Robust Control Design Toolbox for General Time Delay Systems via Structured Singular Value: Unstable Systems with Factorization for Two-Degree-of-Freedom Controller

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***Abstract*—The application of the Robust Control Design Toolbox for General Time Delay Systems via Structured Singular Value: Unstable Systems for the Matlab system to the unstable plant with time delay in numerator and denominator is described in this paper. The uncertain time delays are treated using multiplicative and quotient uncertainty. The algebraic approach part implements evolutionary algorithm Differential Migration and pole placement for general 3rd order system with evaluation via structured singular value. Both, D-K iteration and algebraic approach, implements two-degree-of-freedom feedback loop controller with factorization fixing internal instability. Both procedures are compared in simulations for maximum and half time delay and simple and two-degree-of-freedom feedback loop.**

***Keywords*—Algebraic approach, robust control, RQ-meromorphic functions, structured singular value, Uncertain time delay systems.**

I. Introduction

The paper is focused on control of uncertain time delay systems with time delay in numerator and denominator of the controlled plant. This type of plants is currently solved in the ring of retarded quasipolynomial (RQ) meromorphic functions (see [11] and [12]). However, the robustness is not easy to derive using this approach.

The toolbox presented in this paper implements a method handling the robustness and uncertainty in an easy way giving simple and easy to implement controllers. Typically, for a 1st order system the controller can be described as 4th order transfer function compared to awkward and hard-to-implement controllers obtained from the design in the ring of RQ-meromorphic functions which can treat the uncertainty with difficulties.

The presented method takes into account the uncertainty using the procedure described in [2] and [3] which fully covers the varying time delays and guarantees the robust stability and performance. In order to obtain controllers that satisfy bounded-input bounded-output (BIBO) stability algebraic theory is used for pole placement. The task is accomplished via solving the Diophantine equation in the ring of Hurwitz-stable and proper rational functions ($\mathbf{R}_{\mathrm{PS}}$). As a measure of robust stability and performance, structured singular value denoted μ is used (see [9]).

Due to the multimodality of the cost function in the algebraic approach an algorithm of global optimization is used. For this task evolutionary algorithm Differential Migration (see [1]) appears to be one of the most effective. Therefore, its application was chosen together with Nelder-Mead simplex method as a tool for the final tune-up of the pole placement.

As a reference method, the *D-K* iteration (see [5]) is implemented in the toolbox with entropy, LMI or DGKF formulae as the options in the *D-K* iteration part (see [6], [7] and [8]). The *D-K* iteration controller is compared with the proposed method in the simulations of the response to the step of the reference for different values of uncertain time delays. The controllers are connected in simple and two-degree-of-freedom feedback loop (1DOF and 2DOF, see [10]) with corresponding factorization fixing internal instability of 2DOF feedback loop.

The following notation is used: $\|\cdot\|_\infty$ denotes $\mathbf{H}_\infty$ norm, $\overline{\sigma}(\cdot)$ is maximum singular value, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{\mathrm{PS}}$ denotes the ring of Hurwitz-stable and proper rational functions.

II. Preliminaries

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\mathbf{\Delta} \equiv \{\mathrm{diag}[\delta_1 I_{r_1},\ldots,\delta_S I_{r_S},\Delta_1,\ldots,\Delta_F] : \delta_i \in \mathbf{C}, \Delta_j \in \mathbf{C}^{m_j\times m_j}\} \quad (1)$$

where S is the number of repeated scalar blocks,
F is the number of full blocks,
$r_1,\ldots, r_S$ and $m_1,\ldots, m_F$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n \quad (2)$$

Definition 1: For $\mathbf{M} \in \mathbf{C}^{n\times n}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\Delta}(\mathbf{M}) \equiv \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I}-\mathbf{M}\Delta)=0\}} \quad (3)$$

If no such $\Delta \in \mathbf{\Delta}$ exists making $\mathbf{I} - \mathbf{M}\Delta$ singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M}) = 0$.

