# AN INVESTIGATION ON EVOLUTIONARY IDENTIFICATION OF CONTINUOUS CHAOTIC SYSTEMS


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**Abstract**

This paper discusses the possibility of using evolutionary algorithms for the reconstruction of chaotic systems. The main aim of this work is to show that evolutionary algorithms are capable of the reconstruction of chaotic systems without any partial knowledge of internal structure, i.e. based only on measured data. Algorithm SOMA was used in reported experiments here. Systems selected for numerical experiments here is the well-known Lorenz system. For each algorithm and its version, repeated simulations were done, totaling 20 simulations. According to obtained results it can be stated that evolutionary reconstruction is an alternative and promising way as to how to identify chaotic systems.



**Keywords:** SOMA, Lorenz system, PSO


## 1 Introduction

Identification of various dynamical systems is vitally important in the case of practical applications as well as in theory. A rich set of various methods for dynamical system identification has been developed. In the case of chaotic dynamics, it is for example the well-known reconstruction of a chaotic attractor based on research of [1] who has shown that, after the transients have died out, one can reconstruct the trajectory of the attractor from the measurement of a single component. Since the entire trajectory contains a large amount of information, the series of papers by [2], [3] is introduced to show a set of averaged coordinate invariant numbers (generalized dimensions, entropies, and scaling indices) by which different strange attractors can be distinguished. The method presented in this research is based on Evolutionary Algorithm's (EA's), see [4], which allows the reconstruction of not only chaotic attractors as a geometrical objects, but also their mathematical description. All these techniques belong to the class of genetic programming techniques; see [5], [6]. Generally, when it is used on data fitting, these techniques are called Symbolic Regression (SR). SR represents a process, in which measured data is fitted by a suitable mathematical formula such as $x_2 + C$, $sin(x) + l/e^x$, etc., mathematically, this process is quite well known and can be used when data of an unknown process is obtained. Historically, SR has been in the preview of manual manipulation, however during the recent past, a large inroad has been made through the use of computers. Generally, there are two well-known methods, which can be used for SR by means of computers. The first one is called Genetic Programming (GP), [5], [6] and the other is Grammatical Evolution (GE), [7], [8]. The idea as to how to solve various problems using SR by means of EA's was introduced by John Koza, who used Genetic Algorithms (GA) for GP. GP is basically a symbolic regression, which is done by the use of EA's, instead of a human brain. The ability to solve very difficult problems is now well established, and hence, GP today performs so well that it can be applied, e.g. to synthesize highly sophisticated electronic circuits, [9]. In the last decade of the 20th century, C. Ryan developed a novel method for SR, called GE. GE can be regarded as an unfolding of GP due to some common principles, which are the same for both algorithms. One important characteristic of GE is that it can be implemented in any arbitrary computer language compared with GP, which is usually done (in its canonical form) in LISP. In contrast to other EA's, GE was used only with a few search strategies, for example with a binary representation of the populations in [10]. Another interesting investigation using symbolic regression was carried out by [11] working on Artificial Immune Systems or/and systems which are not using tree structures like linear GP (full text is at https://eldorado.uni-dortmund.de/bitstream /2003/20098/2/Brameierunt.pdf) and another similar algorithm to Analytic Programming (AP), Multi Expression Programming (see http://www.mep.cs.ubbcluj.ro/). Put simply, EA simulates Darwinian evolution of individuals (solutions of given problem) on a computer and are used to estimate-optimize numerical values of defined cost function. Methods of GP are able to synthesize in an evolutionary way complex structures like electronic circuits, mathematical formulas etc. from basic set of symbolic (nonnumeric) elements. In this paper, AP is applied, see [12], [13], [14], [15], [16] for the identification of selected chaotic system. Identification is not done on the "level" of the strange attractor reconstruction, but it produces a symbolic-mathematical description of the identified system. Investigation reported here is a continuation of research done in [12].

Synthesis, identification and control of complex dynamical systems are usually extremely complicated. When classics methods are used, some simplifications are required which tends to lead to idealized solutions that are far from reality? In contrast, the class of methods based on evolutionary principles is successfully used to solve this kind of problems with a high level of precision. In this paper, an alternative method of EA's is used, which has been successfully proven in many experiments like chaotic systems synthesis, neural network synthesis or electrical circuit synthesis. This paper discusses the possibility of using EA's for the identification (reconstruction) of chaotic systems. The


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main aim of this work is to show that EA's are capable of the reconstruction of chaotic systems without any partial knowledge of internal structure, i.e. based only on measured data. Self -Organizing Migrating Algorithm (SOMA) has been used for numerical study reported here. Systems selected for numerical experiments here is the well-known Lorenz at-tractor. For each algorithm and its version, repeated simulations were done, amounting to 20 simulations. According to obtained results it can be stated that evolutionary reconstruction is an alternative and promising way as to how to identify chaotic systems.

