ADAPTIVE CONTROL OF A CONTINUOUS STIRRED TANK REACTOR BY TWO FEEDBACK CONTROLLERS

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Abstract: The paper deals with adaptive control of a continuous stirred tank reactor (CSTR). A nonlinear model of the process is approximated by a continuous-time external linear model. The parameters of the CT external linear model of the process are estimated using a corresponding delta model. The control system with two feedback controllers is considered. The controller design is based on the polynomial approach. The resulting proper controllers ensure stability of the control system as well as asymptotic tracking of step references and step load disturbance attenuation. The adaptive control is tested on the nonlinear model of the CSTR with a consecutive exothermic reaction.

Keywords: Adaptive control, continuous-time model, delta model, parameter estimation, polynomial method.

1. INTRODUCTION

Continuous stirred tank reactors (CSTRs) belong to a class of nonlinear systems where both steady-state and dynamic behaviour are nonlinear. Their models are derived and described in e.g. (Ogunnaike and Ray, 1994), (Schmidt, 2005) and (Corriou, 2004). The process nonlinearities may cause difficulties when controlling using conventional controllers with fixed parameters. One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of an external linear model (ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller's parameters.

The control itself can be either continuous-time or discrete. While for design of a continuous-time controller, it is necessary to know a continuous-time ELM and its parameters, a discrete-time controller requires knowledge of a discrete ELM. Experiences of authors in the field of control of nonlinear technological processes indicate that the continuous-time (CT)approach gives better results when controlling processes with strong nonlinearities. In the case of discrete control in order to cope with the nonlinearity, it is necessary to sample signals very frequently. However, it is well known from the properties of transfer functions in the z-domain that a sampling period cannot be shortened too much.

For the CT ELM parameters estimation, either the direct method or application of an external delta model with the same structure as the CT model can be used. The procedure based on direct CT ELM parameter estimation was described in (Dostál *et al.*, 2001).

The basics of delta models have been described in e.g. (Middleton and Goodwin, 1990), (Mukhopadhyay *et al.*, 1992) and (Goodwin *et al.*, 2001). Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete z-models. In addition, parameters of delta models can directly be estimated from sampled signals. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process). Complete description and experimental verification can be found in (Stericker and Sinha, 1993).

This contribution deals with continuous-time adaptive control of the CSTR as a non-linear single input – single output process. The parameters of its CT ELM are obtained via corresponding delta model parameter estimation. The control system with two feedback controllers is used according to (Ortega and Kelly, 1984). This set-up gives better control results for the reference tracking than using only a feedback controller. Input signals for the control system are step references and step disturbances injected into the input of the controlled process. The resulting controllers derived using polynomial method (Kučera, 1993) guarantee stability of the control system, asymptotic tracking of step references and step load disturbances attenuation. The approach is tested on a nonlinear model of the CSTR with a consecutive exothermic reaction.

2. CT EXTERNAL LINEAR MODEL

The CT external linear model (ELM) is chosen on the basis of some preliminary knowledge of dynamic behaviour of the controlled nonlinear process. This model is described in the time domain by differential equation

$$a(\sigma)\,y(t) = b(\sigma)\,u(t) \tag{1}$$

where $\sigma = d/dt$ is the derivative operator and a, b are polynomials in σ. Considering zero initial conditions, and, using the Laplace transform, the ELM is represented in the complex domain by the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b(s)}{a(s)} \tag{2}$$

where s is the complex variable and both a and b are coprime polynomials in s. The transfer function (2) is considered to be proper ($\deg b \le \deg a$).

3. DELTA MODEL

Establish the delta operator defined by

$$\delta = \frac{q-1}{T_0} \tag{3}$$

where q is the forward shift operator and T_0 is the sampling period. When the sampling period is shortened, then, the delta operator approaches the derivative operator σ so that

$$\lim_{T_0 \to 0} \delta = \sigma \tag{4}$$

and, the δ-model

$$a'(\delta)\,y(t') = b'(\delta)\,u(t') \tag{5}$$

approaches the continuous-time model (1) as shown in (Stericker and Sinha, 1993).

Here, t' is the discrete time, and, a', b' are polynomials in δ.

