Statistical Analysis of Modified Predictive Control of Non-Minimum Phase System

MAREK KUBALCIK, VLADIMIR BOBAL
Department of Process Control
Faculty of Applied Informatics
Tomas Bata University in Zlin
Nad Stranemi 4511, 760 05, Zlin
CZECH REPUBLIC
kubalcik@utb.cz, bobal@utb.cz
http://fai.utb.cz/en

TOMAS BAROT
Department of Mathematics with Didactics
Faculty of Education
University of Ostrava
Frani Sramka 3, 709 00, Ostrava
CZECH REPUBLIC
Tomas.Barot@osu.cz
http://pdf.osu.eu/kmd

Abstract: - Testing statistical hypothesis in comparison of corresponding pairs of signals in control has not been widely used both in practice and research. In this paper, pairs of signals obtained during control before and after a modification of a control algorithm are paired tested by quantitative methods. In previous research, the authors used this type of mathematical analysis with regards to the control quality in an analysis of signals in multivariable MPC with a modified optimization strategy. The control quality was slightly influenced in favour of the decreasing the computational complexity of the MPC control algorithm. In this paper, results of a predictive control of a non-minimum phase system with an elimination of the undershoot are analysed in detail. The control algorithm was modified by a particular setting of control constraints. The aim of this modification was suppression of the undershoot. The quality of control with the original and modified control algorithms is analysed by testing hypothesis. Particular signals are compared using the testing hypotheses on the statistically significant differences.

Key-Words: - Statistical analysis, testing hypotheses, paired test, Model Predictive Control, non-minimum phase system, control quality criterions.

1 Introduction

The Model Predictive Control (MPC) [1]-[5] is a modern control method which can be advantageous for control of various types of controlled systems. It is suitable also for control of non-minimum phase systems [6]. However, control of non-minimum phase systems is characterized by an undesirable undershoot. In this paper, a modification, applied in favour of an elimination of this undesirable undershoot [7] is extended by a detailed statistical analysis using techniques which were presented in [8].

An achievement of a suitable quality of control can be considered as one of the main aims in the process control [4] in general. The quality of control is often examined in order to evaluate which control algorithm reaches the best results in a particular control problem. The control algorithms which yield appropriate control results are often complex and computationally demanding. Therefore, there is an effort to propose modifications of the control algorithms in order to simplify them. These simplifications are obviously at the expense of the control quality. The quality of the control is then examined in order to evaluate whether the modified control algorithms are still suitable for a particular control problem or not. [8]

For these purposes of analysis effect of modification on control, methods of the statistical induction have a significant role in the quantitative research e.g. [9]-[12]. However, in the area of the process control, testing hypothesis has not been widely considered as an established tool for signal analyses, although signals in control loops are suitable for analysis by means of quantitative statistical methods due to their stochastic character. Particularly, a statistical paired comparison [9] can be applied for analysis of control quality achieved with different control algorithms. This comparison can be based on a paired comparison of corresponding signals obtained with different or modified control algorithms. [8]

However, modifications of control algorithms can have also another aim instead of the decreasing of the computational complexity. In this paper, a modification of MPC for purposes of a suppression of the undershoot during control of a non-minimum phase system was assumed. The partial elimination of the undershoot in MPC was described in [7]. A detailed analysis by rules published in [8] will be further presented.

2 Model of Non-Minimum Phase System

In this paper, a linear second order discrete dynamic model of the controlled non-minimum phase system is considered. The non-minimum phase systems are characterized by an undershoot in their dynamical behaviour. [7]

A mathematical model of the system can be described by a discrete transfer function (2) which represents a second-order continuous system (1) discretized with a given sampling period. The model (1) is stable if the poles π_1 and π_2 are negative. The undershoot is caused by some positive roots in the numerator. The roots of the numerator are zeros ϑ. [7]

$$G(s) = \frac{s - \vartheta}{(s - \pi_1)(s - \pi_2)} \tag{1}$$

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{2}$$

3 Control Quality Criterions

Standard criterions which are commonly used for evaluation of control quality are sums of squares of control increments J_1 (3) and sums of squares of control errors J_2 (4). These criterions can result only in descriptive attributes of control quality. In the Model Predictive Control presented in this paper, these criterions are utilized. [13]

$$J_1 = \sum_k \left[\Delta u(k)\right]^2 \tag{3}$$

$$J_2 = \sum_k \left[w(k) - y(k)\right]^2 \tag{4}$$

4 Model Predictive Control

The Model Predictive Control [1]-[5] is a modern control method which can be advantageous for control of various types of controlled systems.