III. ALGEBRAIC μ-SYNTHESIS

The algebraic μ-synthesis can be applied to any control problem that can be transformed to the loop in Fig. 1, where **G** denotes the generalized plant, **K** is the controller, Δ_{del} is the perturbation matrix, r is the reference and e is the tracking error.

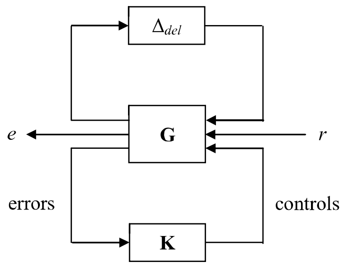

Fig. 1. Closed loop interconnection.

For the purposes of the algebraic μ-synthesis, the MIMO system with l inputs and l outputs is decoupled into l identical SISO plants. The nominal model is defined in terms of transfer functions:

$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & \cdots & P_{1l}(s) \\ \vdots & \ddots & \vdots \\ P_{l1}(s) & \cdots & P_{ll}(s) \end{bmatrix} \tag{4}$$

For decoupling the nominal plant $\mathbf{P}_{nom}$ ($\mathbf{P}_{nom}$ invertible), it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\mathbf{I}_l \det[\mathbf{P}_{nom}(s)]\frac{1}{P_{xy}(s)}[\mathbf{P}_{nom}(s)]^{-1} \tag{5}$$

where P_{xy} is an element of $adj[\mathbf{P}_{nom}(s)] = \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}$ with the highest degree of numerator $\{adj[\mathbf{P}_{nom}(s)]$ denotes adjugate matrix of $\mathbf{P}_{nom}\}$. The choice of the decoupling matrix prevents the controller from cancelling any poles or zeros from the right half-plane so that internal stability of the nominal feedback loop is held. The MIMO problem is reduced to finding a controller $K(s)$ which is tuned via setting the poles of the nominal feedback loop with the plant

$$\begin{aligned} \mathbf{P}_{dec}(s) &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)]\mathbf{I}_l \end{aligned} \tag{6}$$

Define

$$P_{dec} \equiv \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)] \tag{7}$$

Transfer function P_{dec} can be approximated by a system P_{dec}^* with lower order than P_{dec}

$$P_{dec}^*(s) = \frac{b(s)}{a(s)} \tag{8}$$

which can be rewritten in terms of its coefficients and transformed to the elements of $\mathbf{R}_{\mathrm{PS}}$

$$P_{dec}^*(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1}s^{n-1}}{(\alpha_1 + s)(\alpha_2 + s)\cdot\ldots\cdot(\alpha_n + s)}} = \frac{B}{A},\ A,\ B \in \mathbf{R}_{\mathrm{PS}} \tag{9}$$

The controller $K = N_k/D_k$ is obtained by solving the Diophantine equation

$$AD_k + BN_k = 1 \tag{10}$$

with $A, B, D_k, N_k \in \mathbf{R}_{\mathrm{PS}}$. Equation (10) is often called the Bezout identity. All feedback controllers N_K/D_K are given by

$$K = \frac{N_k}{D_k} = \frac{N_{k_0} - AT}{D_{k_0} + BT}, \qquad N_{k_0}, D_{k_0} \in \mathbf{R}_{\mathrm{PS}} \tag{11}$$

where N_{k_0}, $D_{k_0} \in \mathbf{R}_{\mathrm{PS}}$ are particular solutions of (10) and T is an arbitrary element of $\mathbf{R}_{\mathrm{PS}}$.

The controller K satisfying equation (10) guarantees the BIBO (bounded input bounded output) stability of the feedback loop in Fig. 2. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G},\mathbf{K})$ making possible usage of performance weights with integration property implying non-existence of state space solutions using DGKF formulae (see [6]) due to zero eigenvalues of appropriate Hamiltonian matrices. Such methodology results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (10). This technique is neither possible in the scope of the standard μ-synthesis using DGKF formulae nor using LMI approach (see [7]) leading to numerical problems in most of real-world applications.

