Annals of DAAAM for 2009 & Proceedings of the 20th International DAAAM Symposium, Volume 20, No. 1, ISSN 1726-9679
ISBN 978-3-901509-70-4, Editor B. Katalinic, Published by DAAAM International, Vienna, Austria, EU, 2009
Make Harmony Between Technology and Nature, and Your Mind will Fly Free as a Bird
DAAAM World Symposium

OPTIMAL DISTRIBUTION OF LOAD BETWEEN COOPERATING PRODUCTION UNITS WITH COMBINED PRODUCTION OF HEAT AND ELECTRIC ENERGY

BALATE, J[aroslav]; KLAPKA, J[indrich]; KONECNY, P[avel]; NAVRATIL, P[avel] & CHRAMCOV, B[ronislav]

Abstract: *This paper deals with one of the possible accesses to solving optimization of the operation of heat source with combined production of heat and electric energy by method of a non-linear programming. In the paper it is described the creation of non-linear mathematical model of combined heat and electric energy production plant.*
Key words: *heat, electric energy, non-linear model, optimization*

1. INTRODUCTION

The problem of economical distribution of load between cooperating production units arises at optimization of combined production of electric energy and heat production plants which have larger number of cooperating production units. Knowledge of power-economical characteristics of separate production appliances is the basic presumption of economical production, in our specific case it concerns consumption characteristics of boilers which generally have non-linear course. The task of economical distribution of load belongs between the basic tasks of optimal control. In this case its basic principle is to minimize fuel consumption for required heat output. From the mathematical point of view the aim of optimal control will be to achieve extreme value of objective (criterial) function E, in our case minimization of production costs. We will look for the minimum of objective function E, therefore for the minimum of production costs.

In our case the optimization of combined heat and power plant operation (example - Fig. 1) can be carried out by two methods:

- linear replacement of consumption characteristics of boilers
- non-linear replacement of consumption characteristics of boilers

Linear and non-linear mathematical models of a heat source with combined production of heat and electric energy are described in this paper.

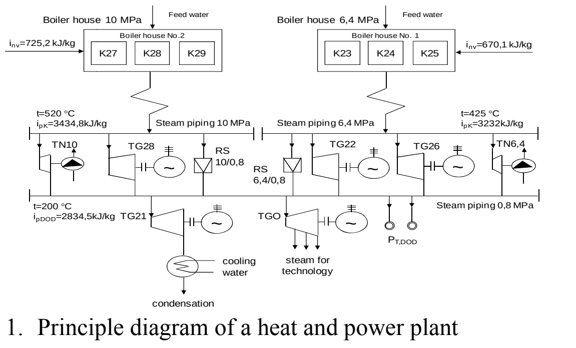

Fig. 1. Principle diagram of a heat and power plant

Legend: TN – Turbo-feed pump, TG – Back-pressure turbo-generator, TG21 - Condensing turbo-generator, TGO - Bleeder steam turbine, RS - Reducing station, $P_{T,DOD}$ - Heat output supply

The total immediate electric output P will be calculated according to determined heat output supplied to the heat network $P_{T,DOD}$. $P_{T,DOD}$ is the independent variable.

2. MATHEMATICAL LINEAR MODEL

This model was formulated in papers (Balate, 1969), (Balate et al., 2006), (Phan, 1996), (Simek, 2007). Essentially the linear model was created on the base of knowledge of balance equations of steam pipings, further of limiting conditions of non-negative values of dependent variables and of objective function. All variables in the used mathematical model and also limiting conditions are linear. The supplied heat output in the heat network $P_{T,DOD}$ *is* independent variable. In order to form the mathematical linear model, it is necessary to perform the following requirements:

- the consumption parameters of production units must have a convex monotonic course
- it is necessary to approximate the convex curves of the consumption parameters by linear sections

The way of linearization of the consumption parameters characteristics is illustrated on Fig. 2a. Consumption characteristics for back-pressure turbines are shown on Fig. 2b.