## 2 Motivation

Motivation of this research is quite simple. As mentioned in the introduction, EA's are capable of hard problem solving. A lot of examples about EA's can be easily found. EA's use with chaotic systems is done for example in [17] where EAs has been used on local optimization of chaos, [18] for chaos control with use of the multi-objective cost function or in [19] and [20], where EA's have been studied on chaotic landscapes. Slightly different approach with EA's is presented in [12], selected algorithms were used to synthesize artificial chaotic systems. In [21], [22], EA's has been successfully used for real-time chaos control and in [23] and [24] EA's was used for optimization of Chaos Control. Another examples of EA's use can be found in [25] which developed statistically robust EA's, and on the opposite side [26] used EA's for fuzzy power system stabilizer which has been applied on single-machine infinite bus system and multi-machine power system. Other research was done by [27]. Parameters of permanent magnet synchronous motors has been optimized by Particle Swarm Algorithm (PSO) and experimentally validated on the servomotor. IIR filter synthesis has been used in swarm intelligence [28] and [29] applied in co-evolutionary Particle Swarm Optimization (CoPSO) approach for the design of constrained engineering problems, particularly for pressure vessel, compression spring and welded beam. The main question in the case of this paper is if EAs are able to identify chaos in symbolic i.e. mathematical description.
All experiments here were designed to check and either confirm or reject this idea.

## 3 Evolutionary Reconstruction of Chaotic systems

Another approach completely different from classical methods, which is demonstrated in this paper is the use of EA's. They are applied on selected examples to demonstrate how evolutions can be applied ton the reconstruction of chaotic systems. Experiments described here are focused on EA's use to reconstruct mathematical description of Lorenz attractor. Preliminary results has been reported in [30], where detailed described of the reconstruction of discrete systems as well as the reconstruction of the $\dot{z}(t)$ of Lorenz system, see Eq. (1).

### 3.1 Continuous systems: Lorenz System Reconstruction

Evolutionary reconstruction of chaotic systems was in our previous numerical studies restricted mainly to discrete systems. Methods of symbolic regression are general enough to be also used on the reconstruction of continuous chaotic systems, as demonstrated in [30]. To check this idea, a well known chaotic system has been selected - Lorenz equation, see Eq. (1). In order to simplify this experiment for the first time in [30], the third equation $\dot{z}(t)$ has been selected to be synthesized, see Eq. (1). Here we demonstrate, that the whole system can be reconstructed by means of EA's, without preliminary or auxiliary information excluding time series. The basic set of objects used in symbolic regression was $\{x(t), y(t), z(t), +, -, x, /\}$. The total number of simulation has been set to 20 and in this case (comparing with [30] where 5 algorithms (Differential Evolution (DE), SOMA, GA, Simulated Annealing (SA), Evolutionary Strategies (ES)) in all 12 versions were used) only one evolutionary algorithm (SOMA) has been used in order to identify (reconstruct) by synthesis a suitable solution.

#### 3.1.1 Experiment Setup

The cost function was defined by Eq. (2), difference between the behavior of the original and identified system has been calculated in the interval $t \in [0, 20]$ with randomly selected initial conditions. Cost value has been calculated in the interval $t \in [0, 20]$. The objective was to minimize this function to 0. One version of SOMA (AllToOne), see [31], has been applied in order with AP and were used for all simulations in this paper. SOMA parameter setting is described in Table 1. Each simulation, focused on the synthesis of $x(t)$, $y(t)$, $z(t)$, has been repeated applied 20 × in order to synthesize an appropriate structure which can serve as models of the observed chaotic system.

Table 1: SOMA setting

| | |
|---|---|
| PathLength | 3 |
| Step | 0.11 |
| PRT | 0.1 |
| PopSize | 100 |
| Migrations | 8 |
| MinDiv | -0.1 |
| Individual Length | 30 |

Table 2: Cost Function Evaluations

| | |
|---|---|
| Maximum | 40 759 |
| Average | 18 549 |
| Minimum | 5 402 |

#### 3.1.2 Continuous systems: Results

Results of this case study are depicted in the Fig's. 1 -7, and Eq's. 3-7. the number of cost function evaluations, needed to get suitable solution is reported in Table 2. Some selected results are depicted in the following figures: Fig's 1-3 shows typical example of the difference between time series of the original Lorenz and identified system, Fig's 4 - gives another example of difference for another identified systems, this time in three dimensional space ($E_3$) and Fig's 6-7 show Lorenz attractor in $E_3$ in two different time slices. Selected results are also reported in Eq's. 3-5 which shows selected identified parts of Lorenz systems and in Eq. 6, where is described as an identified equivalent of Lorenz system, also used to generate Fig's 6-7.

