Research Article

The Kronecker Summation Method for Robust Stabilization Applied to a Chemical Reactor

Radek Matušů,¹ Jana Závacká,² Roman Prokop,¹ and Monika Bakošová²

¹ Department of Automation and Control Engineering, Faculty of Applied Informatics, Tomas Bata University in Zlín, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic

² Department of Information Engineering and Process Control, Faculty of Chemical and Food Technology,

Slovak University of Technology in Bratislava, Radlinského 9, 812 37 Bratislava, Slovakia

Correspondence should be addressed to Radek Matušů, rmatusu@fai.utb.cz

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The paper focuses on robust stabilization where the suitable parameters of a simple continuous-time PI controller are determined through a combination of the Kronecker summation method, sixteen plant theorem, and an algebraic approach to control design in the ring of proper and stable rational functions. The initial theoretical background is followed by an illustrative experiment which includes computation of the controller and verification of control results for a continuous stirred tank reactor with exothermic reaction modelled as a fourth-order interval system.

1. Introduction

A popular and practically preferred approach to appropriate control synthesis for plants with complex properties consists of construction of an uncertain model, which should cover all possible operating points, parameter variations and nonlinear behavior, and consequent robust control design [1, 2]. Although the resulting controllers can have a simple structure and fixed parameters, which is even acknowledged from the practical application viewpoint, both process modelling and control design are generally nontrivial tasks.

Chemical reactors, which belong among the most interesting and critical processes in all chemical engineering, represent the class of systems suitable for robust control applications. Their control is usually affected by very complex behavior and, moreover, bounded with potential safety problems. A common type of reactor is known as a continuous stirred tank reactor (CSTR) [3]. The mathematical model of CSTR, robustly stabilized in this paper, has been constructed in [4]. Moreover, the same work has presented stabilization of the CSTR using technique from [5, 6] embellished with a polynomial control. Besides, robust static output feedback control has been utilized to this CSTR in [7]. The idea of robust stabilization applied in this paper is similar to [4], but the contribution is mainly in the use of alternative methods, that is, combination of the Kronecker summation method [8] and an algebraic approach to control design under the ring of proper and stable rational functions (R_{PS}) allowing the elegant tuning [9–14] which has been already investigated in [15, 16].

This paper deals with design of robustly stabilizing continuous-time PI controllers for a continuous stirred tank reactor (CSTR) in which exothermic reaction occurs. The controlled plant is assumed as a fourth-order interval plant, and easily tunable PI controller is designed in order to robustly stabilize the closed control loop. As a synthesis method, the combination of Kronecker sum method, sixteen plant theorem and an algebraic approach, is utilized. This compound and its application to the chemical reactor model represent the key contribution of the work. The robust stabilization is verified on a simulative example.

The paper is organized as follows. In Section 2, the Kronecker summation method is described. Section 3 then extends the idea for an interval plant. Subsequently, Section 4 briefly outlines the algebraic approach to controller design itself. Next, a chemical reactor description and specific control experiment are provided in the extensive Section 5. And finally, Section 6 offers some conclusion remarks.

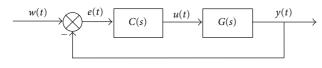


FIGURE 1: The classical closed control loop.

2. Kronecker Summation Method

An interesting technique for computation of stabilizing PI controllers based on the Kronecker summation has been presented in [8]. The main purpose of the method is to find possible variations of PI controller parameters which ensure stability of the classical one-degree-of-freedom closed control loop according to Figure 1, where

$$G(s) = \frac{B(s)}{A(s)} \tag{1}$$

is transfer function of a controlled system with fixed coefficients, and

$$C(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s}$$
(2)

is a PI controller.