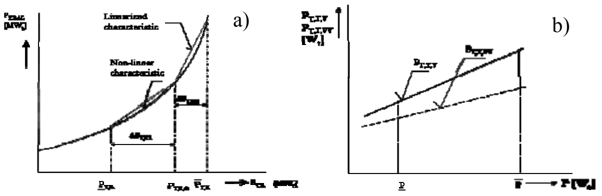

Fig. 2. Economic characteristic of a steam boiler (a), Economic characteristic of back-pressure turbine (b)

Legend: $P_{T,PAL}$ [W_t] - heat output in fuel, $P_{T,K}$ [W_t] - heat output of boiler, $P_{T,K,ek}$ [W_t] - heat output of boiler (economical), $\Delta P_{T,K}$ [W_t] – increment of heat output of boiler, $\underline{P}_{T,K}$ [W_t] - minimum value of heat output of boiler, $\overline{P}_{T,K}$ [W_t] - maximum value of heat output of boiler; $P_{T,T,V}$ [W_t] - turbine heat input, $P_{T,T,VY}$ [W_t] - heat output on turbine outlet, P [W_e] - electric power of turbo-generator unit

The linear mathematical model for linear programming consists of:

- heat balance equations in piping
- equations of produced electric power
- definition of non-negative inequalities
- definition of objective function E
- determination of delivered heat output to heat network

The linear model has a lot of dependent variables and one independent variable - $P_{T,DOD}$.

3. MATHEMATICAL NON-LINEAR MODEL

In this chapter creation of non-linear mathematical model will be described. This model is the main item of this paper.

Derivation of consumption characteristics

Consumption characteristics of boilers have generally non-linear courses (Fig. 2a). These courses can be replaced by exponential approximation

$$y = a.e^{\beta.x} \rightarrow P_{T,PAL} = a.e^{\beta P_{T,K}} \tag{1}$$

their relative increment of heat is expressed by derivation

$$y' = a.\beta.e^{\beta.x} \tag{2}$$

Non-linear mathematical model of the scheme of combined heat and electric energy production plant (Fig. 1) - theoretical part

1. Balance equation for steam piping 10 MPa

$$\sum_{i=1}^{n,10} a_{10,i}\, e^{\beta_{10,i} P_{T,K}^{10,i}} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right)\right) + P_{T,N,V}^{10} + P_{T,R}^{10/0,8}\right] k_q^{10} \tag{3}$$

Left side in this balance equations is expressed by means of exponential approximation (1); separate coefficients are expressed as follows: $a = a_{10,i}$, $\beta = \beta_{10,i}$ and dependent variable which we look for is expressed as: $x = P_{T,K}^{10,i}$.

2. Balance equation for steam piping 6.4 MPa

$$\sum_{i=1}^{n,6.4} a_{6.4,i} e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right)\right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8}\right] k_q^{6.4} \tag{4}$$

Left side of this balance equation is also expressed by means of exponential approximation (1); separate coefficients are expressed as follows: $a = a_{6.4,i}$, $\beta = \beta_{6.4,i}$ and dependent variable which we look for is expressed as: $x = P_{T,K}^{6.4,i}$.

3. Balance equation for steam piping 0.8 MPa

$$\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right) + \sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right) + P_{T,N,VY}^{10} + \tag{5}$$

$$+ P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} = \left[\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j}\left(P^{0.8,j} - \underline{P}^{0.8,j}\right)\right) + P_{T,DOD}\right] k_q^{0.8}$$

4. Total produced electric output

$$P = \sum_{j=1}^{m,10} P^{10,j} + \sum_{j=1}^{m,6.4} P^{6.4,j} + \sum_{j=1}^{m,0.8} P^{0.8,j} \tag{6}$$

5. Objective function

Objective function expresses production costs for dependent variable $P_{T,K}$. It is necessary to calculate the values of this dependent variable for each value of independent variable $P_{T,DOD}$ - heat supplied to heat network (Fig. 1).

Then the objective function E has this form

$$E = \sum_{i=1}^{n,10} a_{10,i}\left[e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}\right] + \sum_{i=1}^{n,6.4} a_{6.4,i}\left[e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}\right] \tag{7}$$

upon the conditions:

$$\underline{P}_{T,K}^{10,i} \le P_{T,K}^{10,i} \le \overline{P}_{T,K}^{10,i} \quad (i = 1, 2, \ldots, n_{10}), \tag{8}$$

$$\underline{P}_{T,K}^{6.4,i} \le P_{T,K}^{6.4,i} \le \overline{P}_{T,K}^{6.4,i} \quad (i = 1, 2, \ldots, n_{6.4}) \tag{9}$$

It is useful to stress that non-linearities of this problem are in difference to these (following) variables:

$e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}$ and $e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}$

Variables $e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}$ and $e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}$ are constant.