$$
\begin{aligned}
\dot{x}(t) &= -a(x(t) - y(t)) \\
\dot{y}(t) &= bx(t) - x(t)z(t) - y(t) \\
\dot{z}(t) &= x(t)y(t) - z(t) \\
&where\ a = 3\ and\ b = 26.5
\end{aligned} \tag{1}
$$

$$
CV = \sum_{t=0}^{t=20} \left( \begin{array}{l} |x_{t,Lorenz} - x_{t,synthesized}| + \\ |y_{t,Lorenz} - y_{t,synthesized}| + \\ |z_{t,Lorenz} - z_{t,synthesized}| \end{array} \right) \tag{2}
$$

$$
\begin{aligned}
\dot{z}(t) &= y(t)\left(x(t) - \frac{z(t)}{y(t)}\right) \\
\dot{z}(t) &= y(t)\left(x(t) - \frac{x(t)+z(t)}{y(t)}\right) + x(t) \\
\dot{z}(t) &= y(t)(x(t)y(t)) + y(t)2z(t)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\dot{x}(t) &= x(t) - y(t) + 4.1(-x(t) + y(t)) \\
\dot{x}(t) &= -2.7x(t) + 2.7y(t) \\
\dot{x}(t) &= 0.32\frac{-\mathrm{x(t)}+\mathrm{y(t)}}{0.32}
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\dot{y}(t) &= 26.5x(t) - y(t) - x(t)]z(t) \\
\dot{y}(t) &= -y(t) - x(t)(-26.5 + z(t)) \\
\dot{y}(t) &= -y(t) + x(t)(26.5 - z[t])
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\dot{x}(t) &= 0.32 + \frac{-x[t]+y[t]}{0.32} \\
\dot{y}(t) &= -y[t] + x[t] * (26.5 - z[t]) \\
\dot{z}(t) &= y[t] * (x[t] - z[t]/y[t])
\end{aligned} \tag{6}
$$

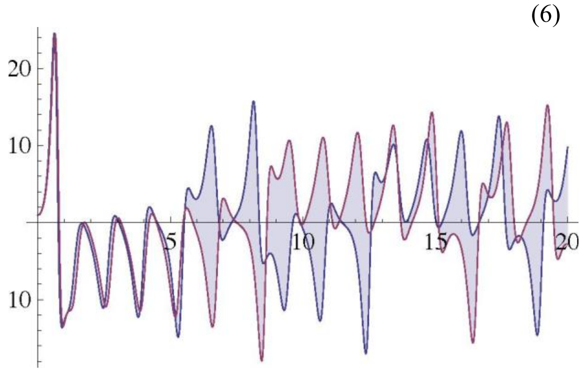

Fig. 1: Difference between $x(t)$ of the original Lorenz system (Eq. (1)) and identified $x(t)$.

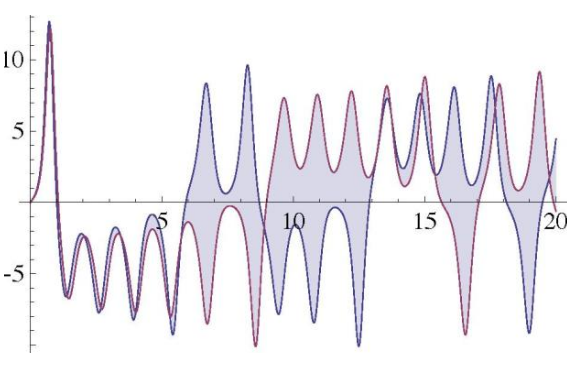

Fig. 2: Difference between $y(t)$ of the original Lorenz system (Eq. (1)) and identified $y(t)$.

It is clear that this approach is also usable, i.e. it can be used to synthesize continuous systems, and however more extensive study is needed.

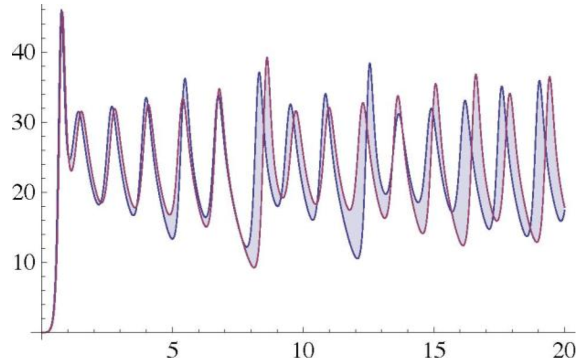

Fig. 3: Difference between $z(t)$ of the original Lorenz system (Eq. (1)) and identified $z(t)$.

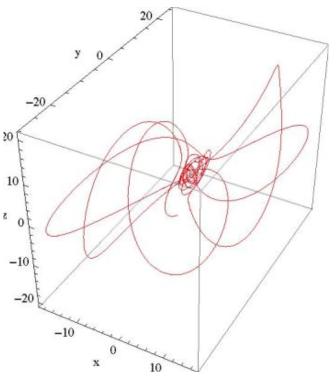

Fig. 4: Difference in the trajectory in $E_3$ between Eq. (1) and Eq. (6).