Remember that Kronecker summation of two general square matrices Q (with size q-by-q) and R (r-by-r) is defined as [17]

$$Q \oplus R = Q \otimes I_r + I_q \otimes R, \tag{3}$$

where I_q , I_r are identity matrices of size *q*-by-*q* and *r*-by-*r*, respectively, and where \otimes stands for the Kronecker product. For example,

$$Q \otimes I_r = \begin{bmatrix} q_{11}I_r & \cdots & q_{1q}I_r \\ \vdots & \ddots & \vdots \\ q_{q1}I_r & \cdots & q_{qq}I_r \end{bmatrix}.$$
 (4)

The significant feature of the final square matrix (3) (qr-by-qr) is that it has qr eigenvalues which are pair-wise combinatoric summations of the q eigenvalues of Q and r eigenvalues of R. In other words, the operation of Kronecker summation induces the "eigenvalue addition" feature to the matrices. One can exploit this property to obtain the equation for which all pairs (k_P, k_I) leading to purely imaginary roots comply.

The characteristic equation of the closed control loop (Figure 1) has the form

$$P_{CL} = A(s)s + B(s)(k_P s + k_I)$$

= $f_n(k_P, k_I)s^n$
+ \cdots + $f_1(k_P, k_I)s + f_0(k_P, k_I)$
= 0. (5)

Now define

$$x'_{1} = x_{2}$$

$$x'_{2} = x_{3}$$

$$\vdots$$

$$x'_{n} = \frac{f_{0}(k_{P}, k_{I})}{f_{n}(k_{P}, k_{I})}x_{1} - \frac{f_{1}(k_{P}, k_{I})}{f_{n}(k_{P}, k_{I})}x_{2} - \dots - \frac{f_{n-1}(k_{P}, k_{I})}{f_{n}(k_{P}, k_{I})}x_{n},$$
(6)

and transform (5) into matrix differential equation

$$X' = MX, (7)$$

where *M* is matrix of size *n*-by-*n*

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\frac{f_0(k_P, k_I)}{f_n(k_P, k_I)} - \frac{f_1(k_P, k_I)}{f_n(k_P, k_I)} & \cdots & \cdots & -\frac{f_{n-1}(k_P, k_I)}{f_n(k_P, k_I)} \end{bmatrix},$$
(8)

and
$$X' = [x'_1, x'_2, \dots, x'_n]^T$$
, $X = [x_1, x_2, \dots, x_n]^T$. Equations (5) and (7) are connected by relation

 $P_{CL} = f_n(k_P, k_I) \det(sI - M) = 0.$ (9)

Evidently, the same complex variable s is both the root of (5) and the eigenvalue of M. Thanks to the fact that M is a constant matrix, the complex conjugates of s must satisfy

also (9), that is,

$$\det(s^*I - M) = 0.$$
 (10)

Consequently, as it was presented in [8], if $s = j\omega$ is the root of (5) it must be the eigenvalue of M. Furthermore, $s^* = -j\omega$ is also the root of (5) and the eigenvalue of M. As the summation of two eigenvalues $s = j\omega$ and $s^* = -j\omega$ is equal to zero, the Kronecker summation of two matrices must be singular when such correspondence of k_P , k_I , and ω occurs. Thus,

$$\det(M \oplus M) = 0 \tag{11}$$

determines the stability boundary in (k_P, k_I) plane, because every couple (k_P, k_I) satisfying (11) means that the same couple inserted into (5) will lead to the pair of conjugate purely imaginary roots or zero roots. These positions together with the line $k_I = 0$ are the only ones where the system stability can shift. Generally, the stability boundary splits the (k_P, k_I) plane into the stable and unstable areas. The selection of the stabilizing region(s) can be performed through a test point and corresponding representative polynomial within each area.

3. Robust Stabilization of Interval Systems Using PI Controller

So far, we could apply the Kronecker sum method to calculate a region of stabilizing PI controller parameters for plant with fixed coefficients. However, the papers [5, 6, 8] extended this (or an alternative) stabilization technique also for interval systems. The simple idea consists of the combination with the so-called sixteen plant theorem [1, 18, 19]. This proposition says that a first-order controller (such as PI controller) robustly stabilizes an interval plant

$$G(s,b,a) = \frac{B(s,b)}{A(s,a)} = \frac{\sum_{i=0}^{m} [b_i^-, b_i^+] s^i}{s^n + \sum_{i=0}^{n-1} [a_i^-, a_i^+] s^i}, \quad m < n, \quad (12)$$

if and only if it stabilizes its sixteen Kharitonov plants. The values b_i^- , b_i^+ , a_i^- , and a_i^+ represents, respectively, lower and upper bounds for parameters in numerator and denominator.

