Robust Stabilization of Interval Plants using Kronecker Summation Method

RADEK MATUŠŮ, ROMAN PROKOP
Department of Automation and Control Engineering
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nad Stráněmi 4511, 76005 Zlín
CZECH REPUBLIC
{rmatusu; prokop}@fai.utb.cz

KATARÍNA MATEJIČKOVÁ, MONIKA BAKOŠOVÁ
Department of Information Engineering and Process Control
Faculty of Chemical and Food Technology
Slovak University of Technology in Bratislava
Radlinského 9, 81237 Bratislava
SLOVAKIA
{katarina.matejickova; monika.bakosova}@stuba.sk

Abstract: - This paper deals with design of continuous-time robustly stabilizing Proportional-Integral (PI) controllers for interval systems using the combination of Kronecker summation method, sixteen plant theorem and an algebraic approach to controller tuning. The effectiveness and practical applicability of the proposed method is demonstrated in both simulation and real experiments including control of a third order nonlinear electronic plant.

Key-Words: - Robust Stabilization, PI Controllers, Interval Systems, Kronecker Summation Method, Algebraic Synthesis, Electronic Model

1 Introduction

Despite the rapid development of many advanced control technologies, the engineers from practice still clearly prefer the application of controllers with simple PI or PID structure. This kind of controllers is very popular because of their easy implementation and sufficient performance at the same time, even under conditions of uncertainty, and thus the investigation of an effective tuning method remains very topical.

The long-term research interest has issued in number of theoretical and application works on classical PI(D) controllers [1], [2], [3], [4], [5] and their various modifications [6], [7], [8], [9]. A possible approach to robust control design [10], [11], [12], [13], [14], [15] for systems with interval uncertainty consists of computation of all robustly stabilizing controllers and consequently the selection of the final one on the basis of user demands. The calculation of robustly stabilizing controllers can be done using the stability boundary locus as published in [16], [17] or alternatively with the assistance of Kronecker summation method [18]. The approach from [16], [17] has been analyzed in [19], [20], while this paper studies alternative method [18] and verifies it on the same laboratory apparatus as in [19], [20]. Furthermore, a technique for controller choice itself can be adopted from algebraic approach [21], [22], [23]. This method is based mainly on general solutions of Diophantine equations in the ring of proper and Hurwitz stable rational functions (R_{PS}). An advantage is that the controller can be further tuned through the only positive scalar tuning parameter *m*.

The contribution is focused on computation of continuous-time robustly stabilizing PI controllers for interval plants using Kronecker summation method, sixteen plant theorem and several algebraic tools. Originality of the proposed approach lies in combination of Kronecker summation method for obtaining the stability boundary and the choice of the final controller via an algebraic methodology. However, the work deals not only with theoretical background but also with the practical application in laboratory conditions. A nonlinear electronic plant, considered as the 3rd order interval system, has been controlled in various operational points with the assistance of the designed PI algorithms which have been realized using the Simatic automation system by Siemens Company.

The structure of this paper is organized as follows. In Section 2, Kronecker summation method and its application in feedback stabilization of fixed plant via PI controllers are presented. The Section 3 enriches this method with sixteen plant theorem in order to make it utilizable for interval systems. Then, Section 4 contains the fundamentals and essential rules which are necessary for algebraic design of PI controllers under R_{PS}. Further, simulation example illustrating application of all described techniques is given in Section 5 and results of real control of nonlinear electronic plant in laboratory conditions are provided in an extensive Section 6. Finally, Section 7 offers some conclusion remarks.

2 Computation of Stabilizing PI Controllers using Kronecker Summation

Consider the traditional closed-loop control system as depicted in fig. 1.