The aim of synthesis is to design a controller which satisfies the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},\mathbf{K})(\omega,\alpha_1,\ldots,\alpha_{n+n_1+n_2},t_1,\ldots,t_{n_2})] \le 1,\ \omega \in (-\infty,+\infty) \tag{12}$$

where $n + n_1 + n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution K_0, t_i are arbitrary parameters in $T = \dfrac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1} + s)\cdot\ldots\cdot(\alpha_{n_1+n_2} + s)}$ and $\mu_{\mathbf{\Delta}}$ denotes the structured singular value of LFT on generalized plant **G** and controller **K** with

$$\Delta \equiv \begin{bmatrix} \Delta_F & 0 \\ 0 & \Delta_{del} \end{bmatrix} \tag{13}$$

where Δ_{del} denotes the perturbation matrix and Δ_F is a full-block matrix corresponding with the robust performance condition.

Tuning parameters are positive and constrained to the real axis since parameters of the transfer function have to be real and due to the fact that non-real poles cause oscillations of the nominal feedback loop.

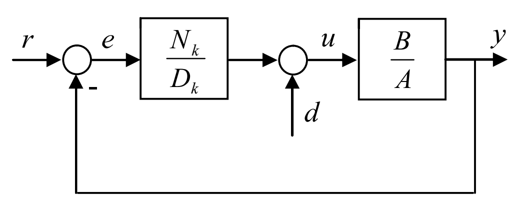

Fig. 2. Nominal feedback loop

A crucial problem of the cost function in (12) is the fact that many local extremes are present. Hence, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary optimization which solves the task very efficiently.

IV. Problem Formulation

The problem to solve is general 1st order system with uncertain time delays:

$$P(s) \equiv \frac{b_0 e^{-\tau_1 s}}{a_1 s + e^{-\tau_2 s}},\ \tau_1 \in [0, T_1],\ \tau_2 \in [0, T_2] \tag{14}$$

This family of plants has uncertain retarded quasi-polynomial in the denominator. The delays vary in the intervals of zero to a predefined value representing the upper bound for each time delay.

This set of plants is treated via LFT using the scheme in Fig. 3. The weights W_{del1} and W_{del2} are obtained from the inequalities:

$$\left|W_{deli}\right| > \left|1 - e^{j\omega T_{di}}\right|, i = 1, 2 \tag{15}$$

The perturbation matrix has the form:

$$\mathbf{\Delta}_{del} \equiv \begin{bmatrix} \delta_{del1} & 0 \\ 0 & \delta_{del2} \end{bmatrix},\ \left|\delta_{del1}\right| < 1,\ \left|\delta_{del2}\right| < 1,\ \delta_{del1}, \delta_{del2} \in \mathbf{C} \tag{16}$$

and performance weight is a 3rd order transfer function:

$$W_1 = \frac{b_{w_1,2}s^2 + b_{w_1,1}s^1 + b_{w_1,0}}{a_{w_1,3}s^3 + a_{w_1,2}s^2 + a_{w_1,1}s^1 + a_{w_1,0}} \tag{17}$$

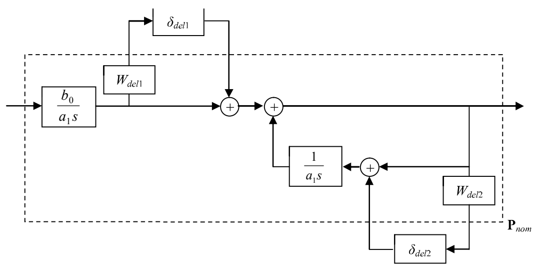

Fig. 3. LFT model of plant

The weights W_{del1} and W_{del2} should satisfy (15) with very low conservatism.

The performance condition is of the form:

$$\left\|W_1 S\right\|_\infty < 1 \tag{18}$$

where S is the sensitivity function and weight W_1 is designed so that the asymptotic tracking is achieved.