4. DELTA MODEL PARAMETER ESTIMATION

Substituting $t' = k - n$ where $k \ge n$, equation (5) may be rewriten as

$$\begin{aligned} \delta^n y(k-n) &= b'_m \delta^m u(k-n) + \dots + b'_1 \delta u(k-n) + \\ &+ b'_0 u(k-n) - a'_{n-1} \delta^{n-1} y(k-n) - \dots \qquad (6) \\ \dots &- a'_{n-1} \delta^{n-1} y(k-n) - \dots - a'_1 \delta y(k-n) - a'_0 y(k-n) \end{aligned}$$

The terms in (6) can be expressed as

$$\delta^i y(k-n) = \sum_{j=0}^{i} \frac{(-1)^j}{T_0^i} \binom{i}{j} y(k-n+i-j) \tag{7}$$

for $i = 0, 1, \dots, n$, and,

$$\delta^l u(k-n) = \sum_{j=0}^{l} \frac{(-1)^j}{T_0^l} \binom{l}{j} u(k-n+l-j) \tag{8}$$

for $l = 0, 1, \dots, m$.

Obviously, an actual value of the controlled output $y(k)$ is included only in the term on the left side of (6) (for $i = n$ in (7)). Now, denoting

$$\begin{aligned} \varphi_y(k) &= \delta^n y(k-n) \\ \varphi_y(k-1) &= \delta^{n-1} y(k-n), \dots \\ \dots, \varphi_y(k-n+1) &= \delta y(k-n), \\ \varphi_y(k-n) &= y(k-n) \\ \varphi_u(k-n+m) &= \delta^m u(k-n), \dots \qquad (9) \\ \dots, \varphi_u(k-n+1) &= \delta u(k-n), \\ \varphi_u(k-n) &= u(k-n) \end{aligned}$$

and, introducing the regression vector

$$\begin{aligned} \boldsymbol{\Phi}_\delta^T(k-1) = \big[&-\varphi_y(k-n) - \varphi_y(k-n+1) \dots - \varphi_y(k-1) \\ &\varphi_u(k-n)\ \varphi_u(k-n+1) \dots \varphi_u(k-n+m) \big] \qquad (10) \end{aligned}$$

then, the parameter vector

$$\boldsymbol{\Theta}_\delta^T = \left[a'_0\ a'_1 \dots a'_{n-1}\ b'_0\ b'_1 \dots b'_m \right] \tag{11}$$

can be estimated recursively from the regression (ARX) model

$$\varphi_y(k) = \boldsymbol{\Theta}_\delta^T(k)\,\boldsymbol{\Phi}_\delta(k-1) + \varepsilon(k) \tag{12}$$

where $\varepsilon(k)$ is the non-measurable random component. For a small sampling interval T_0, the estimated parameters reach the parameters of the CT model so that

$$\begin{aligned} b'_j &\to b_j, \quad j = 0, 1, \dots m \\ a'_i &\to a_i, \quad i = 0, 1, \dots n-1 \end{aligned} \tag{13}$$

5. CONTROL SYSTEM DESCRIPTION

The control system with two feedback controllers is depicted in Fig. 1. In the scheme, w is the reference signal, v denotes the load disturbance, e is the tracking error, u_0 is the output of the controller, y is the controlled output and u is the control input.

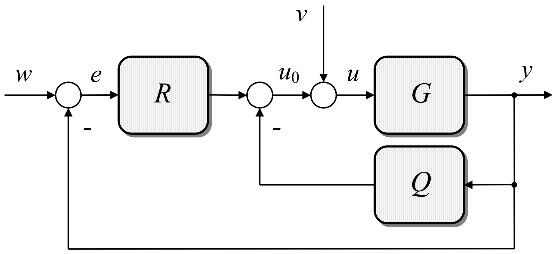

Fig. 1 Control system with two feedback controllers.

Further, G represents the ELM with the transfer function (2), Q and R are feedback controllers. Both w and v are considered to be step functions with transforms

$$W(s) = \frac{w_0}{s}, \; V(s) = \frac{v_0}{s}. \tag{14}$$

The transfer functions of controllers are

$$Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)}, \quad R(s) = \frac{r(s)}{\tilde{p}(s)} \tag{15}$$

where $\tilde{q}, r$ and $\tilde{p}$ are polynomials in s.

6. APPLICATION OF POLYNOMIAL METHOD

The controller design described in this section stems from the polynomial approach. General conditions required to govern the control system properties are formulated as strong stability (in addition to the control system stability, also the stability of controllers is required), internal properness, asymptotic tracking of the reference and load disturbance attenuation.