The models (1)-(2) with one input and one output (SISO) was further considered in the framework of MPC in this paper.

A control law of the MPC can be defined as a set of connected equations on the prediction and control horizons [2] which return results of the future increments of the manipulated variables – vector $\Delta\boldsymbol{u}$ with N_u elements. However, only the first element of this vector is further used in the next sampling period of the control. This is called the receding horizon strategy. The minimum, control and maximum prediction horizons are denoted N_1, N_u and N_2. [1]-[5]

A particular form of this control law can be expressed using the constrained optimization problem (5) with definition of vector $\boldsymbol{b}$ in (6) and of the positive definite matrix $\boldsymbol{H}$ in (7). Matrix $\boldsymbol{M}$ and vector $\boldsymbol{\gamma}$ define constraints conditions. In this paper, the experimentally based constraints condition (8)-(9) is only considered with regards to the presented strategy for suppression of the undershoot in the initial sampling periods [7]. Matrix $\boldsymbol{I}$ is an identity matrix of a given dimension. Variable $w(k)$ is the reference signal in the k-th sampling period. [1]-[5], [14]-[15]

$$min\left\{\frac{1}{2}\Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u} + \boldsymbol{b}^T\Delta\boldsymbol{u} \;\middle|\; \Delta\boldsymbol{u} \in R^{N_u}; \boldsymbol{M}\Delta\boldsymbol{u} \le \boldsymbol{\gamma}\right\} \tag{5}$$

$$\begin{aligned}\boldsymbol{b} = 2(&\boldsymbol{P}\left[y(k), y(k-1), y(k-2), \Delta u(k-1)\right]^T - \\ &-\left[w(k+N_1),..., w(k+N_2)\right]^T)^T\end{aligned} \tag{6}$$

$$\boldsymbol{H} = 2(\boldsymbol{G}^T\boldsymbol{G} + \boldsymbol{I}) \tag{7}$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \boldsymbol{M} \in R^{\boldsymbol{Nu},\boldsymbol{Nu}} \tag{8}$$

$$\boldsymbol{\gamma} = \begin{bmatrix} \Delta u_{max} \\ \Delta u_{max} \\ \vdots \\ \Delta u_{max} \end{bmatrix}, \boldsymbol{\gamma} \in R^{\boldsymbol{Nu},1} \tag{9}$$

In most cases it is necessary to compute multi-step ahead predictions for arbitrary prediction and control horizons. The computation of multi-step ahead predictions is not possible to perform in the simple straightforward way by establishing of previous predictions to later predictions. Recursive expressions for computation of predictions must be derived. Matrices $\boldsymbol{P}$ (11) and $\boldsymbol{G}$ (12) can be evaluated recursively. [1]-[5]

$$\begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+N_2) \end{bmatrix} = \boldsymbol{P}\begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \end{bmatrix} + \boldsymbol{G}\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix} \tag{10}$$