V. Problem Solution

A. Structured Singular Value Framework

The problem defined in previous section can be solved using interconnection in Fig. 4. Here, $\mathbf{G}$ denotes the generalized plant partitioned to

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & G_{22} \end{bmatrix} \tag{19}$$

where the block structure of $\mathbf{G}$ corresponds with the input and output variables in Fig. 1:

$$\begin{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ v \end{bmatrix} = \mathbf{G} \cdot \begin{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix} \\ u \end{bmatrix} \tag{20}$$

Then the transfer function from d to e is the upper linear fractional transformation on $\mathbf{G}$ and Δ

$$e = \mathbf{F}_u(\mathbf{G}, \Delta_{del})r = \mathbf{G}_{22}r + \mathbf{G}_{21}\Delta_{del}(1 - G_{11}\Delta_{del})^{-1}\mathbf{G}_{12}r \tag{21}$$

For stability and performance Theorem 1 and the following Corollary 1 hold:

Theorem 1: The loop in Fig. 4 is well-posed, internally stable and $\left\|\mathbf{F}_L[\mathbf{F}_U(\mathbf{G}, \Delta_{del}), K]\right\|_\infty \le 1$ if and only if

$$\sup_{\substack{\omega \in R \\ K \text{ stabilizing } \mathbf{G}}} \mu_{\mathbf{\Delta}}[\mathbf{F}_L(\mathbf{G}, K)(j\omega)] \le 1 \tag{22}$$

with $\mathbf{\Delta} \equiv \left\{ \begin{bmatrix} \delta_1 & 0 \\ 0 & \Delta_{del} \end{bmatrix}, \left|\delta_1\right| < 1, \delta_1 \in \mathbf{C}, \Delta_{del} \in \mathbf{\Delta}_{del} \right\}$.

Proof: The proof is the same as in [4] and [9] except for the fact that perturbations are complex matrices which simplifies the proof and complies with the definition of μ (Definition 1). ■

Corollary 1: Closed loop in Fig. 5 is stable for all $\Delta_{del} \in \mathbf{\Delta}_{del}$ $\left|\overline{\sigma}(\Delta_{del})\right| < 1$, the performance condition (18) holds and $\left\|F_u(\mathbf{G}, \Delta_{del})\right\|_\infty \le 1$ if and only if conditions (15) hold and

$$\sup_{\substack{\omega \in R \\ K \text{ stabilizing } \mathbf{G}}} \mu_{\mathbf{\Delta}}[\mathbf{F}_L(\mathbf{G}, K)(j\omega)] \le 1 \tag{23}$$

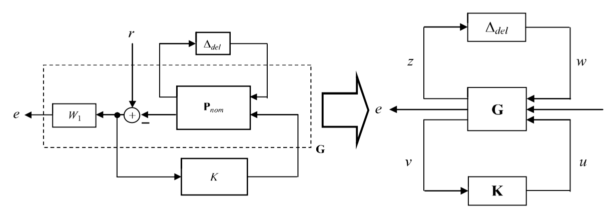

Fig. 4. Closed-loop interconnection for μ-synthesis

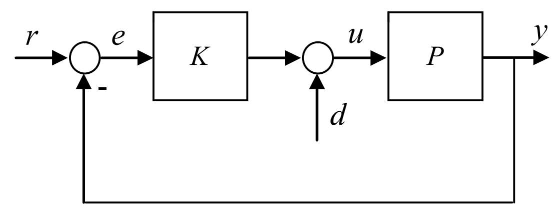

Fig. 5. Feedback loop

The design objective is to find a stabilizing controller K such that

$$\sup_{\omega} \mu_{\Delta}[\mathbf{F}_l(\mathbf{G}, K)] \le 1 \tag{24}$$

where

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G}, K) = \mathbf{G}_{11} + \mathbf{G}_{12}K(1 - G_{22}K)^{-1}\mathbf{G}_{21} \tag{25}$$

is the lower linear fractional transformation on generalized plant $\mathbf{G}$ and controller K (see Fig. 4) and μ_Δ corresponds with the perturbation matrix from the set $\mathbf{\Delta}$.