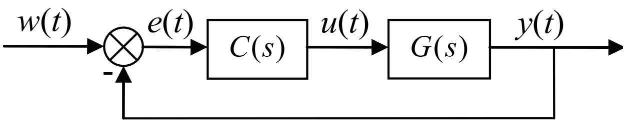

Fig. 1: Feedback Control Loop

The controlled plant is described by:

$$G(s) = \frac{B(s)}{A(s)} \tag{1}$$

and controller is supposed to be in a PI form:

$$C(s) = k_P + \frac{k_I}{s} = \frac{k_P s + k_I}{s} \tag{2}$$

The initial task is to determine the parameters of PI controllers which guarantee stability of the feedback system.

An approach to computation of stabilizing PI controllers which is based on interesting features of Kronecker summation has been published in the paper [18].

First, remind that Kronecker summation of two square matrices Y (of size k-by-k) and Z (l-by-l) is generally defined as:

$$Y \oplus Z = Y \otimes I_l + I_k \otimes Z \tag{3}$$

where I_k, I_l are identity matrices of size k-by-k and l-by-l, respectively, and where $\otimes$ denotes the Kronecker product [24], e.g. concisely:

$$Y \otimes I_l = \begin{bmatrix} y_{11} I_l & \cdots & y_{1k} I_l \\ \vdots & \ddots & \vdots \\ y_{k1} I_l & \cdots & y_{kk} I_l \end{bmatrix} \tag{4}$$

The momentous property of the obtained square matrix $Y \oplus Z$ (kl-by-kl) is that it has kl eigenvalues which are pair-wise combinatoric summations of the k eigenvalues of Y and l eigenvalues of Z. It means the Kronecker summation operation induces the "eigenvalue addition" feature to the matrices. One can exploit this attribute to obtain the equation for which all pairs (k_P, k_I) leading to purely imaginary roots comply.

The characteristic equation of the closed-loop system from fig. 1 is:

$$\begin{aligned} P_{CL} &= A(s)s + B(s)(k_P s + k_I) = \\ &= f_n(k_P,k_I)s^n + \cdots + f_1(k_P,k_I)s + f_0(k_P,k_I) = 0 \end{aligned} \tag{5}$$

Define:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ &\vdots \\ x_n' &= -\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)} x_1 - \frac{f_1(k_P,k_I)}{f_n(k_P,k_I)} x_2 - \cdots - \frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)} x_n \end{aligned} \tag{6}$$

and transform (5) into matrix differential equation:

$$X' = MX \tag{7}$$

where M is n-by-n matrix:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \ \cdots & 0 \\ 0 & 0 & 1 & 0 \ \cdots & 0 \\ \vdots & 0 & 0 & 1 \ \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \ \ddots & 0 \\ 0 & 0 & 0 & \cdots \ 0 & 1 \\ -\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_1(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_2(k_P,k_I)}{f_n(k_P,k_I)} & \cdots \ \cdots & -\frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)} \end{bmatrix} \tag{8}$$

and $X' = [x'_1, x'_2, \ldots, x'_n]^T$, $X = [x_1, x_2, \ldots, x_n]^T$. The equations (5) and (7) are linked via:

$$P_{CL} = f_n(k_P, k_I)\det(sI - M) = 0 \qquad (9)$$

Obviously, the same complex variable s is both the root of (5) and the eigenvalue of M. Owing to the fact that M is a constant matrix, the complex conjugates of s must also satisfy (9), i.e.:

$$\det(s^* I - M) = 0 \qquad (10)$$

On that account, as it has been presented in [18], if $s = j\omega$ is the root of (5) it must be the eigenvalue of M. Moreover, $s^* = -j\omega$ is also the root of (5) and the eigenvalue of M. As the sum of two eigenvalues $s = j\omega$ and $s^* = -j\omega$ equals to zero, the Kronecker summation of two matrices must be singular when such correspondence of k_P, k_I and ω occurs. Consequently:

$$\det(M \oplus M) = 0 \qquad (11)$$

defines the stability boundary in (k_P, k_I) plane, because every couple of (k_P, k_I) satisfying (11) means that the same couple inserted into (5) will lead to the pair of conjugate purely imaginary roots or zero roots. Those are the only positions where the system stability can shift. Generally, the stability boundary splits the (k_P, k_I) plane into the stable and unstable regions. The determination of the stabilizing area (or areas) can be done via a test point, leading to a polynomial to verify, within each region.

