Comparison of Two Approaches to Count Derivations for Continuous-Time Adaptive Control

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1. Introduction

The control of continuous-time systems can be realized by adaptive controllers. Self-tuning controllers are adaptive controllers which call on-line identification and controllers parameters tuning in one step of computation. Supervision enlarges the area of usage of controllers. It is necessary to count derivations of action and output signals during control, which is usually realized by filters. Settings of filters are directly connected with the model of the system. Another approach allows us to use the regression polynomials instead of filters, because the general form of derivations is known before the control. Without filters, this approach keeps the signal unchanged, but the choice of inappropriate length of time interval for polynomial regression increases the amplitude of noise. The chapter shows two examples of control and suggests the appropriate length of time interval for polynomial regression.

Many processes can be viewed in the point of control as continuous-time systems. The implementation of pseudo-continuous model on continuous-time system is called as hybrid system (De Santis et al., 2009). Mostly these systems are nonlinear and specific method of control is needed (Gregorčič and Lightbody, 2010). This chapter uses the method adaptive control, because adaptive control is often used and gives adequate results (Pasik-Duncan, 2001). At adaptive control, the usage of the appropriate identification method is very important. This paper uses recursive instrumental variable method, but there are several other good methods and papers dealing with identification and parameters tuning (Flores and Pastor, 2005, Tzes and Le, 1996, Coello, 2000). The controlled process in this chapter has multi-inputs multi-outputs and the decentralized controller was used. It is common approach in practice (Martínez-Rosas et al., 2006). Decentralized control can be realized by PID controllers. These controllers are very popular due to their advantages, such as simplicity (Vrančić et al., 2010).

The ideas and results obtained in control can be useful in many different areas, for example in robotics or in production systems. Nice paper about spatial ontology for human-robot interaction was written by Belouaer at al. (Belouaer et al., 2010). A special framework to generate configurations in production systems was written by Kanso et al. (Kanso et al., 2010).
2. Theoretical background

2.1 Self-tuning control
Self-tuning controllers (STC) are based on on-line identification and on tuning the controller parameters with respect to identified changes in controlled systems. The self-tuning controllers can be further divided to the STC with explicit identification and the STC with implicit identification, the STC with implicit identification directly identifies the controller parameters. On the other hand, the STC with explicit identification computes the controller parameters using the parameters of the system model (Bobal et al., 2005).

2.2 On-line identification
When self-tuning controller was used, the scheme of input and output signal modification depicted in figure 1 is applied, because the continuous-time system parameters $a_i$ and $b_j$ are estimated using recursive instrumental variable method. The action (input) signal $u(t)$ is continuously approximated by Lagrange regression polynomial on an interval of given length during entire control. The structure of Lagrange regression polynomial (1) together with its derivation (2), (3) is generally known before the start of identification, only the numerical values of their parameters are needed and counted. It is the alternative way to obtain values of derivations needed for identification. After the polynomial approximation, the approximating polynomial derivation $u^{(i)}(t)$ is counted. It is sampled in purpose to count the values of subsystem parameters using recursive identification algorithm.

\[ P_2(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)}f(a) + \frac{(x-a)(x-c)}{(b-a)(b-c)}f(b) + \frac{(x-a)(x-b)}{(c-a)(c-b)}f(c) \] (1)

Fig. 1. Scheme of I/O signals modification for STC.

Lagrange polynomial of second order was used in the paper in the form
The first derivation is

\[
\frac{2x - (b + c)}{(a - b)(a - c)} f(a) + \frac{2x - (a + c)}{(b - a)(b - c)} f(b) + \frac{2x - (a + b)}{(c - a)(c - b)} f(c)
\]

and second derivation is

\[
\frac{2f(a)}{(a - b)(a - c)} + \frac{2f(b)}{(b - a)(b - c)} + \frac{2f(c)}{(c - a)(c - b)}
\]

2.3 Recursive instrumental variable

Instrumental variable method is a modification of the least squares method. The least squares method uses the quadratic criterion and the existence of one global minimum. The instrumental variable method does not allow us to obtain the properties of noise, but it has inferior presumptions than the least square method. It is possible to formulate it recursively (Zhu & Backx, 1993).

\[
\hat{\Theta}^T(k) = \begin{pmatrix} \hat{a}_0, \hat{a}_1, ..., \hat{a}_{\text{deg}(a)}, \hat{b}_0, \hat{b}_1, ..., \hat{b}_{\text{deg}(b)}, d \end{pmatrix}
\]

\[
\Phi^T(k) = \begin{bmatrix} -y(t_k), ..., -y_{L-1}(t_k), u(t_k), ..., u_{L-1}(t_k), 1 \end{bmatrix}
\]

\[
L(k) = \frac{C(k-1)z(k)}{1 + \Phi^T(k)C(k-1)z(k-1)}
\]

\[
C(k) = C(k-1) - \frac{C(k-1)z(k)\Phi^T(k)C(k-1)}{1 + \Phi^T(k)C(k-1)z(k)}
\]

\[
z(k) = \begin{bmatrix} u(t_k), u(t_{k-1}), ..., u(t_{k-n}) \end{bmatrix}
\]

\[
\hat{\epsilon}(k) = y(k) - \Phi^T(k)\hat{\Theta}(k-1)
\]

\[
\hat{\Theta}(k) = \hat{\Theta}(k-1) + L(k)\hat{\epsilon}(k)
\]