B. Algebraic Approach

The plant for which the controller is derived is the nominal system:

$$P_0(s) \equiv \frac{b_0}{a_0 s - 1} \tag{26}$$

Nominal plant P_0 can be transformed to:

$$P_0(s) = \frac{\dfrac{b_0}{\alpha_1 + 1}}{\dfrac{a_1 s - 1}{\alpha_1 + 1}} = \frac{B}{A}, \qquad A, B \in \mathbf{R}_{\mathrm{PS}} \tag{27}$$

The controller is obtained as a solution to the Diophantine equation (10) with BIBO stable feedback controller N_k/D_k given by

$$K = \frac{N_k}{D_k} = \frac{N_{k_0} - AT}{D_{k_0} + BT} = \frac{\dfrac{n_{k_0 1}s + n_{K_0 0}}{(\alpha_2 + s)} - A\dfrac{t_2 s^2 + t_1 s}{(\alpha_3 + s)(\alpha_4 + s)}}{\dfrac{d_{k_0 1}s}{(\alpha_2 + s)} + B\dfrac{t_2 s^2 + t_1 s}{(\alpha_3 + s)(\alpha_4 + s)}} \tag{28}$$

The denominator of (28) is divisible by s so that asymptotic tracking for the stepwise reference signal can be achieved.

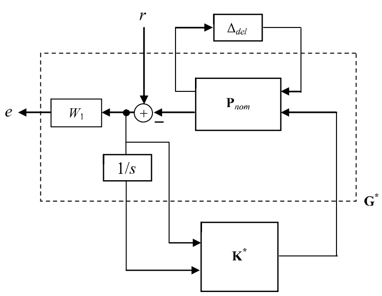

Fig. 6. Closed loop interconnection with integrator cascade

The aim of synthesis is to design a controller which satisfies condition (12). The 1DOF feedback controller obtained from the algebraic approach has the transfer function:

$$K_A(s) = \frac{n_k}{d_k} = \frac{n_{k,4}s^4 + \ldots + n_{k,0}}{s^4 + d_{k,3}s^3 + \ldots + d_{k,1}s} \tag{29}$$

In order to overcome the problem of non-integration structure of the *D-K* iteration controller a scheme with integrator that incorporates the integration property into the controller was used (see Fig. 6). The 1DOF feedback controller obtained from the *D-K* iteration has the transfer function:

$$K_{D-K}(s) = \frac{n_{kdk}}{d_{kdk}} = \frac{n_{kdk,5}s^5 + \ldots + n_{kdk,0}}{s^5 + d_{kdk,4}s^4 + \ldots + d_{kdk,1}s} \tag{30}$$

VI. Time Delay System Control for Uncertain Time Delay in Numerator and Denominator

Consider the set of anisochronic systems with time delay in the numerator and denominator:

$$P(s) \equiv \frac{3e^{-\tau_1 s}}{5s - e^{-\tau_2 s}}, \ \tau_1 \in [0, 4], \ \tau_2 \in [0, 0.8] \tag{31}$$

This set of plants is treated via LFT using the scheme in Fig. 3. Weights W_{del1} and W_{del2} can be obtained from the inequalities:

$$\left|W_{deli}\right| > \left|1 - e^{j\omega T_{di}}\right|, i = 1, 2; T_{d1} = 4, T_{d2} = 0.8 \tag{32}$$

It follows from Fig. 7 and 8 that

$$W_{del1} = \frac{2s}{2s+1}2.5, \ W_{del2} = \frac{0.4s}{0.4s+1}2.5 \tag{33}$$

satisfy (15) with very low conservatism.

Now, it is easy to create an open-loop interconnection with weighted sensitivity function as a performance indicator. Recall the closed-loop interconnection depicted in Fig. 4 with the open loop in dashed rectangle denoted **G**. The perturbation matrix has the form (16) and performance weight is a 3rd order transfer function:

$$W_1 = \frac{0.004}{10s^3 + 100s^2 + s + 1 \cdot 10^{-5}} \tag{34}$$

The weight W_1 has a small factor for s^0 in the denominator so that the DGKF formulae can be used.