3 Robust Stabilization of Interval Plants

The previous Section has outlined calculation of region of stabilizing compensator parameters only for a system with coefficients which are fixed and can not vary. Nevertheless, the works [16], [17], [18] have embellished an arbitrary feedback stabilization technique also for interval plants simply by using its combination with the sixteen plant theorem [10], [25], [26]. According to this rule, a first order controller robustly stabilizes an interval plant:

$$G(s,b,a) = \frac{B(s,b)}{A(s,a)} = \frac{\sum_{i=0}^{m}\left[b_i^-, b_i^+\right]s^i}{\sum_{i=0}^{n}\left[a_i^-, a_i^+\right]s^i}; \quad m < n \qquad (12)$$

where $b_i^-, b_i^+, a_i^-, a_i^+$ represent respectively lower and upper bounds for parameters of numerator and denominator if and only if it stabilizes its 16 Kharitonov plants, which are defined as:

$$G_{i,j}(s) = \frac{B_i(s)}{A_j(s)} \qquad (13)$$

where $i, j \in \{1,2,3,4\}$; and $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval system (12).

Remind that the construction of Kharitonov polynomials e.g. for the numerator interval polynomial:

$$B(s,b) = \sum_{i=0}^{m}\left[b_i^-; b_i^+\right]s^i \qquad (14)$$

is based on use of the lower and upper bounds of interval parameters in compliance with the principle [27]:

$$\begin{aligned} B_1(s) &= b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + \cdots \\ B_2(s) &= b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + \cdots \\ B_3(s) &= b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + \cdots \\ B_4(s) &= b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + \cdots \end{aligned} \qquad (15)$$

As can be seen, the stabilization of an interval plant directly follows from the simultaneous stabilization of all 16 fixed Kharitonov plants. Thus the final area of stability for original interval plant is given by intersection of all 16 related partial areas obtained individually using the Kronecker summation method from the previous Part.

4 Algebraic Design of PI Controller

So far, the methodologies from Sections 2 and 3 allow calculating all robustly stabilizing combinations of proportional and integral gains in PI controller. Nonetheless, the final selection of a controller is another problem. An effective solution is represented by algebraic approach to control design [21], [22], [23], which is based on general solutions of Diophantine equations in R_{PS}, Youla-Kučera parameterization and conditions of divisibility in the specific ring. A merit of the technique is that the controllers can be tuned by the only positive scalar parameter $m > 0$.

Due to the limited space the paper does not provide full details on this method [20], [23]. It exploits only one specific result, i.e. the coefficients of feedback PI controller (2) can be computed according to:

$$k_P = \frac{2m - a_0}{b_0}; \quad k_I = \frac{m^2}{b_0} \tag{16}$$

where the parameters a_0 and b_0 of the first order nominal controlled plant:

$$G_N(s) = \frac{b_0}{s + a_0} \tag{17}$$

are supposed to be known and where the tuning parameter $m > 0$ can be chosen on the basis of several approaches such as trivial "trial-and-error", user knowledge and experience, or using recommendation [28]:

$$m = ka_0 \tag{18}$$

Appropriate coefficient k depends on the size of first overshoot of the output (controlled) variable. Some of its values can be found in table 1.

Table 1: Relation between *k* and overshoot

Overshoot [%]	*k*
0	1.00
1	1.62
2	1.87
3	2.14
5	2.80
10	7.38

5 Simulation Experiment

Assume the controlled process described by third order interval transfer function, which is adopted from [10]:

$$G(s,b,a) = \frac{[0.75, 1.25]s + [0.75, 1.25]}{s^3 + [2.75, 3.25]s^2 + [8.75, 9.25]s + [0.75, 9.25]} \tag{19}$$

The initial aim is to determine set of robustly stabilizing PI controllers and the following objective consists in selecting the final one.