Transforms of the controlled output and the tracking error take forms (for simplification, the argument s is in some polynomials omitted)

$$Y(s) = \frac{b}{d}\left[r\,W(s) + \tilde{p}\,V(s)\right] \tag{16}$$

$$E(s) = \frac{1}{d}\left[(a\tilde{p} + b\tilde{q})W(s) - b\tilde{p}\,V(s)\right]. \tag{17}$$

Here,

$$d(s) = a(s)\,\tilde{p}(s) + b(s)\big(r(s) + \tilde{q}(s)\big) \tag{18}$$

is the characteristic polynomial with roots as poles of the closed-loop.

Establishing the polynomial t as

$$t(s) = r(s) + \tilde{q}(s) \tag{19}$$

and substituting (19) into (18), the condition of the control system stability is ensured when polynomials $\tilde{p}$ and t are given by a solution of the polynomial Diophantine equation

$$a(s)\tilde{p}(s) + b(s)\,t(s) = d(s) \tag{20}$$

with a stable polynomial d on the right side.

With regard to transforms (14), the asymptotic tracking and load disturbance attenuation are provided by divisibility of both terms $a\tilde{p} + b\tilde{q}$ and $\tilde{p}$ in (17) by s. This condition is fulfilled for polynomials $\tilde{p}$ and $\tilde{q}$ in the form

$$\tilde{p}(s) = s\,p(s), \;\; \tilde{q}(s) = s\,q(s). \tag{21}$$

Subsequently, the transfer functions of controllers take forms

$$Q(s) = \frac{q(s)}{p(s)}, \quad R(s) = \frac{r(s)}{s\,p(s)}. \tag{22}$$

A stable polynomial $p(s)$ in denominators of (22) ensures the stability of controllers.

The control system satisfies the condition of internal properness when the transfer functions of all its components are proper. Consequently, the degrees of polynomials q and r must fulfill inequalities

$$\deg q \le \deg p, \;\; \deg r \le \deg p + 1. \tag{23}$$

Now, the polynomial t can be rewritten into the form

$$t(s) = r(s) + s\,q(s). \tag{24}$$

Taking into account solvability of (20) and conditions (23), the degrees of polynomials in (20) and (24) can be easily derived as

$$\deg t = \deg r = \deg a, \;\; \deg q = \deg a - 1,$$
$$\deg p \ge \deg a - 1, \;\; \deg d \ge 2\deg a. \tag{25}$$

Denoting $\deg a = n$, polynomials t, r and q have the form

$$t(s) = \sum_{i=0}^{n} t_i s^i, \;\; r(s) = \sum_{i=0}^{n} r_i s^i, \;\; q(s) = \sum_{i=1}^{n} q_i s^{i-1} \tag{26}$$

where their coefficients fulfill equalities

$$r_0 = t_0, \;\; r_i + q_i = t_i \;\text{ for }\; i = 1, \dots, n \tag{27}$$

Then, unknown coefficients r_i and q_i can be obtained by a choice of selectable coefficients $\beta_i \in \langle 0, 1\rangle$ such that

$$r_i = \beta_i t_i, \;\; q_i = (1-\beta_i)\,t_i \;\text{ for }\; i = 1, \dots, n. \tag{28}$$

The coefficients β_i split a weight between numerators of transfer functions Q and R. With respect to the transform (16), it may be expected that higher values of β_i will speed up control responses to step references.

Remark: If $\beta_i = 1$ for all i, the control system in Fig. 1 simplifies to the 1DOF control configuration. If $\beta_i = 0$ for all i and both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration.

The controller parameters then follow from solution of the polynomial equation (20) and depend upon coefficients of the polynomial d. The next problem here is to find a stable polynomial d that enables to determine acceptable stabilizing and stable controllers.

7. POLE ASSIGNMENT

A required control quality can be achieved by a suitable determination of the polynomial d on the right side of (20). In this paper, the polynomial d is considered as a product of two stable factors

$$d(s) = n(s)\,(s+\alpha)^{\deg d - \deg a}\quad, \alpha > 0 \qquad (29)$$

where n is a stable polynomial given by spectral factorization

$$n^*(s)n(s) = a^*(s)a(s) \qquad (30)$$

where asterisk denotes a conjugate polynomial. Note that the choice of d in the form (29) provides the control of a good quality for aperiodic controlled processes.
Now, it follows from the above introduced procedure that the parameters of both controllers depend upon coefficients β as well as upon the closed-loop pole α. Consequently, tuning of the controllers can be performed by a suitable choice of selectable parameters β and α.