Non-linear mathematical model is described by equations No. (3)-(9).

Meaning of the variables in the equations:

The symbol $\underline{P}$ signs the minimum value and in opposite the symbol $\overline{P}$ signs the maximum one of the appropriate quantity

$P_{T,K}$	[GJ/h] the boilers output
$P_{T,T,V}^{10}$, $P_{T,T,V}^{6.4}$	[GJ/h] the heat input on turbine inlet
$P_{T,T,VY}^{10}$, $P_{T,T,VY}^{6.4}$	[GJ/h] the heat output on turbine outlet
$P_{T,N,V}^{10}$, $P_{T,N,V}^{6.4}$	[GJ/h] the heat input on inlet of the turbo-feed pump
$P_{T,N,VY}^{10}$, $P_{T,N,VY}^{6.4}$	[GJ/h] the heat output on outlet of the turbo-feed pump
$P_{T,R}^{10/0.8}$, $P_{T,R}^{6.4/0.8}$	[GJ/h] the heat output of the reduction stations
$P_{T,DOD}$	[GJ/h] the heat output supplied to the heat network
$k_q^{10}, k_q^{6.4}, k_q^{0.8}$	[-] the coefficients of heat consumption or losses in each phase of technology
$b_{T,V}$, $b_{T,VY}$	[GJ/MWh] relative increment of heat consumption on turbine inlet, outlet
P	[MW] electric output

Non-linear mathematical model of the scheme of combined heat and electric energy production plant (Fig. 1) - practical part

Balance equation for steam piping 10 MPa

$$\sum_{i=1}^{n,10} a_{10,i}\, e^{\beta_{10,i} P_{T,K}^{10,i}} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right)\right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8}\right] k_q^{10} \tag{10}$$

$$\sum_{i=1}^{3,10} a_{10,i}\, e^{\beta_{10,i} P_{T,K}^{10,i}} = \left[\left(\sum_{j=1}^{1}\left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right)\right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8}\right] k_q^{10}$$

$$99.057\, e^{0.0034 P_{T,K}^{10,1}} + 99.057\, e^{0.0034 P_{T,K}^{10,2}} + 99{,}057\, e^{0.0034 P_{T,K}^{10,3}} =$$

$$[(243.02 + 23.84(P^{10,1} - 6)) + 50.91 + 0]\, 1.035$$

Balance equation for steam piping 6.4 MPa

$$\sum_{i=1}^{n,6.4} a_{6.4,i} e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right)\right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8}\right] k_q^{6.4} \tag{11}$$

$$\sum_{i=1}^{3,6.4} a_{6.4,i} e^{\beta_{6.4,i} P_{T,K}^{6.4,3}} = \left[\left(\sum_{j=1}^{2}\left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right)\right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8}\right] k_q^{6.4}$$

$$38.053\, e^{0{,}0092 P_{T,K}^{6.4,1}} + 38.053\, e^{0{,}0092 P_{T,K}^{6.4,2}} + 72.814\, e^{0{,}005 P_{T,K}^{6.4,3}} =$$

$$\left[\left(108.94 + 30.38\left(P^{6.4,1} - 2\right)\right) + \left(157.13 + 27.15\left(P^{6.4,2} - 4\right)\right) + 48.6 + 0\right] 1.015$$

Balance equation for steam piping 0.8 MPa

$$\sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right) + \sum_{j=1}^{m}\left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right) + P_{T,N,VY}^{10} + \tag{12}$$

$$+ P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6,4/0.8} = \left[\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j}.\Delta P^{0.8,j}\right) + P_{T,DOD}\right] k_q^{0.8}$$

$$\sum_{j=1}^{1}\left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right) + \sum_{j=1}^{2}\left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right) + P_{T,N,VY}^{10} +$$

$$+ P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} = \left[\left(\sum_{j=1}^{2}\left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j}\left(P^{0.8,j} - \underline{P}^{0.8,j}\right)\right)\right) + P_{T,DOD}\right] k_q^{0.8}$$

$$213.69 + 20.95\left(P^{10,1} - 6\right) + \left(99.72 + 26.23\left(P^{6.4,1} - 2\right) + 139.53 + 23.46\left(P^{6.4,2} - 4\right)\right) + 43.91$$

$$+ 39.72 + 0 + 0 = \left[\left(57.82 + 69.14(2.75 - 0.8) + 53.63 + 31.43\left(P^{0.8,2} - 2\right)\right) + P_{T,DOD}\right] 1.066$$