$$\left.\begin{array}{c} \boldsymbol{P} \in R^{N_2,4}, \\ \boldsymbol{P_{11}} = 1 - a_1, \boldsymbol{P_{12}} = a_1 - a_2, \boldsymbol{P_{13}} = a_2, \boldsymbol{P_{14}} = b_2, \\ \boldsymbol{P_{21}} = a_1^2 - a_1 - a_2 + 1, \boldsymbol{P_{22}} = a_1 - a_1^2 + a_1 a_2, \\ \boldsymbol{P_{23}} = a_2 - a_1 a_2, \boldsymbol{P_{24}} = b_2 - a_1 b_2, \\ \boldsymbol{P_{31}} = a_1^2 - a_1 + 2a_1 a_2 - a_1^3 + 1 - a_2, \\ \boldsymbol{P_{32}} = a_1 - a_1^2 + a_1^3 - a_1^2 a_2 - a_1 a_2 + a_2^2, \\ \boldsymbol{P_{33}} = a_2 - a_1 a_2 + a_1^2 a_2 - a_2^2, \\ \boldsymbol{P_{34}} = b_2 - a_1 b_2 + a_1^2 b_2 - a_2 b_2, \\ \sum_{r=4}^{N_2} \sum_{s=1}^{4} \begin{pmatrix} \boldsymbol{P_{rs}} = (1 - a_1)\boldsymbol{P_{r-1,s}} + (a_1 - a_2)\boldsymbol{P_{r-2,s}} + \\ + a_2 \boldsymbol{P_{r-3,s}} \end{pmatrix} \end{array}\right\} \quad (11)$$

$$\left.\begin{array}{c} \boldsymbol{G} \in R^{N_2 - N_1 + 1, N_2}, \\ \boldsymbol{G_{11}} = \boldsymbol{G_{22}} = \boldsymbol{G_{33}} = b_1, \boldsymbol{G_{21}} = b_1 - a_1 b_1 + b_2, \\ \boldsymbol{G_{31}} = \boldsymbol{G_{21}} + a_1^2 b_1 - a_1 b_2 - a_2 b_1, \boldsymbol{G_{32}} = \boldsymbol{G_{21}}, \\ \sum_{r=4}^{N_2} \sum_{s=1}^{r} \left(\begin{cases} \boldsymbol{G_{rs}} = b_1, \boldsymbol{G_{r,s-1}} = \boldsymbol{G_{r,s-1}} + b_2; r = s \\ \left.\begin{array}{l} \boldsymbol{G_{rs}} = (1 - a_1)\boldsymbol{G_{r-1,s}} + \\ + (a_1 - a_2)\boldsymbol{G_{r-2,s}} + a_2 \boldsymbol{G_{r-3,s}}; \end{array}\right\} r \neq s \end{cases} \right) \end{array}\right\} \quad (12)$$

5 Statistical Technique for Evaluation of Effect after Application of Modification in Process Control

For purposes of evaluation of control results, the method published in [8] was proposed. In this approach, pairs of discrete signals (controlled variables: original signal u and modified signal u^*, control output variables: original signal y and modified signal y^*) are tested using the testing hypotheses.

A particular type of the paired values comparison corresponds to an individual type of mathematical hypothesis. It can be solved either by the Paired T-test or by the Wilcoxon Paired test. The type of the test is chosen according to the normality property [16]-[17] of the tested data. If the data fulfil normality, the Paired T-test [9] is used. Otherwise the Wilcoxon Paired test [9] is then applied. [8]

An important initial part in testing hypotheses is a declaration of the significance level α. It was considered as 0.05 in this paper. [8]

The second part of testing hypothesis is a declaration of the zero hypothesis. The zero hypothesis is given by the following proposition: "There are not statistically significant differences between pairs of values". An alternative hypothesis is then defined as: "There are statistically significant differences between pairs of values". The results obtained from the testing hypothesis are considered in the form of p values [9]. The zero hypothesis is failed to reject, if the p value of the testing hypothesis is greater or equal to α. Zero hypothesis is rejected in favour of the alternative hypothesis if p is lower than α. [8]

6 Results

In the MATLAB, the predictive control of non-minimum phase systems was realized for the three following continuous models $G_1(s)$-$G_3(s)$ which were transformed to discrete models $G_1(z^{-1})$-$G_3(z^{-1})$ with consideration of a sampling period of 1 second. For the following zeros, continuous respectively discrete models in the form of transfer functions are defined: $\vartheta_1 = 0.5$ (13)-(14), $\vartheta_2 = 0.75$ (15)-(16), and $\vartheta_3 = 1$ (17)-(18). For each model, same poles were considered as π_1= -0.3618, and π_2=-0.1382.