8. EXAMPLE

Consider a CSTR with the first order consecutive exothermic reaction according to the scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ and with a perfectly mixed cooling jacket. Using the usual simplifications, the model of the CSTR is described by four nonlinear differential equations

$$\frac{dc_A}{dt} = -\left(\frac{Q_r}{V_r} + k_1\right)c_A + \frac{Q_r}{V_r}c_{Af} \qquad (31)$$

$$\frac{dc_B}{dt} = -\left(\frac{Q_r}{V_r} + k_2\right)c_B + k_1 c_A + \frac{Q_r}{V_r}c_{Bf} \qquad (32)$$

$$\frac{dT_r}{dt} = \frac{h_r}{(\rho c_p)_r} + \frac{Q_r}{V_r}(T_{rf} - T_r) + \frac{A_h U}{V_r(\rho c_p)_r}(T_c - T_r) \qquad (33)$$

$$\frac{dT_c}{dt} = \frac{Q_c}{V_c}(T_{cf} - T_c) + \frac{A_h U}{V_c(\rho c_p)_c}(T_r - T_c) \qquad (34)$$

with initial conditions $c_A(0) = c_A^s$, $c_B(0) = c_B^s$, $T_r(0) = T_r^s$ and $T_c(0) = T_c^s$. Here, t is the time, c are concentrations, T are temperatures, V are volumes, ρ are densities, c_p are specific heat capacities, Q are volumetric flow rates, A_h is the heat exchange surface area and U is the heat transfer coefficient. The subscripts are denoted $(\cdot)_r$ for the reactant mixture, $(\cdot)_c$ for the coolant, $(\cdot)_f$ for feed (inlet) values and the superscript $(\cdot)^s$ for steady-state values. The reaction rates and the reaction heat are expressed as

$$k_j = k_{0j}\exp\left(\frac{-E_j}{RT_r}\right),\ j = 1,2 \qquad (35)$$

$$h_r = h_1 k_1 c_A + h_2 k_2 c_B \qquad (36)$$

where k_0 are pre-exponential factors, E are activation energies and h are reaction entalpies. The values of all parameters, feed values and steady-state values are given in Table 1.
For the control purposes, the controlled output and the control input are defined as

Table 1. Parameters, inlet values and initial conditions.

$V_r = 1.2$ m^3	$Q_r = 0.08$ m^{3}min^{-1}
$V_c = 0.64$ m^3	$Q_c^s = 0.03$ m^{3}min^{-1}
$\rho_r = 985$ kg m^{-3}	$c_{pr} = 4.05$ kJ kg^{-1}K^{-1}
$\rho_c = 998$ kg m^{-3}	$c_{pc} = 4.18$ kJ kg^{-1}K^{-1}
$A = 5.5$ m^2	$U = 43.5$ kJ m^{-2}min^{-1}K^{-1}
$k_{10} = 5.616 \cdot 10^{16}$ min^{-1}	$E_1/R = 13477$ K
$k_{20} = 1.128 \cdot 10^{18}$ min^{-1}	$E_2/R = 15290$ K
$h_1 = 4.8 \cdot 10^4$ kJ kmol^{-1}	$h_2 = 2.2 \cdot 10^4$ kJ kmol^{-1}
$c_{Af} = 2.85$ kmol m^{-3}	$c_{Bf} = 0$ kmol m^{-3}
$T_f = 323$ K	$T_{cf} = 293$ K
$c_A^s = 0.1649$ kmol m^{-3}	$c_b^s = 0.9435$ kmol m^{-3}
$T_r^s = 350.19$ K	$T_c^s = 330.55$ K

$$y(t) = T_r(t) - T_r^s,\ u(t) = 10\,\frac{Q_c(t) - Q_c^s}{Q_c^s}. \qquad (37)$$

These expressions enable to obtain variables of approximately the same magnitude.
The second order CT ELM has been chosen as

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0 y(t) = b_1\dot{u}(t) + b_0 u(t) \qquad (38)$$

or, in the transfer function representation

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (39)$$

The delta ELM corresponding to (38) has the form

$$\delta^2 y(t') + a_1'\delta y(t') + a_0' y(t') = b_1'\delta u(t') + b_0' u(t'). \qquad (40)$$