In the first instance, consider e.g.:

$$G_{1,1}(s) = \frac{0.75s + 0.75}{s^3 + 3.25s^2 + 8.75s + 0.75} \tag{20}$$

as the first of 16 Kharitonov plants (13). The closed-loop characteristic equation (5) takes the form:

$$\begin{aligned} &s^4 + 3.25s^3 + (8.75 + 0.75k_P)s^2 + \\ &+(0.75 + 0.75k_P + 0.75k_I)s + 0.75k_I = 0 \end{aligned} \tag{21}$$

which means that the matrix (8) is:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.75k_I & -0.75 - 0.75k_P - 0.75k_I & -8.75 - 0.75k_P & -3.25 \end{bmatrix} \tag{22}$$

The stability boundary is given by (11). The position of such pairs (k_P, k_I) which fulfil (11) are shown in fig. 2 and in this specific case it determines two open subsets. Choice of arbitrary point (k_P, k_I) from both sides of stability boundary and subsequent computation of relevant closed-loop characteristic polynomial (21) lead to result that the stabilizing PI controllers for the plant (20) are included in the right-hand area.

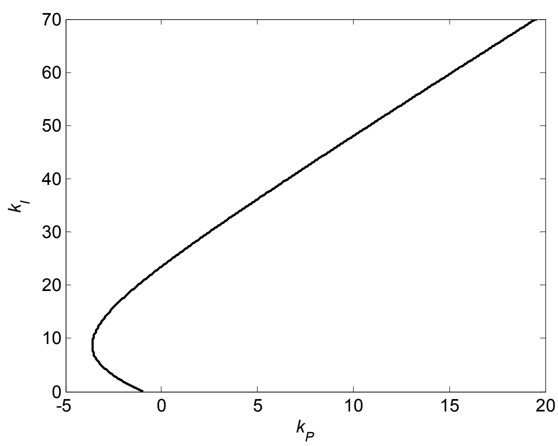

Fig. 2: Stability Boundary for the Plant (20)

Now, one must repeat the analogical procedure for all 16 Kharitonov plants (13). The stability regions for those Kharitonov plants are shown in fig. 3. The highlighted intersection of all particular stability areas determines the final region of robustly stabilizing PI controllers for the original interval plant (19).

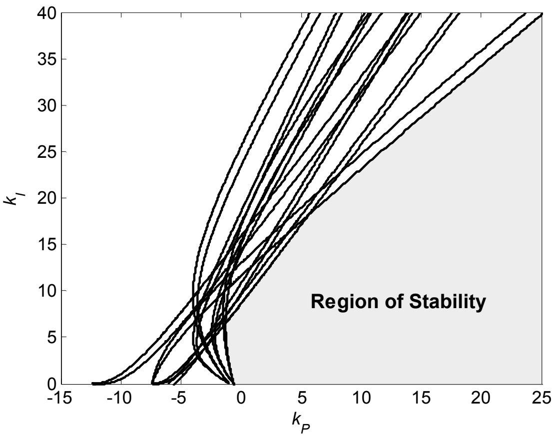

Fig. 3: Stability Areas for 16 Kharitonov Plants and for the Interval System (19)

Quite naturally, the following step brings the question of how to find the practically convenient PI controller from the obtained robust stability region. Among possible methods, the algebraic approach from the Section 4 has been utilized for this purpose.