The plant for which the controller is derived is the nominal system:

$$P_0(s) \equiv \frac{3}{5s - 1} \tag{35}$$

The instability of P_0 does not contradict stability of the nominal *feedback* loop. This is guaranteed by controller K satisfying (10).

Nominal plant P_0 can be transformed to:

$$P_0(s) = \frac{\dfrac{3}{\alpha_1 + 1}}{\dfrac{5s - 1}{\alpha_1 + 1}} = \frac{B}{A}, \qquad A, B \in \mathbf{R}_{\mathrm{PS}} \tag{36}$$

The aim of synthesis is to design a controller satisfying condition (12). Evolutionary optimization by Differential Migration gave the poles and arbitrary parameters as follows:

$$\alpha_1 = 0.023, \alpha_2 = 31.973, \alpha_3 = 23.264, \alpha_4 = 1.771 \tag{37}$$

$$t_1 = 24.50, t_2 = 44.89 \tag{38}$$

and controller

$$K_A(s) = \frac{n_k}{d_k} = \frac{29.16s^4 + 522.7s^3 + 1003s^2 + 389s + 1.159}{s^4 + 39.76s^3 + 538.6s^2 + 862.1s} \tag{39}$$

The *D-K* iteration for the interconnection in Fig. 4 yields the controller

$$K_{D-K}(s) = \frac{21.94s^4 + 210.3s^3 + 105.1s^2 + 1.203s + 0.003}{s^4 + 35.26s^3 + 248.3s^2 + 2.19s + 2 \cdot 10^{-5}} \tag{40}$$

Both controllers satisfy condition (12) (see Fig. 10) with maximum values:

$$\sup_{\omega} \mu_{\Delta}[\mathbf{F}_l(\mathbf{G}, K_A)] = 0.995, \ \sup_{\omega} \mu_{\Delta}[\mathbf{F}_l(\mathbf{G}, K_{D-K})] = 0.989.$$

The controllers for 2DOF feedback loop (Fig. 9a, 9b - algebraic approach and *D-K* iteration, respectively) have the compensator (n_{k2}, d_{k2}, n_{kdk2}, d_{kdk2}) defined as fraction of the factors corresponding with most stable zero and least stable pole of K_A and K_{D-K} and feedback (n_{k1}, d_{k1}, n_{kdk1}, d_{kdk1}) and feed-forward part (n_{FW}, d_{k1}, n_{FWdk}, d_{kdk1}) defined by the fraction of the factors corresponding with remaining zeros and poles of K_A and $K_{D\text{-}K}$ with $n_{FW} = n_{k1,0}$ and $n_{FWdk} = n_{kdk1,0}$ ($n_{k1,0}, n_{kdk1,0}$ being the coefficients of n_{k1} and n_{kdk1} of zero exponent of s):

$$\frac{n_{k1}}{d_{k1}} = \frac{29.16s^3 + 61.94s^2 + 24.62s + 0.07335}{s^3 + 39.76s^2 + 538.6s + 862.1}, \quad \frac{n_{FW}}{d_{k1}} = \frac{0.07335}{s^3 + 39.76s^2 + 538.6s + 862.1}, \quad \frac{n_{k2}}{d_{k2}} = \frac{s + 15.8}{s} \tag{41}$$

$$\frac{n_{kdk1}}{d_{kdk1}} = \frac{21.94s^3 + 11.59s^2 + 0.1342s + 0.000336}{s^3 + 35.25s^2 + 247.9s + 2.215}, \quad \frac{n_{FWdk}}{d_{kdk1}} = \frac{0.000336}{s^3 + 35.25s^2 + 247.9s + 2.215}, \quad \frac{n_{kdk2}}{d_{kdk2}} = \frac{s + 9.039}{s + 1.012 \cdot 10^{-5}} \tag{42}$$