Now, the regression vector takes the form

$$\Phi_\delta^T(k-1) = \big(-\varphi_y(k-2) \quad -\varphi_y(k-1) \quad \varphi_u(k-2) \quad \varphi_u(k-1)\big). \qquad (41)$$

with elements calculated according to (9).
The vector of delta model parameters

$$\boldsymbol{\Theta}_\delta^T(k) = \left[a_0'\ a_1'\ b_0'\ b_1'\right] \qquad (42)$$

is recursively estimated from the equation (12) where

$$\varphi_y(k) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2}. \qquad (43)$$

The recursive estimation of delta model parameters was performed with the sampling interval T_0 = 0.2 min. Here, the recursive identification method with exponential and directional forgetting according to (Bobál *et al.*, 2005) was used.
Now, estimated parameters (42) approximate parameters of the CT model (38) and (39), respectively.
For the second order model (39) with $\deg a = 2$, the transfer functions of the controllers take forms

$$Q(s) = \frac{q_2 s + q_1}{s + p_0},\quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)} \qquad (44)$$

where

$$r_1 = \beta_1 t_1,\ r_2 = \beta_2 t_2,$$
$$q_1 = (1-\beta_1)t_1,\ q_2 = (1-\beta_2)t_2 \qquad (45)$$

and, parameters t and p are calculated from (20) where

$$d(s) = n(s)\,(s + \alpha)^2 \,. \qquad (46)$$

The polynomial n is in the form

$$n(s) = s^2 + n_1 s + n_0 \qquad (47)$$

with coefficients obtained by spectral factorization (30).

9. SIMULATIONS

For the start (the adaptation phase), a P controller with a small gain was used in all simulations.

The effect of the pole α on the control responses is transparent from Figs. 2 and 3. Here, three values of α were selected. The control results show sensitivity of the controlled output and control input to α. Obviously, careless selection of this parameter can lead to controlled output responses of a poor quality or even to unstability. Further, a decreasing α leads to higher values and changes of the control input.

The simulated control responses for different values β including their limiting values ($\beta_1 = \beta_2 = 0$, $\beta_1 = \beta_2 = 1$) are shown in Figs. 4 – 7. The effect of parameters β is evident. Their increase speeds up the control, but, it can lead to overshoots of the controlled output. Moreover, their greater values cause higher control inputs and their changes (derivatives). This fact is important for a practical control where greater input changes may be undesirable.

The presence of the integrating part in the controller R enables not only attenuation of a step disturbance loaded on u, but also rejection of other step disturbances entering into the process. Here, step disturbances in the component A inlet concentration $v(t) = \pm 0.142\ \mathrm{kmol\ m^{-3}}$ at times $t_v = 300\,\mathrm{min}$ and $t_v = 430\,\mathrm{min}$ are considered. The controller parameters were estimated only in the first (tracking) interval $t < 300$ min. During interval $t \geq 300$ min, fixed parameters were used.

The disturbances and controlled output responses are shown in Fig. 8.

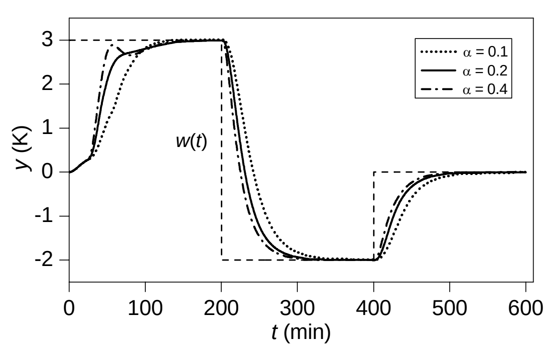

Fig. 2 Controlled output responses ($\beta_1 = 0.2$, $\beta_2 = 0$).

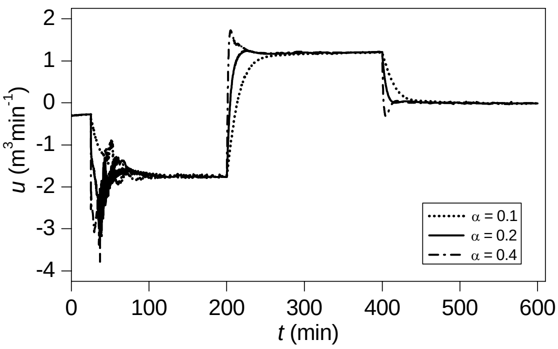

Fig. 3 Control input responses ($\beta_1 = 0.2$, $\beta_2 = 0$).