Remember that the Kharitonov plants are defined as

$$G_{i,j}(s) = \frac{B_i(s)}{A_j(s)},\tag{13}$$

where $i, j \in \{1, 2, 3, 4\}$; $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval system (12), that is [20],

$$B_{1}(s) = b_{0}^{-} + b_{1}^{-}s + b_{2}^{+}s^{2} + b_{3}^{+}s^{3} + \cdots$$

$$B_{2}(s) = b_{0}^{+} + b_{1}^{+}s + b_{2}^{-}s^{2} + b_{3}^{-}s^{3} + \cdots$$

$$B_{3}(s) = b_{0}^{+} + b_{1}^{-}s + b_{2}^{-}s^{2} + b_{3}^{+}s^{3} + \cdots$$

$$B_{4}(s) = b_{0}^{-} + b_{1}^{+}s + b_{2}^{+}s^{2} + b_{3}^{-}s^{3} + \cdots$$
(14)

and analogically

$$A_{1}(s) = a_{0}^{-} + a_{1}^{-}s + a_{2}^{+}s^{2} + a_{3}^{+}s^{3} + \cdots$$

$$A_{2}(s) = a_{0}^{+} + a_{1}^{+}s + a_{2}^{-}s^{2} + a_{3}^{-}s^{3} + \cdots$$

$$A_{3}(s) = a_{0}^{+} + a_{1}^{-}s + a_{2}^{-}s^{2} + a_{3}^{+}s^{3} + \cdots$$

$$A_{4}(s) = a_{0}^{-} + a_{1}^{+}s + a_{2}^{+}s^{2} + a_{3}^{-}s^{3} + \cdots$$
(15)

Thus, robust stabilization of interval system directly follows from the simultaneous stabilization of all sixteen fixed Kharitonov plants. The final stability region for the original interval system is given by the intersection of sixteen partial regions obtained individually via the Kronecker summation method.

4. Algebraic Tuning of PI Controller

Recapitulate that for now we are able to compute all possible robustly stabilizing variations of proportional and integral parts in PI compensator. However, the final choice of the controller from the obtained stability region represents another task. A very good survey on PI(D) control issues is provided, for example, in [21]. For the purpose of this paper, a simple but effective solution offers an algebraic approach to control synthesis [9-11]. This fractional design is grounded in general solutions of Diophantine equations in R_{PS}, Youla-Kučera parameterization of controllers, and conditions of divisibility in R_{PS}. One of main advantages of this method can be seen in the existence of the only tuning parameter m > 0 which serves for influencing the control behavior. This paper is not going to explain the details of this approach. Interested readers can find them, for example, in [11–14]. This work only takes advantage of one simple tuning rule. The parameters of PI controller (2) can be computed in compliance with

$$k_P = \frac{2m - a_0}{b_0}, \qquad k_I = \frac{m^2}{b_0},$$
 (16)

where a_0 and b_0 come from the first-order nominal controlled plant

$$G_N(s) = \frac{b_0}{s+a_0},$$
 (17)

and where the tuning parameter m > 0 can be selected, for example, using the recommendation from [12]

$$m = ka_0. \tag{18}$$

Suitable coefficient k depends on the size of first overshoot. For example, the choice k = 2.14, which is applied in the following simulation experiments, leads to 3% overshoot.