However, this algebraic synthesis requires the model of controlled system in the form of first order transfer function in order to obtain the final controller of appropriate (first) order (PI type). So the simplest approximation of (19) taking advantage of neglecting the higher order members has been applied. It leads to:

$$G_A(s,b,a) = \frac{[0.75, 1.25]}{[8.75, 9.25]s + [0.75, 9.25]} \quad (23)$$

Then, computing the average values of interval parameters has resulted in the nominal plant for control design:

$$G_N(s) = \frac{1}{9s+5} = \frac{0.\bar{1}}{s+0.\bar{5}} \quad (24)$$

The requirement of 1% first overshoot in controlled signal for the case of nominal system (24) leads to the choice of relevant tuning parameter m. It has been specified according to relation (18) and data from table 1:

$$m = ka_0 = 1.62 \cdot 0.\bar{5} = 0.9 \quad (25)$$

Next, the equations (18) and (16) give the transfer function of the controller:

$$C(s) = \frac{11.2s + 7.29}{s} \quad (26)$$

As can be clearly verified, this controller is located inside the region of stability – see its position in fig. 4.

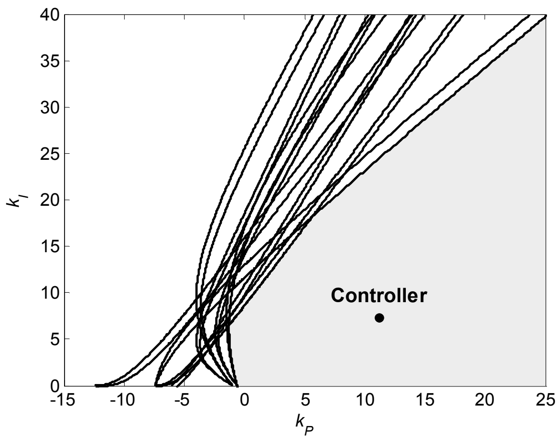

Fig. 4: Position of Controller (26) in Stability Region

Thus, the regulator must robustly stabilize the plant (19).

On the top of that, fig. 5 simply demonstrates robust stability. It shows the control responses of the loop with the PI controller (26) and 1024 “representative” systems from the interval family (19). Each interval parameter has been divided into 3 subintervals and thus these 4 values and 5 parameters result in $4^5 = 1024$ systems for simulation. Moreover, the red curve represents the output variable for the nominal system (24). The prescribed 1%-size of overshoot has been really observed for this case. Besides, it has been assumed the step reference signal changing from 1 to 2 in one third of simulation time and the step load disturbance -5 which influences the input to the controlled plant during the last third of simulation. As can be seen, all “representative” systems are successfully stabilized which confirms that the controller (26) really robustly stabilizes the interval plant (19).

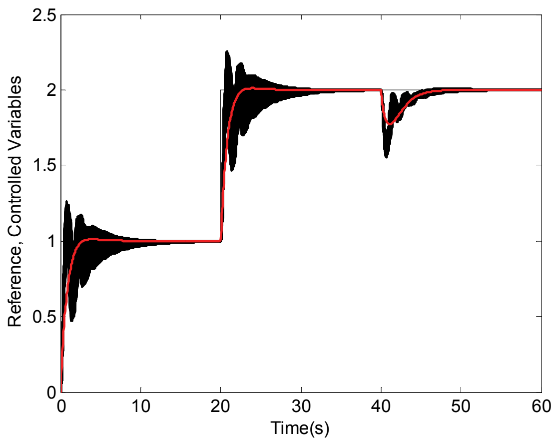

Fig. 5: Output Signals of 1024 "Representative" Plants and Nominal System

6 Real Control Experiments under Laboratory Conditions

The presented theoretical tools have been tested also in laboratory conditions during robust control of a nonlinear electronic model while the control loop has been realized using Simatic S7-300 automation system.

