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A MODEL OF A LABORATORY PLANT WITH INTERNAL DELAYS

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Abstract: *An unconventional approach to modeling of systems with internal delays utilized on a thermal laboratory plant is presented in this paper. The so-called anisochronic models are characterized by the existence of state (internal) delays, possible both distributed and lumped, as an important part of system dynamics. The studied laboratory plant was designed as a thermal heating circuit system at Tomas Bata University in Zlin, Czech Republic, and it represents a model with dynamic properties similar to those of some real heating system. The modeling methodology is based on assumption of all significant delays and latencies in the model. The motivation for the modeling of this plant was double. First, the dynamics of the plant exhibits unconventional step responses which cannot be explained by a standard analytic means. Second, the authors of this contribution intend to use the obtained anisochronic mathematical description of the plant with the view of the verification of algebraic control algorithms designed earlier.*
Key words: *time delay systems, anisochronic models, heating systems, modelling*

1. INTRODUCTION

The task of modeling of systems with internal delays is still opened and unsatisfactorily solved problem, the appropriate solution of which depends on the particular engineering problem. Heating and/or thermal circuits are typical examples of such class of systems in practical industrial as well as real-life applications and they represent a favorite research area as it reveals from recent studies, see e.g. (Zitek, 1986; Hacia & Domke, 2007) to name but a few. Many of commonly presented models and principles are a rather complicated and yield e.g. excessive partial differential equations.

This contribution utilizes anisochronic modeling principle assuming all significant delays as an essential part of system dynamics. This modeling approach was introduced already in (Bellman & Cooke, 1963) and subsequently developed for heating systems e.g. in (Zitek,1986; Zitek & Vitecek, 1999). In this contribution, a laboratory heating plant assembled at the Faculty of Applied Informatics of Tomas Bata University in Zlin is studied. The description of the apparatus framework and its electronic circuits can be found in (Dostalek et al., 2008). Motivations for the construction of a mathematical model of the plant were the following: the dynamics of the plant exhibits unconventional step responses which cannot be explained by standard analytic means; moreover, the authors of this contribution intend to use the obtained anisochronic mathematical description for the verification of algebraic control algorithms in the special ring designed for delayed systems earlier, see e.g. (Pekar & Prokop, 2007; Pekar, 2007).

2. LABORATORY HEATING PLANT DESCRIPTION

A sketch of the laboratory apparatus is pictured in Fig. 1. A heat transferring fluid is transported using a continuously controllable DC pump (6) into a flow heater (1).

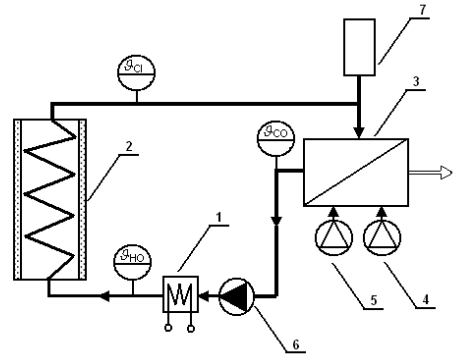

Fig. 1. A sketch of the plant

The temperature of the fluid at the heater output is measured by a platinum thermometer giving value of ϑ_{HO} . Warmed liquid then goes through a long insulated coiled pipeline (2) which causes the significant delay in the system. The air-water heat exchanger (cooler) (3) is equipped with two cooling fans (4, 5) represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers as ϑ_{CI} and ϑ_{CO}, respectively. The expansion tank (7) compensates for the expansion effect of the fluid. Obviously, the plant contains internal delays due to the circuit and thus it is suitable for testing control approaches for anisochronic systems. The laboratory model is connected to a standard PC via serial bus RS232 and a portable data acquisition unit. All tasks relating to the monitoring and control of the plant are served by software running in Matlab 6.5 environment.

3. MATHEMATICAL MODEL

The aim of this contribution is to find a sufficient anisochronic model which can be used for the verification of some control algorithms rather than searching of an exact description of the model. The methodology is based on comprehension of all significant delays and latencies in the model. Each part of the plant is modeled separately first, the whole model is then assembled using these particular descriptions. Particularly, heat balance equations are used while construction the model.

3.1 Model of the heater

$$cM_H \frac{\mathrm{d}\vartheta_{HO}(t)}{\mathrm{d}t} = P(t-0.5\tau_H) + c\dot{m}(t)\left[\vartheta_{HI}(t-\tau_H) - \vartheta_{HO}(t)\right] - K_H(t)\left[\frac{\vartheta_{HO}(t)+\vartheta_{HI}(t-\tau_H)}{2} - \vartheta_A\right] \qquad (1)$$

$$\vartheta_{HI}(t) = \vartheta_{CO}(t-\tau_{CH}) \qquad (2)$$

$$K_H(t) = \frac{h_0 P^2(t) + h_1 \dot{m}^2(t) + h_2 P(t)\dot{m}(t) + h_3}{h_4 P(t) + h_5 \dot{m}(t)} \qquad (3)$$

where c = 4180 [J kg^{-1} K^{-1}] is the specific heat capacity of a fluid, h_0, h_1, h_2, h_3, h_4, h_5 are weighting coefficients for the estimation of the overall heat transmission coefficient of the heater, $K_H(t)$ [W K^{-1}] is the overall heat transmission coefficient of heater wastage energy, $\dot{m}(t)$ [kg s^{-1}] is the mass flow rate of a fluid, M_H [kg] is the overall mass of a fluid in the heater, $P(t)$ [W] is an electric power input into the heater, $\vartheta_A(t)$ [°C] is ambient temperature, $\vartheta_{CO}(t)$ [°C] is output temperature of the cooler, $\vartheta_{HI}(t)$ [°C] is input temperature of the heater, $\vartheta_{HO}(t)$ [°C] is output temperature of the heater, τ_{CH} [s] is a delay of a fluid flow between the cooler and the heater, τ_H [s] is a delay of a fluid flow through the heater.