5. Application to a Chemical Reactor

5.1. CSTR Description. The controlled process adopted from [4, 7, 22] represents hydrolysis of propylene oxide to

propylene glycol in a CSTR. More specifically, the chemical reaction of the process is

$$C_3H_6O + H_2O \longrightarrow C_3H_8O_2. \tag{19}$$

Besides propylene oxide and water, methanol is also added to the CSTR in order to improve the solubility of propylene oxide in water. The excess of water ensures higher selectivity to propylene glycol and eliminates consecutive reactions of propylene oxide with nascent propylene glycol. Dependence of the rate constant of chemical reaction on the temperature can be described by the well-known Arrhenius equation

$$k = k_{\infty} e^{-E/RT_r},\tag{20}$$

where k means reaction rate constant, k_{∞} is the preexponential factor, E represents the activation energy, R signifies the universal gas constant, and T_r is the temperature of the reaction mixture.

Under assumption of ideal mixing in the CSTR, constant reacting volume, and the identical volumetric flow rates of the inlet and outlet streams, the mass balance of the system can be given by

$$V_r \frac{dc_i}{dt} = q_r (c_{i0} - c_i) + V_r \nu_i r \quad i = 1, 2, 3,$$
(21)

where V_r stands for the reacting volume, c_i means the molar concentration of the *i*th component, c_{i0} is the feed molar concentration of the *i*th component, q_r represents the volumetric flow rate of the reaction mixture, v_i determines the stoichiometric coefficient of the *i*th component, and $r = kc_{C_3H_6O}$ is the molar rate of the chemical reaction.

Further, independency of the specific heat capacities, densities, and volumetric flow rates on temperature or mixture composition has been supposed. Moreover, the mixing volume and the heat of mixing have been neglected. So, the simplified enthalpy balance of the reaction mixture and the simplified enthalpy balance of the cooling medium introduced in the monograph [23] and subsequently in the papers [4, 7] can be formulated as, respectively,

$$V_r \rho_r c_{pr} \frac{dT_r}{dt} = q_r \rho_r c_{pr} (T_{r0} - T_r) - UA(T_r - T_c) + V_r (-\Delta_r H^o) r$$
(22)
$$V_c \rho_c c_{pc} \frac{dT_c}{dt} = q_c \rho_c c_{pc} (T_{c0} - T_c) + UA(T_r - T_c),$$

where *T* means the temperature, ρ represents the density, c_p is the specific heat capacity, $\Delta_r H^0$ stands for the reaction enthalpy, *U* is the overall heat transfer coefficient, and *A* is

the heat exchange area. Furthermore, meaning of subscripts is as follows: 0 is for the feed, c for the cooling medium, and r for the reaction mixture. Equation (22) represents a standard in CSTR design. Interested readers can find the specific values of all constant parameters and steady-state inputs of the CSTR in tables in [4, 7].

In fact, the physical parameters (such as reaction enthalpy, pre-exponential factor, and overall heat transfer coefficient) of this CSTR are not known exactly but they are supposed to vary within some intervals (see again [4, 7]). However, this paper particularly takes advantage of the final mathematical model of the CSTR introduced in [4], where it is obtained in the linearized form of an interval transfer function:

$$G(s,b,a) = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0},$$
 (23)

with parameters which can vary within the following bounds:

$$b_{2} \in \langle -0.0291, -0.0245 \rangle$$

$$b_{1} \in \langle -0.0199, -0.0127 \rangle$$

$$b_{0} \in \langle -0.0005740, -0.0003549 \rangle$$

$$a_{3} \in \langle 0.5801, 0.9030 \rangle$$

$$a_{2} \in \langle 0.1002, 0.2299 \rangle$$

$$a_{1} \in \langle 0.0062, 0.0142 \rangle$$

$$a_{0} \in \langle 0.0001094, 0.0002412 \rangle.$$

$$(24)$$

5.2. Control Experiments. The interval transfer function (23) with parameters (24) describing the CSTR is considered to be the controlled plant. The first of its sixteen Kharitonov plants (13) is

$$G_{1,1}(s) = \frac{-0.0245s^2 - 0.0199s - 0.000574}{s^4 + 0.903s^3 + 0.2299s^2 + 0.0062s + 0.0001094}.$$
(25)