The utilized plant, constructed at Slovak University of Technology in Bratislava, has included a 3rd order system with a variable time constant, adjustable from 5s to 20s, and a model of nonlinear valve. The real visual appearance of this model is shown in fig. 6 and the block diagram of the process is in fig. 7, where signals are denoted as follows:

V – control signal for valve opening (0 – 10V)
F – signal representing the valve opening (0 – 10V)
P – output of the process (0 – 10V)
U – disturbance (0 – 10V)

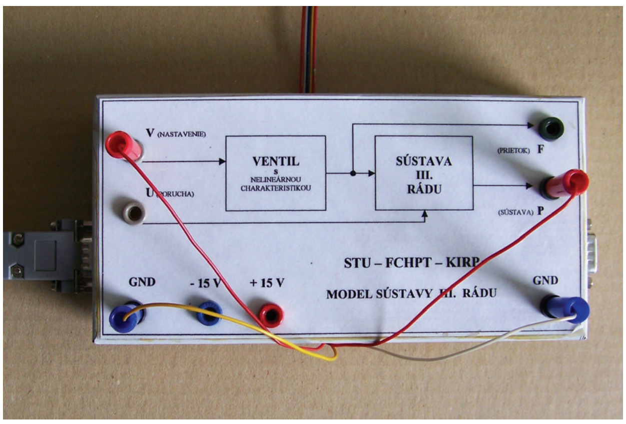

Fig. 6: Electronic Laboratory Model

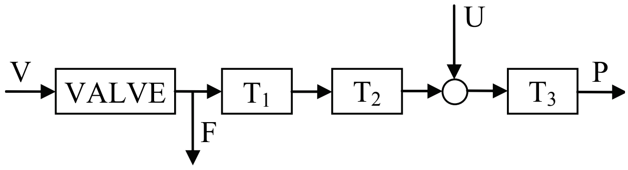

Fig. 7: Block Diagram of Laboratory Model

The plant has been identified as the third order interval system which has led to the approximate mathematical model [19], [20], [29]:

$$G_I(s,b,a) = \frac{[0.35, 5.5]}{[83, 268]s^3 + [104, 171]s^2 + [19, 25]s + 1} \quad (27)$$

The first of its 16 Kharitonov plants (13) can be simply constructed:

$$G_{1,1}(s) = \frac{0.35}{268s^3 + 171s^2 + 19s + 1} \quad (28)$$

The closed-loop characteristic equation (5) is:

$$268s^4 + 171s^3 + 19s^2 + (1 + 0.35k_P)s + 0.35k_I = 0 \quad (29)$$

From here, the matrix (8) follows:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.001306k_I & -0.003731 - 0.001306k_P & -0.0709 & -0.6381 \end{bmatrix} \quad (30)$$

The stability boundary determined according to (11) is depicted in fig. 8. As can be verified, the stabilizing PI controllers for the plant (28) are in the inner space.

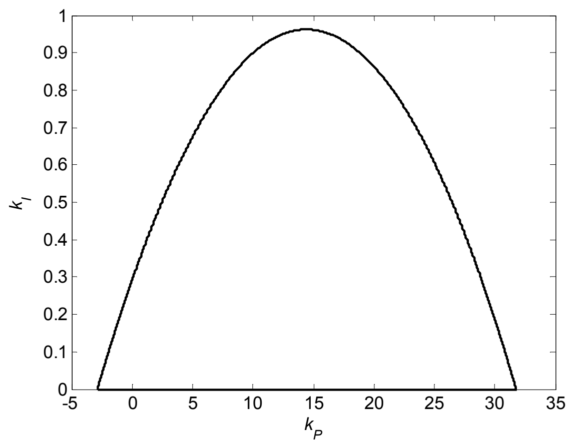

Fig. 8: Stability Boundary for the Plant (28)

Generally, we have to repeat the procedure for all 16 Kharitonov plants (13) as in the previous simulation example. Nonetheless, in such specific case, only 8 plants are enough to test. It is thanks to the fact that the numerator of (27) represents just zero order polynomial with two extreme values and thus it is not necessary to deal with all 4 Kharitonov polynomials for this numerator. The regions of stability for all 8 Kharitonov plants are plotted in fig. 9.