$$G_1(s) = \frac{-s + 0.5}{s^2 + 0.5s + 0.05}; \vartheta_1 = 0.5 \quad (13)$$

$$G_1(z^{-1}) = \frac{-0.5682\,z^{-1} + 0.9601\,z^{-2}}{1 - 1.567\,z^{-1} + 0.6065\,z^{-2}} \quad (14)$$

$$G_2(s) = \frac{-s + 0.75}{s^2 + 0.5s + 0.05}; \vartheta_2 = 0.75 \quad (15)$$

$$G_2(z^{-1}) = \frac{-0.4621\,z^{-1} + 1.05\,z^{-2}}{1 - 1.567\,z^{-1} + 0.6065\,z^{-2}} \quad (16)$$

$$G_3(s) = \frac{-s + 1}{s^2 + 0.5s + 0.05}; \vartheta_3 = 1 \quad (17)$$

$$G_3(z^{-1}) = \frac{-0.356\,z^{-1} + 1.14\,z^{-2}}{1 - 1.567\,z^{-1} + 0.6065\,z^{-2}} \quad (18)$$

In the performed MPC, the minimum, control and prediction horizons were chosen as N_1=1, N_u=30 and N_2 =30. The optimization problem was solved by the Hildreth method [15].

Continuous and discrete step functions, which demonstrates the undershoots in the controlled process behaviour, are displayed in Fig. 1-3 for each defined system $G_1(s)$-$G_3(s)$ resp. $G_1(z^{-1})$-$G_3(z^{-1})$. For each model, pairs of signals u and u^* (Fig. 4-6), respectively y and y^* (Fig. 7-9), can be seen in the initial part with 49 samples of the MPC.

