Performance of Multi-chaotic PSO on a Shifted Benchmark Functions Set

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Abstract. In this paper the performance of Multi-chaotic PSO algorithm is investigated using two shifted benchmark functions. The purpose of shifted benchmark functions is to simulate the time-variant real-world problems. The results of chaotic PSO are compared with canonical version of the algorithm. It is concluded that using the multi-chaotic approach can lead to better results in optimization of shifted functions.

Keywords: Particle swarm optimization, PSO, Chaos, Shifted functions, Multi-chaotic, Pseudo-random numbers

INTRODUCTION

In recent years there has been a significant development in the area of evolutionary computational techniques (ECTs) such as the PSO algorithm [1-4]. One of the promising approaches is the implementation of chaotic sequences as Pseudo-random number generators (PRNGs) [5 - 11]. In this research the performance of PSO algorithm with multi-chaotic PRNG [9] is investigated on two shifted benchmark functions. The shifted benchmark functions are designed in order to better simulate the time-variant real-world problems.

PARTICLE SWARM OPTIMIZATION ALGORITHM

The PSO algorithm is inspired in the natural swarm behavior of birds and fish. It was introduced by Eberhart and Kennedy in 1995 [1]. Each particle in the population represents a candidate solution for the optimization problem that is defined by the cost function (CF). In each iteration of the algorithm, a new location (combination of CF parameters) for the particle is calculated based on its previous location and velocity vector (velocity vector contains particle velocity for each dimension of the problem). Within this research the PSO algorithm with global topology (GPSO) [6] was utilized. The chaotic PRNG is used in the main GPSO formula (1), which determines a new "velocity", thus directly affects the position of each particle in the next iteration.

$$v_{ij}^{t+1} = w \cdot v_{ij}^{t} + c_1 \cdot Rand \cdot (pBest_{ij} - x_{ij}^{t}) + c_2 \cdot Rand \cdot (gBest_j - x_{ij}^{t}) \tag{1}$$

Where:
v_i^{t+1} - New velocity of the *ith* particle in iteration t+1.
w – Inertia weight value; v_i^t - Current velocity of the *ith* particle in iteration t.; c_1, c_2 - Priority factors; $pBest_i$ – Personal best solution found by the *ith* particle; *gBest* - Best solution found in a population; x_{ij}^t - Current position of the *ith* particle (component j of dimension D) in iteration t.; *Rand* – Pseudo random number, interval (0, 1). CPRNG is applied only here.
The maximum velocity was limited to 0.2 times the range as it is usual. The new position of each particle is then given by (2), where x_i^{t+1} is the new particle position:

$$x_i^{t+1} = x_i^{t} + v_i^{t+1} \tag{2}$$

Finally the linear decreasing inertia weight [3, 4] is used in the typically referred GPSO design that was used in this study. The inertia weight has two control parameters wstart and wend. A new w for each iteration is given by (3), where t stands for current iteration number and n stands for the total number of iterations. The values used in this study were wstart = 0.9 and wend = 0.4.

$$w = w_{start} - \frac{((w_{start} - w_{end}) \cdot t)}{n} \tag{3}$$

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CHAOTIC MAPS

In this section two discrete chaotic systems that were used as CPRNGs are presented.

Lozi Map

The Lozi map is a simple discrete two-dimensional chaotic map. The map equations are given in (4). The parameters used in this work are: a = 1.7 and b = 0.5 with respect to [11]. For these values, the system exhibits typical chaotic behavior and with this parameter setting it is used in the most research papers and other literature sources.

$$X_{n+1} = 1 - a\left|X_n\right| + bY_n \qquad (4)$$
$$Y_{n+1} = X_n$$

Arnold's Cat Map

The Arnold's Cat map is a simple two dimensional discrete system that stretches and folds points (x, y) to (x+y , x+2y) mod 1 in phase space. The map equations are given in Eq. 5. This map was used with parameter k = 0.1.

$$X_{n+1} = X_n + Y_n \,(\mathrm{mod}\,1) \qquad (5)$$
$$Y_{n+1} = X_n + kY_n \,(\mathrm{mod}\,1)$$

TEST FUNCTIONS

In order to investigate on the performance of multi-chaotic PSO algorithm on functions closer to real problem than static test function, two shifted function were chosen. Shifted function global optimum moves with each start of the algorithm but keeps their basic characteristic thus simulates the time-variant real problems. Following shifted test functions were used in this study.

Shifted Sphere function is given by (6).

$$f(x) = \sum_{i=1}^{\dim} (x_i - shift_i)^2 \qquad (6)$$

Function minimum: Position for E_n: $(x_1,x_2\ldots x_n)$ = *shift;* Value for E_n: y = 0;

Shifted Rastrigin`s function is given by (7).

$$f(x) = 10\dim + \sum_{i=1}^{\dim} (x_i - shift_i)^2 - 10\cos(2\pi x_i - shift_i) \qquad (7)$$

Function minimum: Position for E_n: $(x_1,x_2\ldots x_n)$ = *shift;* Value for E_n: y = 0;

Shift$_i$ is an random number from interval <-5.11,5.11>. Where <-5.11,5.11> are the low and high bounds for the population individuals. *Shift* value is generated on the start of optimization process.