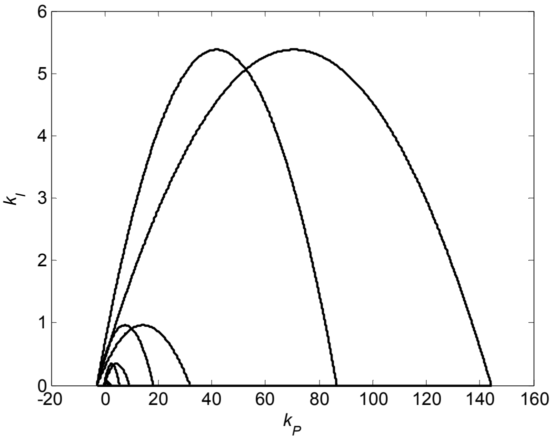

Fig. 9: Stability Areas for 8 Kharitonov Plants

The intersection of all these stability areas is zoomed and depicted in fig. 10. It determines the final region of robustly stabilizing PI controllers for the original interval model (27).

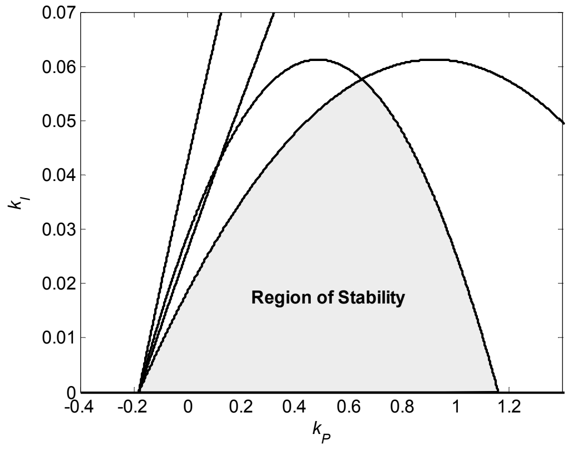

Fig. 10: Stability Region for the Interval System (27)

Analogically to the previous simulation Section, the suitable PI controller from the region of robust stability has been found using the algebraic synthesis. Thus, the original model (27) has been approximated to the first order one:

$$G_A(s,b,a)=\frac{[0.35, 5.5]}{[19, 25]s+1} \tag{31}$$

and the mean values of interval parameters in order to obtain the nominal plant for controller design has been computed:

$$G_N(s)=\frac{2.925}{22s+1}\doteq\frac{0.133}{s+0.04545} \tag{32}$$

First, the assumption of 0% first overshoot in output variable for the case of nominal system, application of appropriate parameter k from table 1, and furthermore equations (18) and (16) give the transfer function of the controller:

$$0\%\Rightarrow m=0.04545\Rightarrow C_1(s)\doteq\frac{0.3417s+0.01553}{s} \tag{33}$$

Then analogically, 1% first overshoot requirement results in:

$$1\%\Rightarrow m\doteq 0.07363\Rightarrow C_2(s)=\frac{0.7655s+0.04076}{s} \tag{34}$$

The fig. 11 depicts the positions of the controllers (33) and (34) in the stability area from fig. 10. As can be seen, they lie on the curve hypothetically connecting the other potential controllers tuned by various parameters $m>0$.

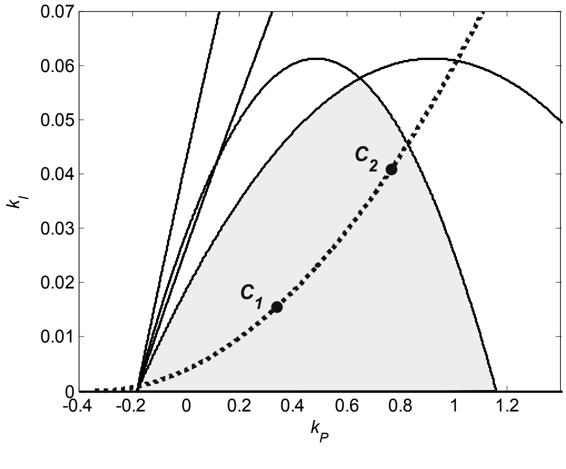

Fig. 11: Positions of Controllers (33) and (34) in Stability Region

Finally, three control experiments have been executed under different working points using the chosen controllers and PLC Simatic S7-300.