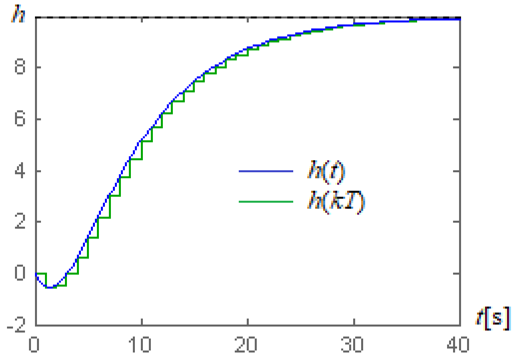

Fig. 1 Continuous and Discrete Step Functions of Model G_1

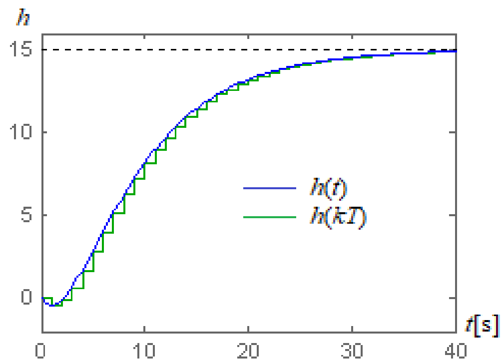

Fig. 2 Continuous and Discrete Step Functions of Model G_2

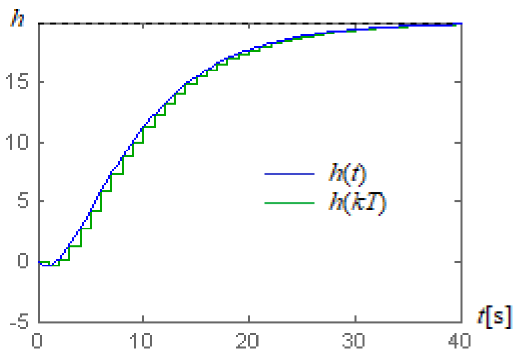

Fig. 3 Continuous and Discrete Step Functions of Model G_3

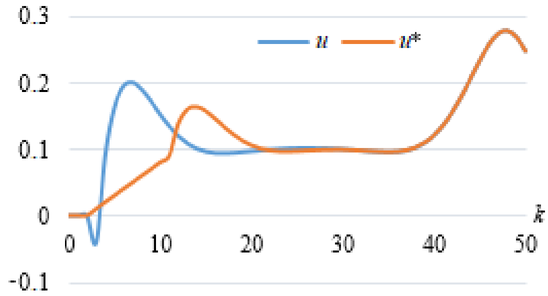

Fig. 4 Initial Samples of Signals u and u^* for G_1

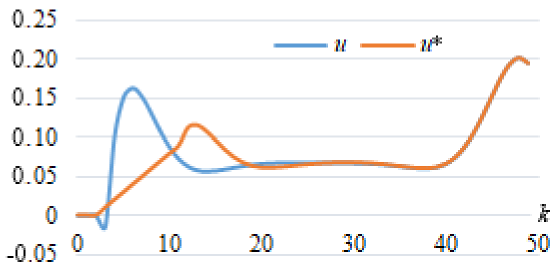

Fig. 5 Initial Samples of Signals u and u^* for G_2

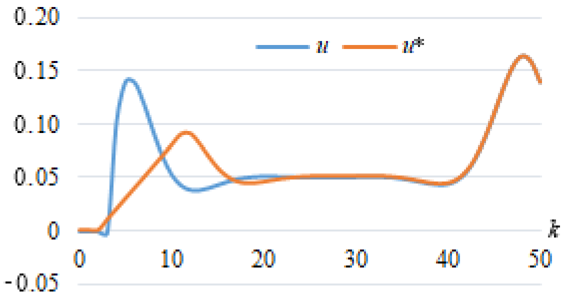

Fig. 6 Initial Samples of Signals u and u^* for G_3

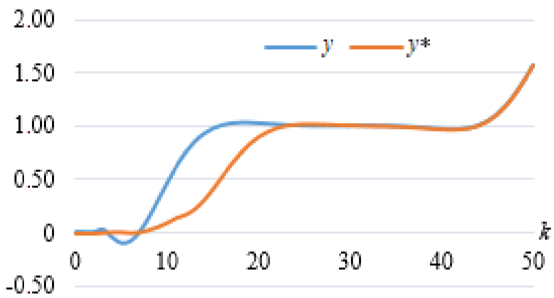

Fig. 7 Initial Samples of Signals y and y^* for G_1

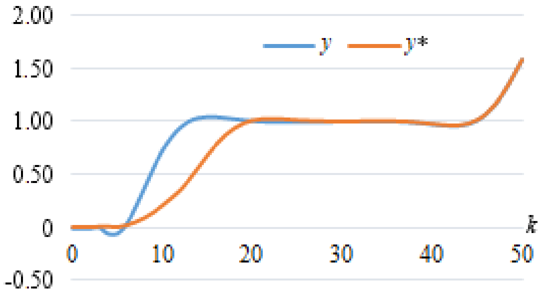

Fig. 8 Initial Samples of Signals y and y^* for G_2

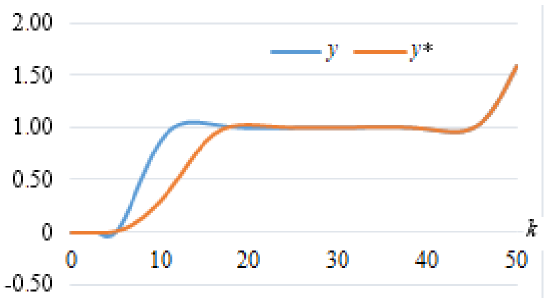

Fig. 9 Initial Samples of Signals y and y^* for G_3

For each model G_i, the statistical analysis (Table 1-2) of signals (with 49 initial values) of u, and y before and after applied modification was realized in the form of p value for the first step change of the reference signal (49 values). In Table 1, the normality of data [16]-[17] of signals was tested with selection of an appropriate statistical test according to achieved result of normality of data using the Shapiro-Wilk method.

In Table 2, achieved results of testing the existence of statistically significant differences between pairs of signal are displayed in the form of p-values using appropriate tests (determined in Table 1). All statistical tests were provided in the PAST Statistics version 2.17 [18]. The significance level was defined as α= 0.05.