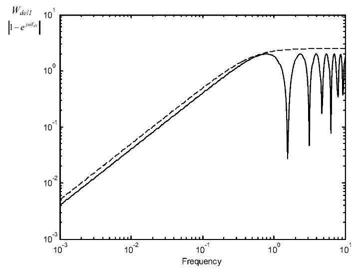

Fig. 7. Bode plot W_{del1} (dashed) and the right side of (15) (solid)

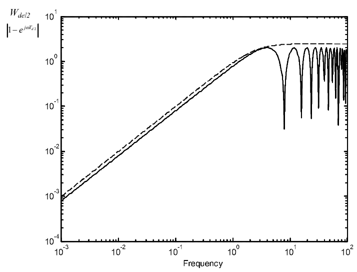

Fig. 8. Bode plot W_{del2} (dashed) and the right side of (15) (solid)

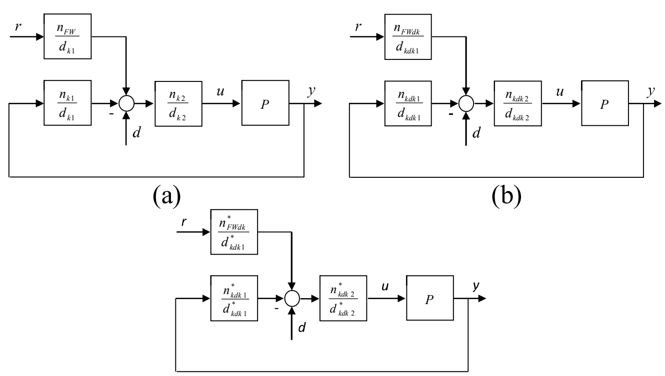

Fig. 9. 2DOF feedback loop

In order to overcome the problem of non-integration structure of the *D-K* iteration controller a scheme with integrator incorporating the integration property into the controller was used (see Fig. 6). The controller for 1DOF (Fig. 5) and 2DOF (Fig. 9c) feedback loop has the transfer functions:

$$K^*_{D-K}(s) = \frac{23.38s^5 + 95.84s^4 + 138.3^3 + 68.59s^2 + 10.37s + 0.0326}{s^5 + 32.14s^4 + 108.7s^3 + 118.0s^2 + 24.41s} \tag{43}$$

$$\frac{n^*_{kdk1}}{d^*_{kdk1}} = \frac{23.38s^4 + 84.3s^3 + 96.66s^2 + 20.89s + 0.066}{s^4 + 32.14s^3 + 108.7s^2 + 118s + 24.41}, \quad \frac{n^*_{FWdk}}{d^*_{kdk1}} = \frac{0.066}{s^4 + 32.14s^3 + 108.7s^2 + 118s + 24.41}, \quad \frac{n^*_{kdk2}}{d^*_{kdk2}} = \frac{s + 0.4935}{s} \tag{44}$$

and $\sup_{\omega} \mu_\Delta[\mathbf{F}_l(\mathbf{G}, K^*_{D-K})] = 1.002$ consequent upon approximation with lower order transfer function. The higher value of μ can be fixed by relaxing the performance weight.

Simulations have been performed for 1DOF and 2DOF feedback loop with real plant P, i.e. with transport delays present in the simulation model. The interconnection of 2DOF system is in Fig. 9. For details on 2DOF controllers in $\mathbf{R}_{\text{PS}}$ see [10].

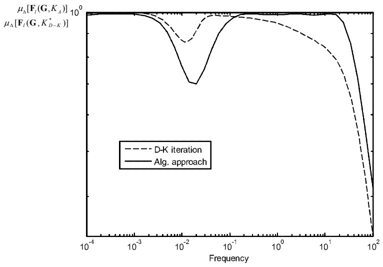

Fig. 10. Mu-plot for the *D-K* iteration with $\mathbf{G}^*$ and 1DOF structure (dashed) and algebraic approach (solid)