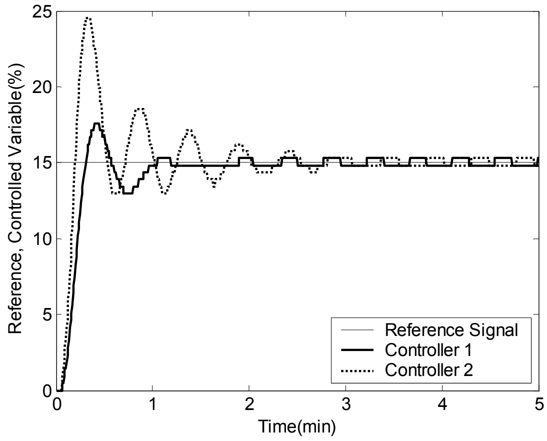

Fig. 12: Real Control Results (for 15% Reference Point)

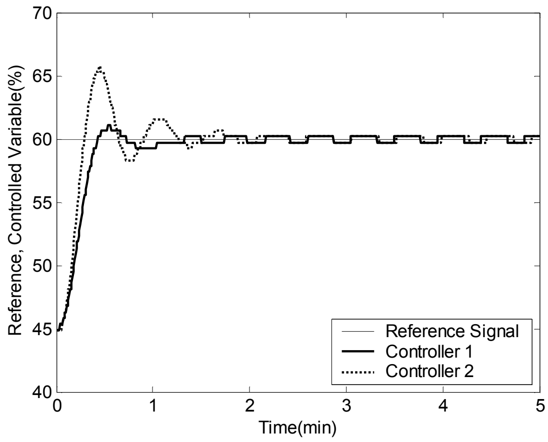

Fig. 13: Real Control Results (for 60% Reference Point)

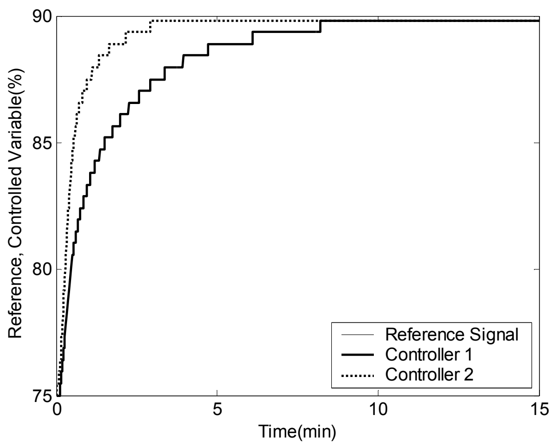

Fig. 14: Real Control Results (for 90% Reference Point)

The nominally prescribed overshoots have not been measured in real conditions. Actually it was expected, because the controlled plant has had highly nonlinear behaviour and these recommendations strictly hold true only for the nominal linear system. Figs. 12-14 indicate that the "less aggressive" controller C_1 provides very good results mainly in the mean set points, but it has comparatively long settling time in higher operational areas. On the other hand, the controller C_2 is much "faster" here, however it is more oscillating in the lower levels. Altogether, both compensators have been able to control the nonlinear process robustly stable and with acceptable performance. The definitive selection of the controller would depend on the main operational area.

7 Conclusion

The paper has dealt with an approach to computation of robustly stabilizing PI controllers. The proposed method has been based on combination of calculating the stability boundary via Kronecker summation, its extension for interval systems using sixteen plant theorem, and the choice of the final regulator through the single-parameter tuning algebraic approach. The developed synthesis represents easy but effective way of designing the controllers for interval systems. On the other hand, coincident nominal performance and robust stability can not be assured in advance. They have to be verified during the design process which can be considered as a method demerit. However, the applicability has been shown first on simulation example with open region of stability and subsequently also on laboratory experiments in which a nonlinear 3rd order electronic model has been successfully controlled in various operational points.

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