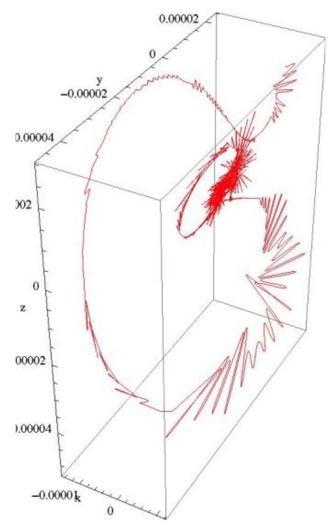

Fig. 5: Another difference in the trajectory in $E_3$ between original and identified Lorenz system.

## 4 Conclusion

Based on recorded data and results, it may be stated that simulations provided promising results, which shows that EAs are capable of model reconstruction of continuous chaotic systems. In this participation the EA SOMA [31] in version AllToOne was used and tested. Descriptions of identified systems are reported in Eq. 3-5 and Eq. 6. Based on previously mentioned facts and all experimental results, conclusions and statements can be made for Lorenz system reconstruction as follows

- **Number of successful reconstruction.** In this research, all 20 numerical simulations has returned acceptable reconstructions, which less or more has fitted the original Lorenz system behaviour. It is important to note that the behavior of Lorenz attractor was identical only in the interval $t \in [0, 20]$. Outside this interval behavior of synthesized "Lorenz system" has been less or more divergent, which is obvious.
- **Experiment overview.** The cost function, defined by Eq. (2) has been used for the continuous case. This cost function calculates the difference between original behavior of Lorenz system and the just identified one. In the time interval [0,20] with sampling period 0.01s, i.e. 2000 sampled points was used to calculate the differences. Simulation has been repeated 20 x and number of cost functions is depicted in the Table 2.
- **Number of successful reconstruction.** In this research, all 20 numerical simulations has returned acceptable reconstructions, which less or more has fitted the original Lorenz system behaviour. It is important to note that the behavior of Lorenz attractor was identical only in the interval $t \in [0, 20]$. Outside this interval behavior of synthesized "Lorenz system" has been less or more divergent, which is obvious.
- **Behavior preciseness.** Based on the figures depicted in this research, it seems that reconstruction was not as successful, because differences (see Fig's 1 - 3 or 4 - 5) are sometimes large. However we would like to say that our aim was not absolute identification (if theoretically possible) but rather identification that reconstructed system will fill state space by the same attractor as the original one. See for example Fig's 6-7. In each of both figures is depicted attractor of the original as well as the reconstructed system. One can see that both attractors occupy the same state space.
- **Other evolutionary techniques.** In this research, the so-called AP has been used, however we have to say that another and more well known techniques like GP, see [5], [6] or GE, see [7], should give similar results as reported here.

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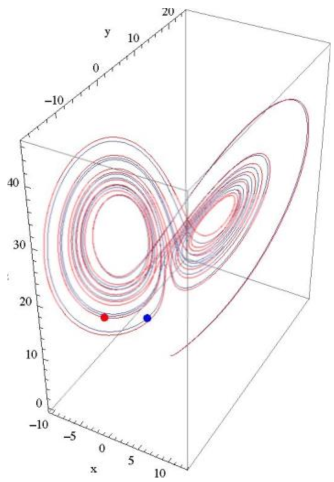


Fig. 6: Chaotic attractors of original Lorenz system (Eq. (1)) and identified system Eq. (6).

Note the position of two points (for $t$ = 10s) on trajectories, compare with. Fig 7. Identified system generates an attractor filling the same space as Lorenz, however with phase shift, resulting in Fig 4 or Fig 5.

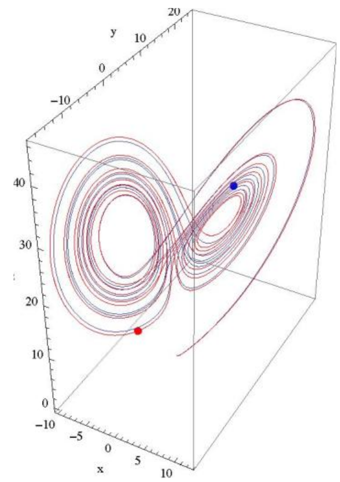


Fig. 7: Chaotic attractors of original Lorenz system (Eq. (1)) and identified system Eq. (6).

Note position of two points (for $t$ = 16s) on trajectories, compare with Fig 6. Identified system generates an attractor filling the same space as Lorenz, however with phase shift, resulting in Fig 4 or Fig 5


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