Total produced electric output

$$P = \sum_{j=1}^{m,10} P^{10,j} + \sum_{j=1}^{m,6.4} P^{6,4,j} + \sum_{j=1}^{m,0.8} P^{0.8,j} = \sum_{j=1}^{1,10} P^{10,j} + \sum_{j=1}^{2,6.4} P^{6.4,j} + P_{TGO}^{0.8,1} + P_{TG21}^{0.8,2} \tag{13}$$

$$= P^{10,1} + P^{6.4,1} + P^{6.4,2} + 2.75 + P_{TG21}^{0.8,2}$$

Objective function

$$E = \sum_{i=1}^{n,10} a_{10,i}\left[e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}\right] + \sum_{i=1}^{n,6.4} a_{6.4,i}\left[e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}\right]$$

$$= \sum_{i=1}^{3,10} a_{10,i}\left[e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}\right] + \sum_{i=1}^{3,6.4} a_{6.4,i}\left[e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}\right] \tag{14}$$

$$= 99.057\left[e^{0.0034 P_{T,K}^{10,1}} - e^{0.0034 \cdot 206.15}\right] + 99.057\left[e^{0.0034 P_{T,K}^{10,2}} - e^{0.0034 \cdot 171.79}\right] +$$

$$+ 99.057\left[e^{0.0034 P_{T,K}^{10,3}} - e^{0.0034 \cdot 171.79}\right] + 38.053\left[e^{0.0092 P_{T,K}^{6.4,1}} - e^{0.0092 \cdot 64{,}95}\right] +$$

$$+ 38.053\left[e^{0.0092 P_{T,K}^{6,4,2}} - e^{0.0092 \cdot 80.87}\right] + 72.814\left[e^{0.005 P_{T,K}^{6.4,3}} - e^{0.005 \cdot 161.73}\right]$$

Limiting non-negative conditions: (15)

$206.15 \le P_{T,K}^{10,1} \le 429.48$	[GJ/h]	$171.79 \le P_{T,K}^{10,2} \le 343.58$	[GJ/h]
$171.79 \le P_{T,K}^{10,3} \le 343.58$	[GJ/h]	$64.95 \le P_{T,K}^{6.4,1} \le 145.81$	[GJ/h]
$80.87 \le P_{T,K}^{6.4,2} \le 161.73$	[GJ/h]	$161.73 \le P_{T,K}^{6.4,3} \le 243.02$	[GJ/h]
$6 \le P^{10,1} \le 25.5$	[MW]	$2 \le P^{6.4,1} \le 5.1$	[MW]
$4 \le P^{6.4,2} \le 9$	[MW]	$2 \le P_{TG21}^{0.8,2} = P^{0.8,2} \le 6$	[MW]

4. CONCLUSION

This paper has explained two methods of forming mathematical model of a heat and power plant which produces heat and electric energy in order to optimize its operation. Certainly, the differences between both models consist in reduction of the base variables number, the consumption characteristic approximation of non-linear equations and in forming of the other objective function. In the case of the linear model, the objective function is linear and it is non-linear in the second one. In order to gain the values for the general coefficients in the above mentioned mathematical models, it is necessary to process the whole rank of base and consumption parameters of the production units. The obtained results would be used as a base for operative planning and also for own operation control of production of heat and electric energy in real time.

5. ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Education of the Czech Republic under grant No. MSM 7088352102 and in part by the project of National Research Programme II No. 2C06007.

6. REFERENCES

Balate, J. (1969). *Design of optimal control of heat output of district heating systems sources in Brno locality.* Dissertation work, Brno (in Czech)

Balate, J. Chramcov, B. & Navratil, P. (2006). Non-linear model of optimization of operation of the heat source with combined production of heat and power, *Proceedings of 17th International DAAAM Symposium*, Katalinic, B. (Ed.), pp.027-028, ISBN 3-901509-57-7, Vienna, Austria, November 2006, Published by DAAAM International Vienna, Vienna

Phan, T.D. (1996). *Algorithms of optimal control in production and heat distribution.* Diploma work, Zlin 1996 (in Czech)

Šimek, J. (2007). *Optimal control of the heat source with combined production of heat and power*. Diploma work, Zlin 2007 (in Czech)

*** (2006) Optimization Toolbox, MATLAB