – THE POLYNOMIAL TOOLBOX

Consider the family of polynomials given by:

$$\begin{aligned} p(s,q) = s^3 + \left[\cos(q_1 q_2)\right] s^2 + \\ + \left[5\sqrt{|q_1|} - 3\sin q_2 - 3\cos(q_1 q_2) + 6\right] s + \\ + \left[-2\sqrt{|q_1|} + 4\sin q_2 + 2\cos(q_1 q_2) + 0.1\right] \end{aligned} \tag{5}$$

and $q_1, q_2 \in \langle -1;1 \rangle$.

The visualization of the value set such systems can be conveniently done in The Polynomial Toolbox for Matlab (Šebek, *et al.*, 2000; www.polyx.com) through the routines “vset” and “vsetplot”. More specifically, the set of the following commands have been used in this case:

```
pinit
p0=0.1+6*s+s^3;
p1=-2+5*s;
p2=4-3*s;
p3=2-3*s+s^2;
q1=-1:.005:1;
q2=-1:.005:1;
expr='p0+sqrt(abs(q1))*p1+sin(q2)*p2+cos(q1*q2)*p3';
V=vset(q1,q2,expr,p0,p1,p2,p3,j*(0:.25:5));
vsetplot(V)
```

Fig. 1 shows value sets of (6) for frequencies $\omega = \langle 0;5 \rangle$ with step 0.25 while the detailed version is in fig. 2. The origin of the complex plane is included in the value sets and consequently the polynomial (6) is not robustly stable.

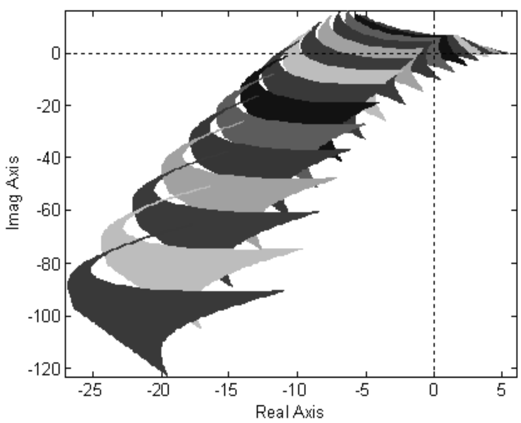

Fig. 1. The value sets of family (5) – full view

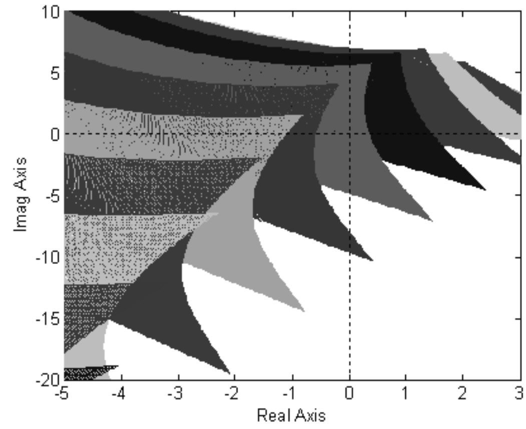

Fig. 2. The value sets of family (5) – zoomed view

5. CONCLUSION

The contribution has been focused on problems connected with robust stability analysis for systems with nonlinear uncertainty structure. Instead of very complex or not existing analytic tools the highly universal graphical test comprising the value set concept and the zero exclusion condition has been used. This principle can be generalized not only for continuous-time systems, but also for discrete-time ones or even for other stability areas. The practical investigation of robust stability based on these key instruments can be handy done in The Polynomial Toolbox for Matlab. Its possible utilization has been demonstrated on an illustrative example for continuous-time polynomial with relatively complicated general structure of uncertainty.

6. ACKNOWLEDGEMENT

The work was supported by the Ministry of Education, Youth and Sports of the Czech Republic under Research Plan No. MSM 7088352102. This assistance is very gratefully acknowledged

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