Table 1: Testing Normality of Data of Signals Before and After Applied Modification

Model G_i	*u*	*y*
Model G_1	*Non-modified u*: $p = 2.36\times10^{-5}$ *Modified u**: $p = 5.41\times10^{-4}$	*Non-modified y*: $p = 0.7124$ *Modified y**: $p = 0.7982$
Stat.Method	**Wilcoxon Test**	**Paired T-Test**
Model G_2	*Non-modified u*: $p = 0.8252$ *Modified u**: $p = 0.8396$	*Non-modified y*: $p = 0.6685$ *Modified y**: $p = 0.7685$
Stat.Method	**Paired T-Test**	**Paired T-Test**
Model G_3	*Non-modified u*: $p = 0.7906$ *Modified u**: $p = 0.8053$	*Non-modified y*: $p = 0.6393$ *Modified y**: $p = 0.7363$
Stat.Method	**Paired T-Test**	**Paired T-Test**

Table 2: Testing Significant Differences Between Pairs of Signals (Before and After Modification)

Model G_i	*u, u**	*y, y**
Model G_1	$p = 0.5273$	$p = 0.001082$
Sign. Differences	**Failed to Reject** on $\alpha = 0.05$	**Rejected** on $\alpha = 0.05$
Model G_2	$p = 0.2601$	$p = 0.00164$
Sign. Differences	**Failed to Reject** on $\alpha = 0.05$	**Rejected** on $\alpha = 0.05$
Model G_3	$p = 0.3633$	$p = 0.002832$
Sign. Differences	**Failed to Reject** on $\alpha = 0.05$	**Rejected** on $\alpha = 0.05$

Finally, the control quality criterions J_1 and J_2 both without and with the proposed modification (denoted by *) were computed according to (3)-(4). The resulting values of the criterions are in Table 3. Complete courses of signals u and y during the whole MPC can be seen in the Fig. 10-11.

Table 3: Monitoring Control Quality Criterions Before and After Applied Modification

Model G_i	*J_1*	*J_2*
Model G_1	*Non-modified J_1:* *0.08222* *Modified J_1*:* *0.059832*	*Non-modified J_2:* *11.17881* *Modified J_2*:* *14.79767*
Model G_2	*Non-modified J_1:* *0.047731* *Modified J_1*:* *0.031527*	*Non-modified J_2:* *9.210172* *Modified J_2*:* *12.4431*
Model G_3	*Non-modified J_1:* 0.033936 *Modified J_1*:* *0.021231*	*Non-modified J_2:* *8.126307* *Modified J_2*:* *10.97569*

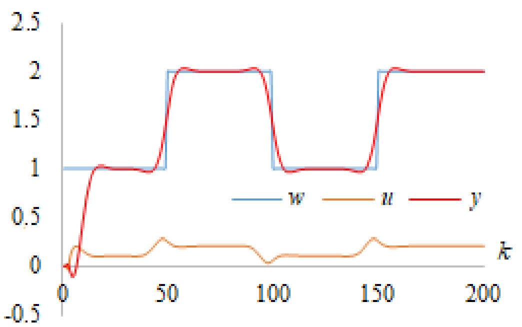

Fig. 10 Signals in MPC before Modification (for Model G_1)

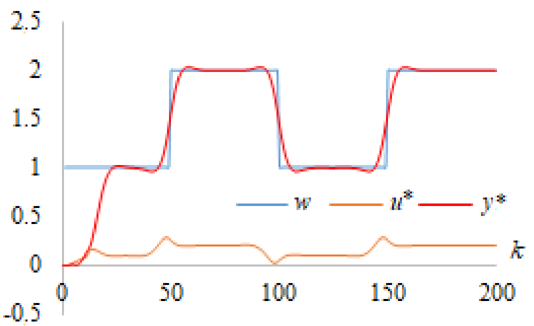

Fig. 11 Signals in MPC after Modification (for Model G_1)