Corresponding closed-loop characteristic (5) can be computed as

$$s^{5} + 0.903s^{4} + (0.2299 - 0.0245k_{P})s^{3} + (0.0062 - 0.0199k_{P} - 0.0245k_{I})s^{2} + (0.0001094 - 0.000574k_{P} - 0.0199k_{I})s - 0.000574k_{I} = 0.$$
(26)

which means that the matrix (8) takes the following form:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{f_0(k_P, k_I)}{f_5(k_P, k_I)} & -\frac{f_1(k_P, k_I)}{f_5(k_P, k_I)} & -\frac{f_2(k_P, k_I)}{f_5(k_P, k_I)} & -\frac{f_3(k_P, k_I)}{f_5(k_P, k_I)} & -\frac{f_4(k_P, k_I)}{f_5(k_P, k_I)} \end{bmatrix},$$
(27)

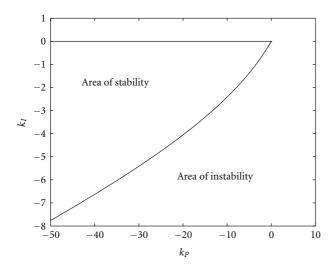


FIGURE 2: Areas of stability/instability for the Kharitonov plant (25).

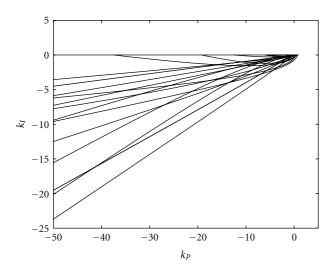


FIGURE 3: Areas of stability for sixteen Kharitonov plants.

where

$$-\frac{f_0(k_P, k_I)}{f_5(k_P, k_I)} = 0.000574k_I$$

$$-\frac{f_1(k_P, k_I)}{f_5(k_P, k_I)} = -0.0001094 + 0.000574k_P + 0.0199k_I$$

$$-\frac{f_2(k_P, k_I)}{f_5(k_P, k_I)} = -0.0062 + 0.0199k_P + 0.0245k_I \qquad (28)$$

$$-\frac{f_3(k_P, k_I)}{f_5(k_P, k_I)} = -0.2299 + 0.0245k_P$$

$$-\frac{f_4(k_P, k_I)}{f_5(k_P, k_I)} = -0.903.$$

The stability boundary is determined by (11). The positions of such pairs (k_P, k_I) which fulfill (11) are plotted in Figure 2. The decision on area of stability and instability can be simply done with the assistance of an arbitrary testing

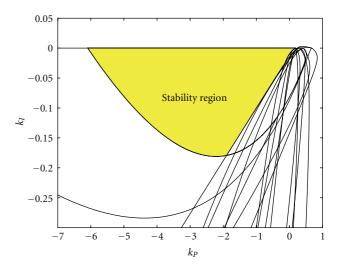


FIGURE 4: Zoomed robust stability region for the interval system (23), (24).

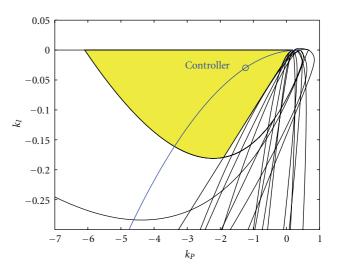


FIGURE 5: Position of controller (32) in stability region.

point from the appropriate set. Moreover, the figure is supplemented with the half line $k_I = 0$ bordering the "upper" part of the stabilizing area.

Further, we must repeat the analogical procedure for all sixteen Kharitonov plants. The stability areas for the Kharitonov plants are depicted in Figure 3, and its zoomed version with highlighted intersection of all particular areas of stability is shown in Figure 4. The highlighted area represents the final region of robustly stabilizing PI controller parameters for the original interval transfer function of CSTR (23), (24).

The following question is how to find the practically convenient PI controller from the obtained robust stability region. This paper utilizes the algebraic-based approach outlined in the Section 5. Nevertheless, this method requires a first-order fixed nominal model (17) of controlled plant in order to compute the final controller of appropriate (first) order, that is, with PI structure. So, the paper employs very $\begin{array}{c} 2.5 \\ 2 \\ 1.5 \\ y \\ 1 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 200 \\ 400 \\ 600 \\ 800 \\ 1000 \\ Time (min) \end{array}$

FIGURE 6: Output signals of 128 "representative" plants and nominal system.