EXPERIMENT

The control parameters of PSO algorithm were set as follows:

Pop. size: 40; N. of iterations: 5000; w_{start}: 0.9; w_{end}: 0.4; Dimension: 10; Runs: 50. Two different instances of multi-chaotic PSO [9] are investigated here. In the multi-chaotic approach two different CPRNGs are switched when the algorithm seems to stagnate (for details see [9]). In the first design in this study the optimization starts with Lozi map based CPRNG and it is switched to CPRNG based on Arnold´s Cat map. In the second design the CPRNGs are used in opposite order. The results are summarized in Tables 1 and 2. Furthermore mean gBest history for each function is depicted in Fig. 1 and Fig. 2.

TABLE 1. Mean results comparison – Shifted Sphere function

Sphere	GPSO	Lozi - Arnold	Arnold - Lozi
Mean CF Value:	0.000289213	0.00183071	**0.000042082**
Std. Dev.:	0.00128856	0.012187	0.000171946
CF Value Median:	0.	0.	0.
Max. CF Value:	0.00863989	0.0870667	0.00121531
Min. CF Value:	0.	0.	0.

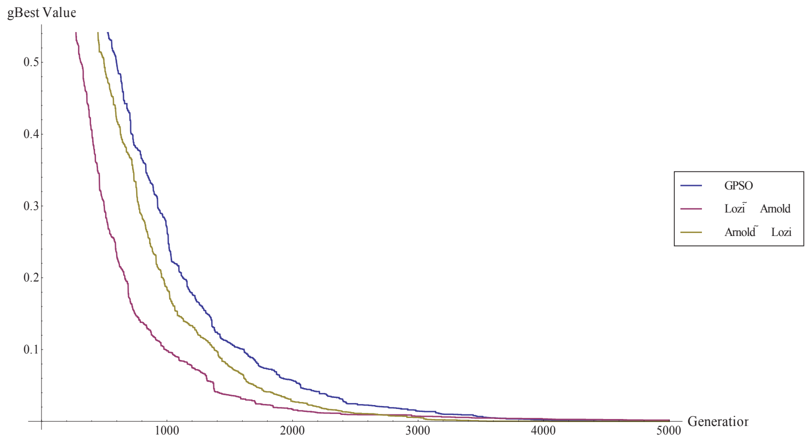

FIGURE 1. Mean gBest history for the Sphere function

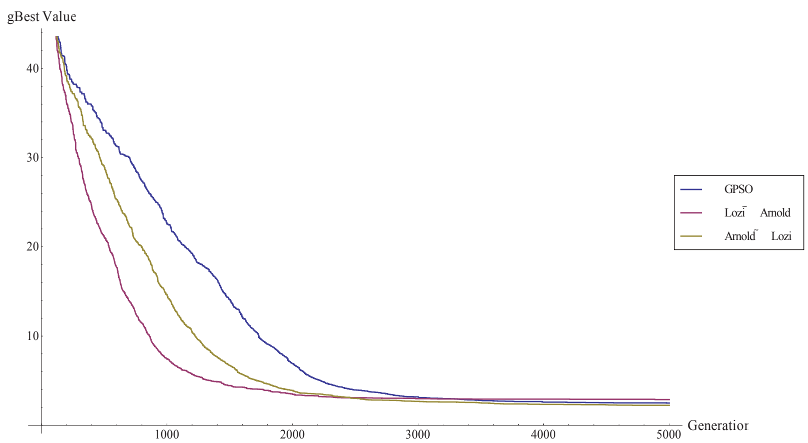

FIGURE 2. Mean gBest history for the Rastrigin´s function

TABLE 2. Mean results comparison – Shifted Rastrigin´s function

Rastrigin	GPSO	Lozi - Arnold	Arnold - Lozi
Mean CF Value:	2.49478	2.88599	**2.22995**
Std. Dev.:	1.53435	1.64208	1.52427
CF Value Median:	1.99001	2.98488	1.98992
Max. CF Value:	6.96471	6.96471	6.2411
Min. CF Value:	0.	0.	0.

CONCLUSION

In this study the performance of multi-chaotic PSO was investigated on two different shifted benchmark functions. The aim was to investigate the performance of this design on closer to real-world problems. Results presented in this work support claim that using two different CPRNGs within one run of the algorithm may improve the performance of PSO algorithm on various optimization tasks. The second designed combination of Arnold´s Cat map based CPRNG and Lozi map based CPRNG outperformed the canonical version in both cases. This promising result should motivate future research of this approach.

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