3.2 Model of the coiled pipeline

$$cM_P \frac{\mathrm{d}\vartheta_{CI}(t)}{\mathrm{d}t} = c\dot{m}(t)\left[\vartheta_{HO}(t-\tau_{HC}) - \vartheta_{CI}(t)\right] - K_P\left[\frac{\vartheta_{CI}(t)+\vartheta_{HO}(t-\tau_{HC})}{2} - \vartheta_A\right] \tag{4}$$

where K_P [W K^{-1}] is the overall heat transmission coefficient of the pipeline, M_P [kg] is the overall mass of a fluid in the pipeline, $\vartheta_{CI}(t)$ [°C] input temperature of the cooler, τ_{HC} [s] is a delay of a fluid flow between the heater and the cooler.

3.3 Model of the of the heat exchanger

$$cM_C \frac{\mathrm{d}\vartheta_{CO}(t)}{\mathrm{d}t} = c\dot{m}(t)\left[\vartheta_{CI}(t-\tau_C) - \vartheta_{CO}(t)\right] - K_C(t)\left[\frac{\vartheta_{CO}(t)+\vartheta_{CI}(t-\tau_C)}{2} - \vartheta_A\right] \tag{5}$$

$$K_C(t) = c_2 u_C^2(t-\tau_{KC}) + c_1 u_C(t-\tau_{KC}) + c_0 \tag{6}$$

where c_0 [W K^{-1}], c_1 [W K^{-1} V^{-1}], c_2 are weighting coefficients for the estimation of the overall heat transmission coefficient of the cooler, $K_C(t)$ [W K^{-1}] is the overall heat transmission coefficient of the cooler, M_C [kg] is the overall mass of a fluid in the cooler, $u_C(t)$ [V] is a voltage input to the cooling fan, τ_C [s] is a delay of a fluid flow through the cooler, τ_{KC} [s] is a delay between a control signal to the cooling fan and the output temperature of the cooler.

3.4 Model of the of the pump

$$\dot{m}(t) = p_0\left[u_P(t) + p_1\right]^{p_2} \tag{7}$$

where p_0 [m^3 s^{-1}], p_1 [V], p_2 are weighting coefficients for the estimation of the mass flow rate of a fluid, is a voltage input to the pump.

3.5 Model linearization

The well-known Taylor series expansion at an operation point was used where the first two terms were taken into account.

4. PARAMETERS ESTIMATION

Unknown parameters of the model can be estimated by measurements of static (i.e. steady states) and dynamic (i.e. step responses) characteristics, and from steady forms of equations (1) – (7). Least mean square criterion together with simulations was used for estimation of masses from step responses. The results are the following: h_0 = 4.1927, h_1 = -0.0017, h_2 = -15008, h_3 = -130020, h_4 = 716.468, h_5 = 77.7722; p_0 = 5.077 10^{-3}, p_1 = 0.266, p_2 = 0.274; K_P = 0.39; c_0 = 11.8, c_1 = 2.755, c_2 = -0.19; τ_H = 3, τ_{HC} = 115, τ_C = 22, τ_{KC} = 12, τ_{CH} = 8; M_H = 0.08, M_P = 0.22, M_C = 0.27.

A comparison of calculated (simulated) and measured step responses is displayed in Fig. 2.

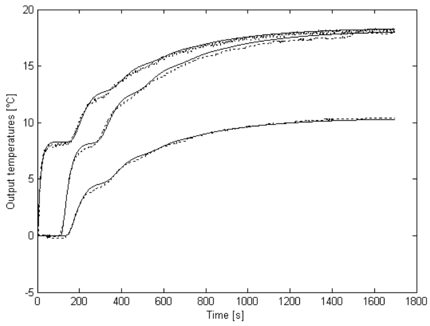

Fig. 2. Comparison of measured (dotted) and calculated (solid) step responses for u_P = 5V, u_C = 3V, ΔP = 300 W, on/off fan is on

5. CONCLUSIONS

The presented paper studied an approach to the modeling of a laboratory heating plant with both input-output and internal delays. The goal was to find an anisochronic model of the appliance so that it can be used for the verification of control algebraic algorithms, developed by the authors earlier. Anisochronic modeling philosophy comprises delays as an important factor in the plant dynamics. Unknown parameters were further estimated experimentally. The final graphical comparison of the step responses demonstrates a very good agreement of measured and calculated data and thus verifies the applicability of the model.

6. ACKNOWLEDGEMENT

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