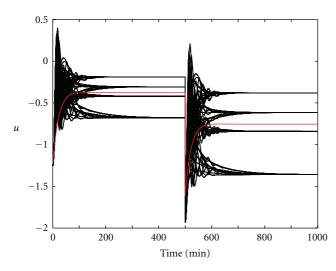


FIGURE 7: Control signals for 128 "representative" plants and nominal system.

simple but effective method of obtaining such model. In the first step, the fourth-order model with fixed parameters has been computed using the average values of interval parameters in (23), (24):

$$G_A(s) = \frac{-0.0268s^2 - 0.0163s - 0.00046445}{0.74155s^3 + 0.16505s^2 + 0.0102s + 0.0001753}.$$
(29)

Next, the simplest possible approximation takes advantage of neglecting the higher than zero-order powers of s in numerator and higher than first-order powers of s in denominator, that is, the corresponding coefficients are supposed to be zero. This approximation is easy but efficient enough as can be seen in the following steps. Obviously, it results in a first-order nominal model suitable for the applied synthesis:

$$G_N(s) = \frac{-0.00046445}{0.0102s + 0.0001753} = \frac{-0.045534}{s + 0.017186}.$$
 (30)

Further, (16) has been used for the calculation of PI controller parameters while the tuning parameter m has been chosen according to (18)

$$m = 2.14 \cdot a_0 = 2.14 \cdot 0.017186 \doteq 0.036778, \tag{31}$$

in order to obtain 3% first overshoot for the nominal case. Consequently, the resulting controller is given by

$$C(s) = \frac{-1.238s - 0.029706}{s}.$$
 (32)

This controller is located inside the stability region as can be seen in Figure 5. It means that the regulator robustly stabilizes the CSTR (23), (24). Besides, the controller lies on the curve hypothetically connecting the other potential controllers tuned by various parameters m > 0.

Finally, the robust stability and control performance is confirmed and demonstrated in Figure 6. It shows the output signals of the control loop with designed PI controller (32) and 128 "representative" systems from the interval family describing the CSTR (23), (24). The minimum and maximum values of each interval parameter have been used. It results in $2^7 = 128$ systems for simulation. On top of that, the red curve represents the output for the nominal system (30). Furthermore, Figure 7 depicts corresponding 128 + 1 control (actuating) signals. As far as control quality is considered, the results from Figure 6 might not be impressive at first sight, but there is a tradeoff between the simplicity of applied control algorithm and the performance here. As can be seen, the only one off-line tuned feedback PI controller with fixed coefficients has been utilized for controlling the chemical reactor with all the possible variations of uncertain parameters. We can observe that the CSTR is robustly stabilized successfully.

6. Conclusion

The main aim of the paper has been to present a possible approach to robust stabilization of a CSTR with exothermic reaction modelled as a fourth-order interval system. The developed easy but effective method of PI controller design has combined the Kronecker summation method with sixteen plant theorem and the algebraic tools. The proposed technique is applicable to a wide range of real processes, provided that they can be expressed by means of interval system and subsequently temporarily approximated by a first order model. The demerit of the method can be seen in a missing guarantee of coincident nominal performance and robust stability before the design process itself. They have to be verified during or after the design. Nevertheless, the applicability has been clearly demonstrated on the example where the CSTR has been successfully robustly stabilized.

Acknowledgments

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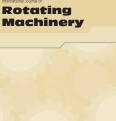
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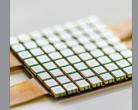


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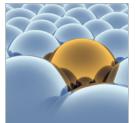




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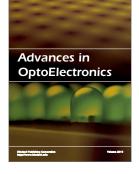
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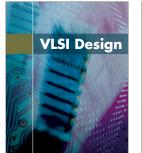


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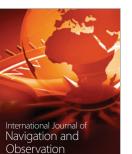


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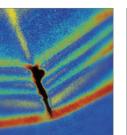


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