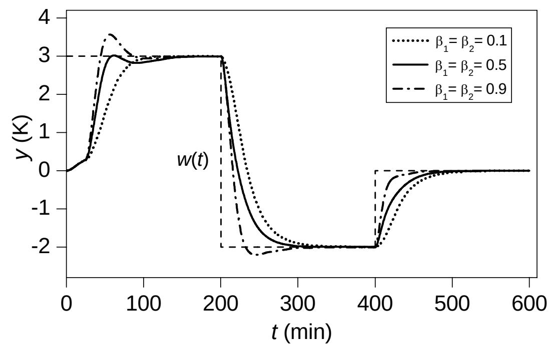

Fig. 4 Controlled output responses ($\alpha = 0.2$).

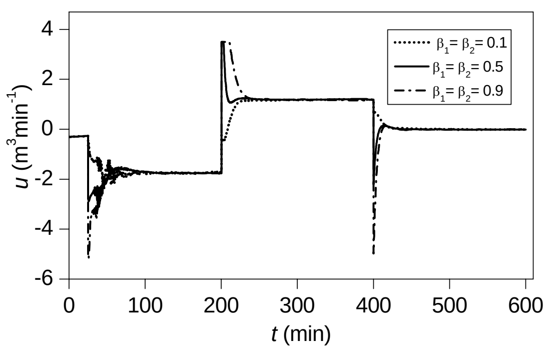

Fig. 5 Control input responses ($\alpha = 0.2$).

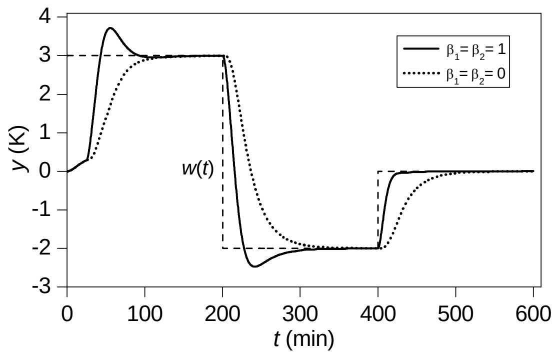

Fig. 6 Controlled output responses ($\alpha = 0.2$).

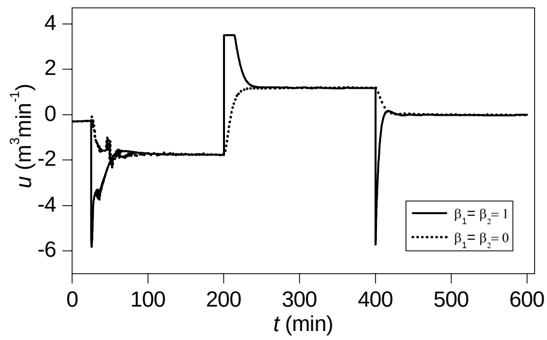

Fig. 7 Control input responses ($\alpha = 0.2$).

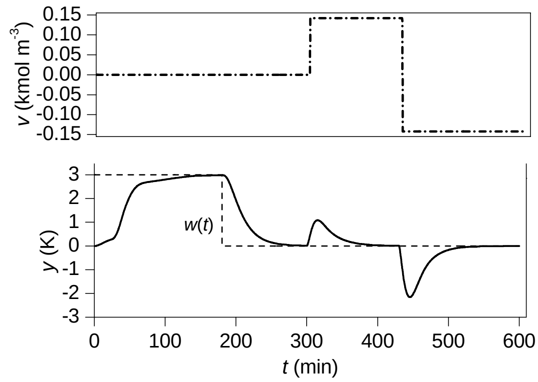

Fig. 8 Step disturbance attenuation ($\alpha = 0.2$, $\beta_1 = 0.2$, $\beta_2 = 0$).

10. CONCLUSIONS

In this paper, one approach to the continuous-time adaptive control of a continuous stirred tank reactor was proposed. The presented strategy uses two feedback controllers and enables to create an effective control algorithm. This algorithm is based on an alternative continuous-time external linear model with parameters obtained through recursive parameter estimation of a corresponding delta model. Both resulting continuous-time controllers are derived using the polynomial approach and given by a solution of a polynomial Diophantine equation. Tuning of their parameters is possible either via closed-loop pole assignment or by a choice of selectable coefficients splitting a weight between numerators of controllers' transfer functions. The presented method has been tested by computer simulation on the nonlinear model of the CSTR with a consecutive exothermic reaction. The results demonstrate the applicability of the presented control strategy. It can be deduced that the described adaptive strategy is also suitable for other similar technological processes.

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