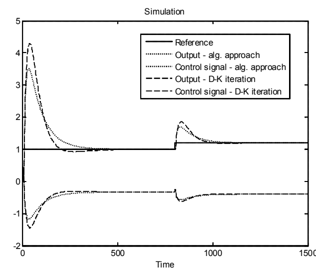

Fig. 11. Simulation for 1DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$)

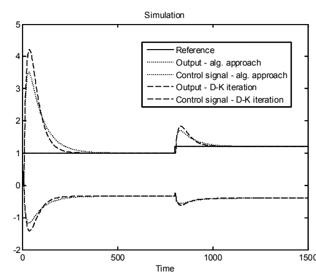

Fig. 12. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 1DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$)

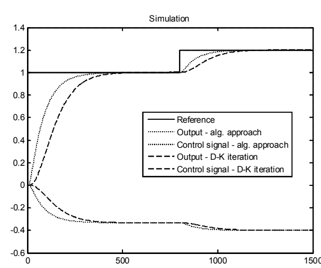

Fig. 13. Simulation for with 2DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$).

Simulation for both controllers with 1DOF structure and stepwise reference signal is in Fig. 11. Simulation for 2DOF structure and the same reference signal is in Fig. 13. It is apparent that the *D-K* iteration has a non-zero steady-state error for both 1DOF and 2DOF interconnection which is not the case of the algebraic approach. Set point tracking is faster for the algebraic approach with lower overshoot for 1DOF controller structure. The steady-state error is not present for the *D-K* iteration and generalized plant $\mathbf{G}^*$ with integrator cascade included (Fig.12 and 14). The standard procedure yields faster tracking, however, the complexity of the controller is higher than for the algebraic approach and *D-K* iteration with no cascade in generalized plant.

The same simulations but with half time delays are depicted in Fig. 15 and 16. It can be observed that the properties of feedback loop do not degrade if the time delays vary in the intervals of 0 to 4 and 0 to 0.8 for τ_1 and τ_2, respectively, except the overshoot for *D-K* iteration with integrator cascade and half time delays (Fig. 16). For the 2DOF structure, no overshoot is present which is not true for 1DOF feedback loop.

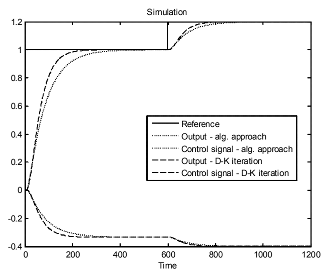

Fig. 14. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 2DOF structure ($\tau_1 = 4$, $\tau_2 = 0.8$)

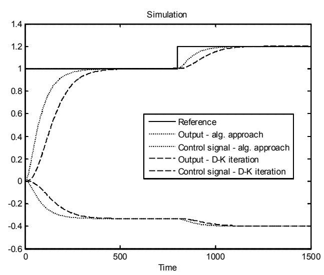

Fig. 15. Simulation for 2DOF structure ($\tau_1 = 2$, $\tau_2 = 0.4$)

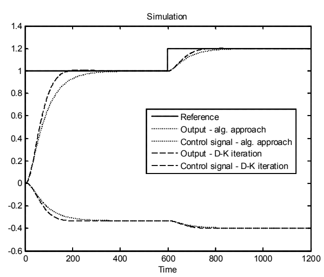

Fig. 16. Simulation for *D-K* iteration with $\mathbf{G}^*$ and 2DOF structure ($\tau_1 = 2$, $\tau_2 = 0.4$).

VII. Download

The Robust Control Toolbox for Time Delay Systems with Time Delay in Numerator and Denominator toolbox can be downloaded from:

http://dlapa.cz/homeeng.htm

VIII. Conclusion

The Robust Control Toolbox for Time Delay Systems with Time Delay in Numerator and Denominator has been applied to unstable time delay system with uncertain time delays in both numerator and denominator of the controlled plant. The simulation proved functionality of the algebraic approach and the method of treating uncertain time delays using linear fractional transformation and structured singular value even in the case of uncertain time delay in the denominator of the control plant as well as the functionality of factorization for both approaches in two-degree-of-freedom feedback loop fixing internal instability

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