4 Conclusion

For the purpose of application of testing hypothesis into a field of analysis of signals, the control quality achieved by two different algorithms was analyzed and compared using testing hypothesis. In this hypothesis testing, partial values of signals were analyzed in each sampling period of MPC. It would not be possible to consider statistical significance of differences in achieved control quality only from the descriptive attributes given by standardly used control quality criterions. By using of methods of testing hypotheses on existence of the statistical significant differences between two discrete signals, analysis of a control quality was successfully complemented by more mathematically supported results. The paired comparison was performed by the Wilcoxon paired test and Paired T-test on the significance level 0.05. Therefore, the achieved results had relevant informational value based on mathematical statistics. A realization was presented on a simulation of the predictive control of the non-minimum phase systems. It was statistically proved that there are statistically significant differences between pairs of controlled variables. This means that the quality of control was significantly improved. However, there were not statistically significant differences between the values of the manipulated variables on the defined significance level.

References:

[1] E. F. Camacho, and C. Bordons, *Model predictive control*, Springer, 2007.
[2] W. H. Kwon, *Receding horizon control: model predictive control for state models*, Springer, 2005.
[3] J. A. Rossiter, *Model Based Predictive Control: a Practical Approach*, CRC Press, 2003.
[4] J. P. Corriou, *Process control: theory and applications*, Springer, 2004.
[5] A. Abraham, N. Pappa, D. Honc, et.al., "Reduced order modelling and predictive control of multivariable nonlinear process", *Sadhana - Academy Proceedings in Engineering Sciences*, vol. 43, no. 3, 2018.
[6] J. B. Hoagg, and D. S. Bernstein, "Nonminimum-phase zeros - much to do about nothing - classical control - revisited part II.", *Control Systems*, vol. 27, no. 3, pp. 45-57, 2007.
[7] T. Barot, and M. Kubalcik, "Predictive control of non-minimum phase systems", presented at the 2014 15th International Carpathian Control Conference, Velke Karlovice, Czech Republic.
[8] M. Kubalcik, V. Bobal, and T. Barot, "Statistical analysis of control quality of MPC using testing hypothesis", presented at the 2019 23rd International Conference on Circuits, Systems, Communications

and Computers (CSCC 2019), Marathon Beach, Athens.

[9] D. Stockemer, *Quantitative Methods for the Social Sciences*, Springer, 2019.

[10] M. Vaclavik, Z. Sikorova, and I. Cervenkova, “Analysis of Independences of Normality on Sample Size with Regards to Reliability”, *Advances in Intelligent Systems and Computing: Computational Statistics and Mathematical Modelling Methods in Intelligent Systems*, 2019, to be published.

[11] J. Pivarc, “Ideas of Czech primary school pupils about intellectual disability”, *Educational Studies*, 2018, to be published.

[12] L. Korenova, T. Barot, R. Krpec, I. Veress-Bagyi, “Analysis of Applied Mobile Technologies in University Math Education”, presented at the 2019 11th Annual International Conference on Education and New Learning Technologies, Palma de Mallorca, Spain.

[13] M. Kubalcik, V. Bobal, and T. Barot, “Analysis of Modified Optimization in Multivariable Predictive Control”, *Int. Journal of Mechatronics and Applied Mechanics*, no. 5, pp. 7-12, 2019.

[14] Z. Dostal, *Optimal Quadratic Programming Algorithms: With Applications to Variational Inequalities*, Springer, 2009.

[15] L. Wang, *Model Predictive Control System Design and Implementation Using MATLAB*, Springer, 2009.

[16] H. Alizadeh Noughabi, “Two Powerful Tests for Normality”, *Annals of Data Science*, vol. 3, no. 2, pp. 225-234, 2016.

[17] M. Vaclavik, Z. Sikorova, and T. Barot, “Particular Analysis of Normality of Data in Applied Quantitative Research”, *Advances in Intelligent Systems and Computing: Computational and Statistical Methods in Intelligent Systems*, vol. 859, pp. 353-365, 2019.

[18] O. Hammer, D. A. T. Harper, and P. D. Ryan, “PAST: Paleontological statistics software package for education and data analysis”, Palaeontologia Electronica, vol. 4, no. 1, 2001.