2.4 Suboptimal linear quadratic controller

The used suboptimal method was introduced by Dostal (Dostal, 1997). Let us minimize quadratic functional

\[
f = \int_0^\infty \left\{ \mu u^2(t) + \varphi \dddot{u}^2(t) \right\} dt
\]

where \( \mu \geq 0, \varphi > 0 \) are penalty constants. Stable polynomials \( g \) and \( n \) are counted as results of spectral factorizations.
Solving the following diophantic equation

\[ asp + bq = gn \]

(13)
gives the parameters of controller. If the system transfer function has the form

\[ G(s) = \frac{b_0}{s^2 + a_1 s + a_0} \]

(14)
The controller is

\[ FQ = -\frac{q_2 s^2 + q_1 s + q_0}{s(p_2 s^2 + p_1 s + p_0)} \]

(15)
and polynomials \( g \) and \( n \) are

\[ g(s) = g_3 s^3 + g_2 s^2 + g_1 s + g_0 \]

(16)
\[ n(s) = s^2 + n_1 s + n_0 \]

(17)
Their coefficients obtained by spectral factorization are in the form

\[ g_0 = \sqrt{\mu b_0^2} \]

(18)
\[ g_1 = \sqrt{2g_2 g_0 + \varphi a_0^2} \]

(19)
\[ g_2 = \sqrt{2g_3 g_1 + \varphi (a_1^2 - 2a_0)} \]

(20)
\[ g_3 = \sqrt{\varphi} \]

(21)
\[ n_0 = \sqrt{a_0^2} \]

(22)
\[ n_1 = \sqrt{2n_0 - a_1^2 - 2a_0} \]

(23)

2.5 Supervisor

The used supervisor is based on the supervisor introduced by Perutka (Perutka, 2007) and it is used in this paper for the first time.

Supervisor is used for decentralized or decoupled control of multi-input multi-output systems, number of inputs and outputs are the same and denoted as \( n \). Such system is controlled by \( n \) sub-controllers. Let us suppose the existence of bits field with \( n \times n \) dimension. The initial values of the field form the identity matrix. Each row the field corresponds to one subsystem of controlled system.
Step 1. Go through the bits field row by row. The row which gives the highest number after conversion also gives the number of the subsystem in which goes one step of the on-line identification.

Step 2. When the subsystem is set, the last bit in the row of identified subsystem is set to 1 and in remaining rows the last bit is set to 0.

Step 3. One bit left rotation of all rows in bits field.

Step 4. Go through the bits field row by row. The row which gives the lowest number after conversion also gives the number of the subsystem in which goes one step of the on-line identification.


Repeat Step 1 to 6 \(n/2\)-times at even \(n\) and \(n/2\)-times without Step 4 to 6 at last calling at odd \(n\) after the change of set-point. After this tuning, run the self-tuning control without supervisor until the new change of the set-point when the supervisor is called.

3. Experimental part

In figures 2-7, there are obtained results of control of two inputs two outputs systems by two controllers. Counting step was 0.2 s. In these figures, the meaning of the symbols is following: \(w_1\) – set-point of first subsystem, \(u_1\) – action signal of first subsystem, \(y_1\) – output signal of first subsystem, \(w_2\) – set-point of second subsystem, \(u_2\) – action signal of second subsystem, \(y_2\) – output signal of second subsystem, \(p_{11}, p_{01}, q_{21}, q_{11}, q_{01}\) – parameters of first sub-controller, \(b_{01}, a_{11}, a_{01}\) – parameters of the model of the first controlled subsystem.

Fig. 2. History of control – too small interval for approximation by Lagrange polynomial.
Fig. 3. History of controller parameters for 1st subsystem – too small interval for approximation by Lagrange polynomial.

Fig. 4. History of subsystem model parameters for 1st subsystem - too small interval for approximation by Lagrange polynomial.
Fig. 5. History of control – adequate interval for approximation by Lagrange polynomial.

Fig. 6. History of controller parameters for 1st subsystem – adequate interval for approximation by Lagrange polynomial.
Figures 2 - 4 provide the results obtained for time interval of approximation 0.41 s. Figures 5 - 7 provide the results obtained for time interval of approximation 4.1 s.

From the illustratively shown results, it is clear that it is important to correctly choose the appropriate length of time interval which is used for regression by Lagrange polynomial. By repeating several experiments with different controlled systems it was verified that it is appropriate to use 20 counting steps in the presence of noise.

![Graph showing history of subsystem model parameters](image)

**Fig. 7.** History of subsystem model parameters for 1st subsystem - adequate interval for approximation by Lagrange polynomial.

### 4. Conclusions

The chapter presented simulation results of self-tuning control with polynomial regression used for derivations counting. It was shown that inappropriate selection of regression interval makes more noise. The recommended length of the interval was given. Future work
will focus on the exact mathematical derivation of the time appropriate interval with the combination of the dynamical filter of noise.

5. Acknowledgements

The author would like to mention MSM7088352101 grant, from which the work was supported.

6. References


The book "Cutting Edge Research in New Technologies" presents the contributions of some researchers in modern fields of technology, serving as a valuable tool for scientists, researchers, graduate students and professionals. The focus is on several aspects of designing and manufacturing, examining complex technical products and some aspects of the development and use of industrial and service automation. The book covered some topics as it follows: manufacturing, machining, textile industry, CAD/CAM/CAE systems, electronic circuits, control and automation, electric drives, artificial intelligence, fuzzy logic, vision systems, neural networks, intelligent systems, wireless sensor networks, environmental technology, logistic services, transportation, intelligent security, multimedia, modeling, simulation, video techniques, water plant technology, globalization and technology. This collection of articles offers information which responds to the general goal of technology - how to develop manufacturing systems, methods, algorithms, how to use devices, equipments, machines or tools in order to increase the quality of the products, the human comfort or security.

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